

String Graphs – Part 4

Student

7 8 9 **10** 11 12



Aims

- Locate the points of intersection between consecutive lines.
- Determine a rule connecting consecutive points of intersection using Difference Tables.

Difference Tables

In this activity you will locate the successive points of intersection for the lines in Strings 1 using simultaneous equations and establish a general rule for these points of intersection using difference tables.



For linear functions a constant change in x brings about a constant change in y . This is **only** true for linear relationships. For a special group of relationships called “polynomials” difference tables can be used to calculate successive differences which eventually produce a constant change and hence a rule for the relationship.

Suppose you want to identify the rule for this pattern:

X	1	2	3	4	5	6	7	8	9
Y	6	11	18	27	38	51	66	83	102

The rule and pattern are not obvious. It is clear that while the x values are going up by the same amount (1 unit), the Y values are not changing by the same amount. The first difference is $11 - 6 = 5$; the second difference is $18 - 11 = 7$; the third difference is $27 - 18 = 9$. You may start to notice a pattern in the ‘differences’.

Question: 1.

The top two rows table below show the original relationship between X and Y . The third row in the table is the ‘first order differences’, the Δ means difference. The difference between consecutive Y values, as described above has been entered for the first three differences. The fourth row shows the ‘second order differences’, the *differences of the differences*. Finish off the difference table.

X	1	2	3	4	5	6	7	8	9
Y	6	11	18	27	38	51	66	83	102
	Δ_1	5	7	9					
		Δ_2	2	2					
			Δ_3	0					

Question: 2.

Which row is the first to display constant differences?

Polynomials

Polynomials are of the form: $a_n x^n + \dots + a_3 x^3 + a_2 x^2 + a_1 x + a_0$. The **degree** of the polynomial is the equal to the highest power of x .

Examples:

This polynomial $5x^3 - 9x^2 + 4x + 7$ is degree 3 because there is an x^3 term in the polynomial, the highest power. Polynomials of degree 3 are also referred to as 'cubic' functions.

This polynomial $7x^2 + 2x - 9$ is degree 2 because there is an x^2 term in the polynomial, the highest power. Polynomials of degree 2 are also referred to as 'quadratic' functions.

This polynomial $3x + 11$ is degree 1 because there is an x term in the polynomial, the highest power is one. These polynomials are generally called 'linear' functions.

Difference tables can tell us a lot about a polynomial. In the simple case of a degree 1 polynomial (straight line graph), we notice that for a constant change in x the polynomial changes by a constant amount, that is to say that Δ_1 is constant. In the case of a degree 2 polynomial a constant difference occurs in the 'differences of the differences', Δ_2 is constant. In general, the first row where a constant difference appears tells us the degree of the polynomial.

Question: 3.

What is the degree of the polynomial according to the difference table in Question 1?

Question: 4.

The leading edge of difference table is like a DNA print of the Polynomial. The leading edge of the difference table in Question 1 is emphasised by light grey shading and red digits. Write down the leading edge of this difference table. (Relationship)

Genetic Coding of Basic Polynomials

The zero polynomial (degree 0) is trivial from a calculation perspective, but is a necessary consideration for subsequent calculations.

X	1	2	3	4	5	6	7	8	9
Y	1	1	1	1	1	1	1	1	1
	Δ_1	0	0	0	0	0	0	0	0
		Δ_2	0	0	0	0	0	0	0

We can see that all subsequent differences will also be zero. The leading edge of the zero polynomial is therefore: 1, 0, 0, 0 ...

Question: 5.

Complete the difference table for the simple relationship: $y = x$.

X	1	2	3	4	5	6	7	8	9
Y	1	2	3	4	5	6	7	8	9
	Δ_1	1	1	1					
		Δ_2	0						

Question: 6.

Complete the difference table for the simple relationship: $y = x^2$.

X	1	2	3	4	5	6	7	8	9
Y	1	4	9	16	25	36	49	64	81
	Δ_1	3	5	7					
		Δ_2	2						
			Δ_3	0					

Question: 7.

Complete the difference table for the simple relationship: $y = x^3$.

X	1	2	3	4	5	6	7	8	9
Y	1	8	27	64	125	216	349	512	729
	Δ_1	7	19	37					
		Δ_2	12	18					
			Δ_3	6					

The calculator instructions that follow will help you connect the information about Polynomials and Difference tables to help establish a rule for the original relationship (Question 1). Once you have established the relationship, we will head back to our string graphs and establish some rules there.

Determining Equations

Start a new document and insert a **Notes Application**.

The Notes application will contain a mixture of text and calculations. The beauty of the Notes Application is that calculations can be automatically updated!

Mathematics in the Notes Application is contained within Maths Boxes. You can use the menu to insert a Maths Box or simply press **Ctrl + M**.

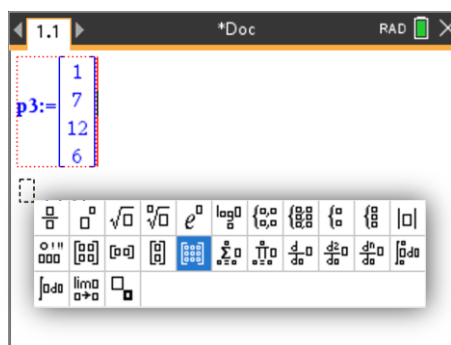
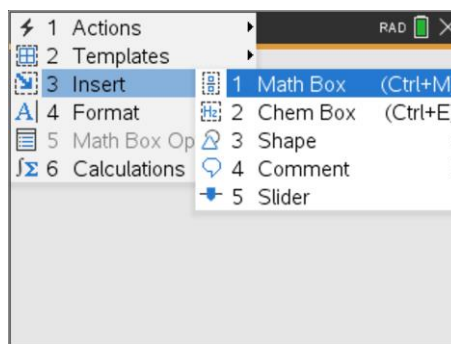
The **leading edge** from each of the difference tables in Questions 5 to 7 produced some of the fundamental building blocks that can be used to determine the equation to the polynomial.

In the maths box enter the following leading edge 'matrix'.

Note that it matches the leading edge for $y = x^3$.

The matrix tool is located in the Maths Templates key. Our matrix consists of 4 rows and 1 column.

We are using P3 (or P_3) to represent that this matrix is the leading edge of the simplest polynomial of degree 3.



Now enter the leading edge matrix for the $y = x^2$ relationship.

Two more leading edge matrices need to be added.

$$p_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } p_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The leading edge of our original relationship looks like this:

$$\begin{bmatrix} 6 \\ 5 \\ 2 \\ 0 \end{bmatrix}$$

Our aim is to use multiples of our 'DNA' building blocks to create this leading edge matrix. Our difference table tells us that the relationship is a degree 2 polynomial, so we can take an almost random guess for the coefficients.

In a new maths box type the following: $2P_2 + 5P_1 + 1P_0$

This corresponds to the rule: $2x^2 + 5x + 1$

Notice that the result looks nothing like the one we want, but we can use our guess to guide us towards the solution.

Change the coefficient (number in front) of P_2 to a 1 then press Enter to see the result.

Notice that the number in row 3 now matches the leading edge we are trying to create.

Experiment with the coefficient of P_1 until you get the number in the second row to match the leading edge we are trying to create. (5)

Once you have the second row matching, try varying the coefficient of P_0 until the first row also matches.

Question: 8.

Use your results from the exploration on the **Notes Application** to write a rule for the first relationship:

X	1	2	3	4	5	6	7	8	9
Y	6	11	18	27	38	51	66	83	102

Substitute values in for x to show that your rule works.

Question: 9.

Here is another, much more complicated relationship.

X	1	2	3	4	5	6	7	8	9
Y	8	18	44	92	168	278	428	624	872
	Δ_1	10	26	48	76				
		Δ_2	16	22	28				
			Δ_3	6	6				

- Complete the difference table.
- Record the leading edge.
- Change the rule in your the Notes Application to: $P_3 + 4P_2 - 5P_1 + 2P_0$. The resulting leading edge matrix will NOT match your above results. Change the coefficients, starting with the coefficient of P_3 , working your way up, until you match the leading edge matrix for this question.
- Write down the rule and show that it produces the relationship between x and y.

Returning to the Strings

In the first activity, the point (0, 10) was connected to (1, 0). The point (0, 9) was connected to (2, 0), a family of points were connected by a family of straight lines. Activity 2 connected a different family of points and activity 3 found the points of intersection of consecutive straight lines. It is now time to find the consecutive points of intersection for the lines from activity 1.

Question: 10.

Find the point of intersection between the first pair of lines in activity 1: $y = -10x + 10$ and

$$y = -\frac{9}{2}x + 9.$$

Question: 11.

Find the point of intersection between the second pair of lines in activity 1: $y = -\frac{9}{2}x + 9$ and

$$y = -\frac{8}{3}x + 8.$$

Question: 12.

Find the points of intersection for the remaining pairs of straight lines in activity 1 and tabulate the results.

Question: 13.

Populate the difference table using the equation pair (t) and the abscissa for the point of intersection.

t	1	2	3	4	5	6	7	8	9
x	$\frac{2}{11}$								
	Δ_1								
		Δ_2							
			Δ_3						

Question: 14.

Use the results from the difference table and the leading edge matrices in the Notes application to find a relationship between the equation pair and the abscissa.

Question: 15.

Populate the difference table using the equation pair (t) and the ordinate for the point of intersection.

t	1	2	3	4	5	6	7	8	9
y	$\frac{90}{11}$								
	Δ_1								
		Δ_2							
			Δ_3						

Question: 16.

Use the results from the difference table and the leading edge matrices in the Notes application to find a relationship between the equation pair and the ordinate.

Question: 17.

Activity 3 introduced “Parametric” graphing. Your answers to Questions 14 and 16 represent equations for x and y in terms of a parameter: ‘t’. Graph the parametric relationship along with the family of straight lines from Activity 1, in quadrant 1. [Paste a copy of your graph as your answer]

Note: For the parametric graph use $t: 0 \leq t \leq 10$ and a t step value of 0.1

Question: 18.

For your graph in Question 17, change the range of values for t to $-10 \leq t \leq 20$ and ‘zoom out’ to see more of the graph. Compare this graph to the one obtained in Activity 3.