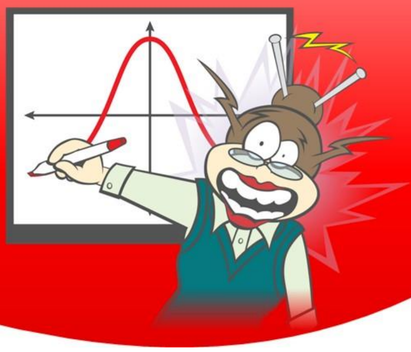
STUDENT REVISION SERIES



Vector Functions

Question: 1.

The position of two particles are described by the position vectors:
$\boldsymbol{r}(t) = (4\,t - 3)\,\boldsymbol{i} + (2\,t\, - 1)\,\boldsymbol{j}\,, t \geq 0$ and
$\boldsymbol{s}(t) = (2 - t)\,\boldsymbol{i} + (t - 2)\,\boldsymbol{j}\,, t \geq 0$

a. Find the distance of the particle r from the origin when $t = 2$

b. Find an expression for the distance between the two particles at time t

c. Hence, find the minimum distance between the two particles and the value of t for which this occurs.

Question: 2.

The path of an object is defined parametrically by $\boldsymbol{r}(t) = (5 - 4\cos(t))\,\boldsymbol{i} + (3\sin(t) - 4)\,\boldsymbol{j}$

Show that the cartesian equation of the path is $\frac{(x-5)^2}{16} + \frac{(y+4)^2}{9} = 1$

Question: 3.

Two ships A and B are observed from a lighthouse at origin O. Relative to O, their positions at a time t hours are given by
$\boldsymbol{r}_A(t) = (t + 2)\,\boldsymbol{i} + (t^2 - 5t + 6)\,\boldsymbol{j}$
$\boldsymbol{r}_B(t) = (2t - 4)\,\boldsymbol{i} + (t^2 - 4t)\,\boldsymbol{j}$

a. Sketch the paths of each ship for t≥0. Show the direction of motion.

b. Find the cartesian equation for the path of ship A.

c. Find where the ships paths cross.

d. Find when and where the ships collide.

Question: 4.

The cartesian equation of the path of the object described by $\boldsymbol{r}(t) = 2\sec^2(t)\ \boldsymbol{i} + 4\tan^2(t)\ \boldsymbol{j}$ is:

a. $y = 2x - 4$
b. $4x^2 - y^2 = 16$
c. $\frac{x^4}{4} + \frac{y^2}{16} = 1$
d. $4x^2 + y^2 = 16$
e. $\frac{x^4}{4} - \frac{y^2}{16} = 1$

Question: 5.

The cartesian equation of the path of the object described by $\boldsymbol{r}(t) = \left(\sqrt{t - 3}\right)\boldsymbol{i} + 4t^2\,\boldsymbol{j}$ for $t \geq 3$ is:

a. $y = 4x^2$
b. $y = 4\sqrt{x - 3}$
c. $y = 4(x^2 + 3)$
d. $y = 4(x^2 + 3)^2$
e. $y = 16(x^2 + 3)^2$

Author: J Mott

Question: 6.

Two people, Adam and Barry, are jogging on an oval. Their position vectors, relative to an origin O, at time t minutes after midday are given by

$\boldsymbol{r_A} = (1 - 2t)\,\boldsymbol{i} + (2t^2 - t - 1)\,\boldsymbol{j}$

$\boldsymbol{r_B} = (2t - 3)\,\boldsymbol{i} + (2 - 2t)\,\boldsymbol{j}$

Where displacements are measured in metres.

a. State the initial position of Adam. Give your answer in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.

b. Sketch and label the path of both Adam and Barry on the axes below.
Show the direction of motion of each person with an arrow.

c. Find where Adam and Barry cross paths. Give your answer as coordinates.

d. Find when and where Adam and Barry collide.

Question: 7.

The position of two trams, A and B at time t, can be described by the position vectors

$\boldsymbol{r_A} = 3(2 - t)\,\boldsymbol{i} + 2(t + 2)\,\boldsymbol{j}$

$\boldsymbol{r_B} = (t - 1)\,\boldsymbol{i} + (3t - 1)\,\boldsymbol{j}$

Where displacements are measured in kilometres and time is measured in hours.

a. Show that the two trams do not collide.

b. Find where the paths of the two trams cross. Give your answer as coordinates

c. Find the minimum distance between the two trams, and the value of t for which this occurs. Give your answers correct to two decimal places.

Question: 8.

Let i and j be unit vectors in the east and north directions respectively. At time t, the position vector of particle G is given by $\boldsymbol{r_G} = (4t - 6)\,\boldsymbol{i} + (t^2 - 4t + 3)\,\boldsymbol{j}$, and the position vector of particle H is given by $\boldsymbol{r}_H = (t^2 - 3t)\,\boldsymbol{i} + (2 - 2t)\,\boldsymbol{j}$. Particle G is directly north of particle H for t equal to:

a. 1
b. 2
c. 4
d. 6
e. 8

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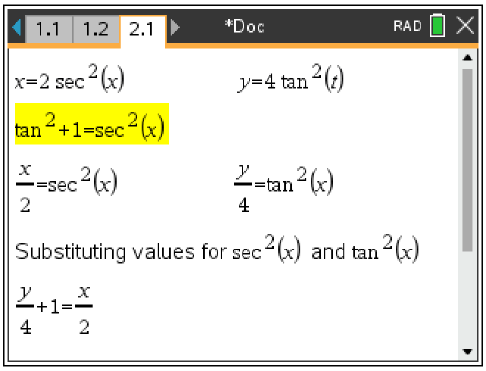
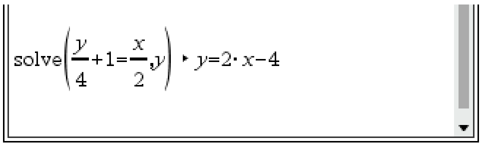
Answers

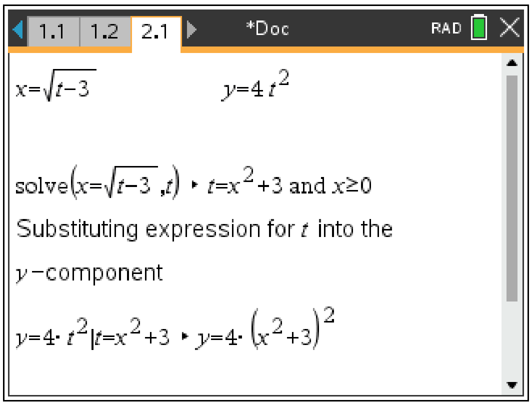
Question 1 to 3 Answered on YouTube.

Question 4 Option A





Question 5 Option D



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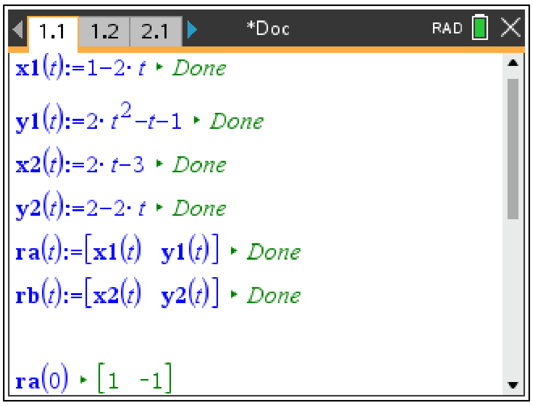
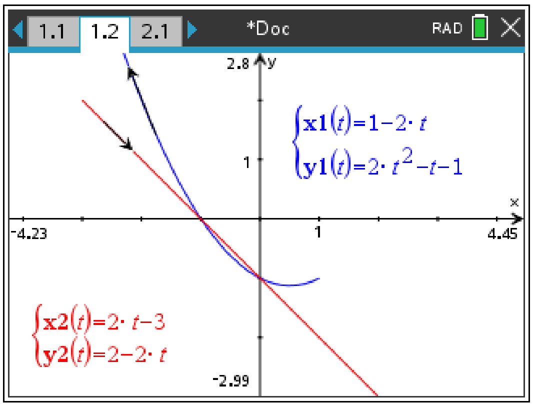
Question 6

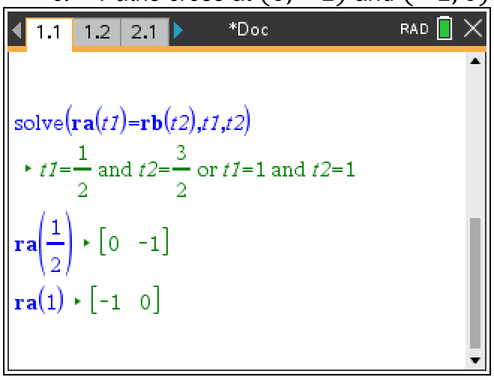
a. Initial position of Adam is $\boldsymbol{i} - \boldsymbol{j}$



b.



c. Paths cross at $(0, -1)$ and $(-1, 0)$



To determine where the paths cross, we need Adam and Barry to be at the same location, but they can be there at different times (e.g. $t1$ and $t2$)

We can find the positions of where the paths cross by evaluating $r_A\left(\frac{1}{2}\right)$ and $r_A(1)$.

Equivalently we can evaluate $r_B\left(\frac{3}{2}\right)$ and $r_B(1)$ (try this yourself to check if it works).

d. The particles will collide at $t = 1$ and at $(-1,0)$

Adam and Barry will collide when they are at the same location at the same time. From the previous result we see that they are at the same location for $t = 1$ (i.e. $t1 = t2 = 1$)

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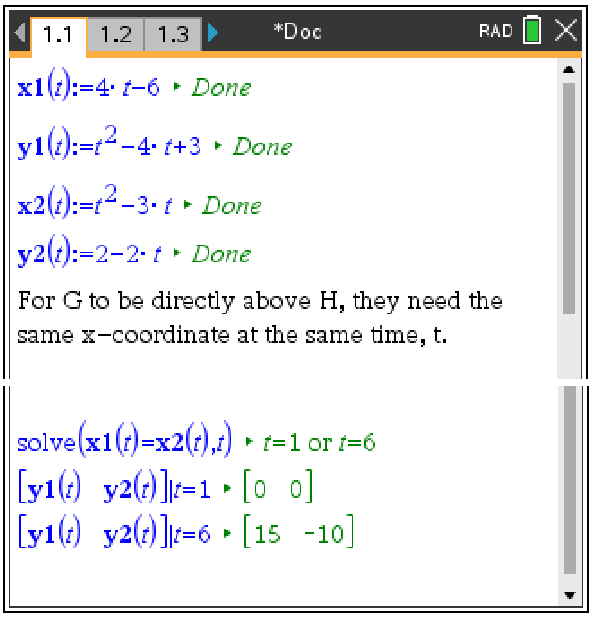
Question 7

a. $x1(t):=3\cdot(2-t)$ ▸ *Done* $y1(t):=2\cdot(t+2)$ ▸ *Done* $x2(t):=t-1$ ▸ *Done* $y2(t):=3\cdot t-1$ ▸ *Done* $ra(t):=[x1(t)\ \ y1(t)]$ ▸ *Done* $rb(t):=[x2(t)\ \ y2(t)]$ ▸ *Done* $\text{solve}(ra(t1)=rb(t2),t1,t2)$ ▸ $t1=\frac{16}{11}$ and $t2=\frac{29}{11}$	To find where the trams, we need them to be at the same location at the same time. To determine this, we can solve $r_A(t1)=r_B(t2)$ If $t1=t2$, then the trams will be at the same location at the same time. If $t1\neq t2$, then They will pass through the same location, but at different times. From the output we can see that $t1\neq t2$, hence the trams do not collide (thankfully).
b. Paths will cross at $\left(\frac{18}{11},\frac{76}{11}\right)$ $\text{solve}(ra(t1)=rb(t2),t1,t2)$ ▸ $t1=\frac{16}{11}$ and $t2=\frac{29}{11}$ $ra\left(\frac{16}{11}\right)$ ▸ $\left[\frac{18}{11}\ \ \frac{76}{11}\right]$ $rb\left(\frac{16}{11}\right)$ ▸ $\left[\frac{5}{11}\ \ \frac{37}{11}\right]$	
c. Min distance is 3.15, and occurs for $t=1.94$ $\text{dist}(t):=\text{norm}(ra(t)-rb(t))$ ▸ *Done* $\text{solve}\left(\begin{cases}\frac{d}{dt}(\text{dist}(t))=0\\ d=\text{dist}(t)\end{cases},\{t,d\}\right)$ ▸ $t=1.94118$ and $d=3.15296$	We first need to find an expression for the distance between the two position vectors. This is given by $\lvert r_A-r_B\rvert$ Note: $\lvert r_A-r_B\rvert=\lvert r_B-r_A\rvert$ (i.e.the distance from A to B is the same from B to A). Also, on CAS, the required syntax is the *norm* command to give the length of a vector. To obtain the minimum, we can solve the derivative of the distance expression equal to zero.

Question 8 Answer D



At $t = 1$, the particles at the same position.

At $t = 6$, $G_y = 15$ and $H_y = -10$, which indicates that G is directly above H at $t = 6$