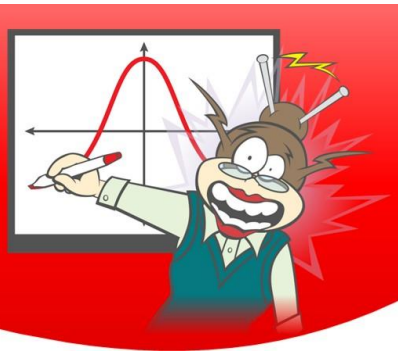


Vector Calculus

Poll Question

The position vector of a particle at time t seconds, $t \geq 0$, is given by $\underset{\sim}{r}(t) = (3-t)\underset{\sim}{i} - 6\sqrt{t}\,\underset{\sim}{j} + 5\underset{\sim}{k}$.
The direction of motion of the particle when $t = 9$ is:

A. $-6\underset{\sim}{i} - 18\underset{\sim}{j} + 5\underset{\sim}{k}$
B. $-\underset{\sim}{i} - \underset{\sim}{j}$
C. $-\underset{\sim}{i} - 6\underset{\sim}{j}$
D. $-\underset{\sim}{i} - \underset{\sim}{j} + 5\underset{\sim}{k}$
E. $-13.5\underset{\sim}{i} - 108\underset{\sim}{j} + 45\underset{\sim}{k}$

Question: 1.

The position vector of an object moving in a plane is given by:

$\underset{\sim}{r}(t) = t^3\underset{\sim}{i} + t^2\underset{\sim}{j}$

Find its velocity, speed, and acceleration when t = 1 and illustrate geometrically.

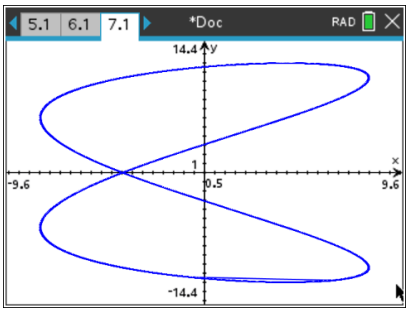
Question: 2.

The position of an object, $\underset{\sim}{r}$ metres, is given by $\underset{\sim}{r}(t) = 3\sin 2t\,\underset{\sim}{i} + 3\cos 2t\,\underset{\sim}{j}$.

(a) Find the speed at any time, t seconds.

(b) Show that the velocity vector is always perpendicular to the acceleration vector.

Question: 3.



The motion of a figure skater relative to a fixed origin O, at time t minutes is modelled using the vector equation:

$$\underset{\sim}{r}(t) = 8\cos(20t)\,\underset{\sim}{i} + 12\sin\left(10t - \frac{\pi}{3}\right)\underset{\sim}{j},\ t \geq 0.$$

a) Find the velocity vector and the acceleration vector of the figure skater.

b) Find the speed of the figure skater after $\frac{\pi}{10}$ minutes.

c) Find the times in one full cycle (when she completes 'an eight') at which velocity is perpendicular to acceleration.

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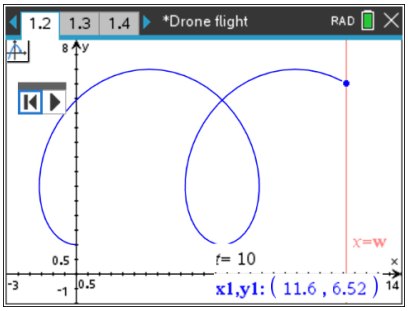
Question: 4.

The position vector of a projectile is:
$\underset{\sim}{r}(t) = 15t\,\underset{\sim}{i} + (29.4t - 4.9t^2)\,\underset{\sim}{j}$ metres. The maximum height of the projectile is equal to:

A 45 m
B 29.4 m
C 36.2 m
D 40 m
E 44.1 m

Question: 5.



In a trial first flight, an experimental drone follows the trajectory
$\underset{\sim}{r}(t) = (t - 3\sin t)\,\underset{\sim}{i} + (4 - 3\cos t)\,\underset{\sim}{j},\ t \geq 0$
but crashes into a wall at time $t = 10$.
Assume all distances are in metres and time is in seconds. Give all answers to two decimal places.

a) At what times was the drone flying vertically?
b) At what times was the drone flying horizontally?
c) Calculate the total distance travelled by the drone until it crashes into the wall.
d) Find the speed of the drone at $t = 10$.
e) Find the angle at which the drone hit the wall.
f) Find the maximum and minimum speeds of the drone during its flight.
g) At what times the velocity was perpendicular to acceleration?
h) Find the magnitude of the acceleration.

Question: 6.

The angle between the direction of two objects with respective position vectors
$\underset{\sim}{r}_1 = \sin \pi t\,\underset{\sim}{i} + 4t\,\underset{\sim}{j}$ and $\underset{\sim}{r}_2 = t^2\,\underset{\sim}{i} - 3\,\underset{\sim}{j}$, when $t = 1$, is nearest to:

A 180°
B 0°
C 162°
D 60°
E 128°

Question: 7.

At time t a particle has position vector $\underset{\sim}{r} = (3\sin t + \sin 2t)\,\underset{\sim}{i} + (3\cos t - \cos 2t)\,\underset{\sim}{j} + t\underset{\sim}{k},\ t \geq 0.$
Find the maximum and minimum speeds of the particle.

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Question: 8.

The displacement of a particle from the origin at time t, $t \geq 0$, is given by $\underset{\sim}{r}(t) = e^{-2t}\underset{\sim}{i} + \sin(\pi t)\,\underset{\sim}{j} + 2\underset{\sim}{k}$.

The initial direction of motion of the particle is:

A. $4\underset{\sim}{i}$ B. $\underset{\sim}{i} + 2\underset{\sim}{k}$ C. $-2\underset{\sim}{i} + \underset{\sim}{j}$ D. $-2\underset{\sim}{i} + \pi\underset{\sim}{j}$ *E.* $-2\underset{\sim}{i} + \pi\underset{\sim}{j} + 2\underset{\sim}{k}$

Question: 9.

An object is thrown in the air and its position is described by the following:

$$\underset{\sim}{r}(t) = 10.5t\,\underset{\sim}{i} + \left(\frac{\pi}{2}t - 4\sin\left(\frac{\pi t}{8}\right)\right)\underset{\sim}{j} + \left(2 + 19.5t - 5t^2\right)\underset{\sim}{k}$$

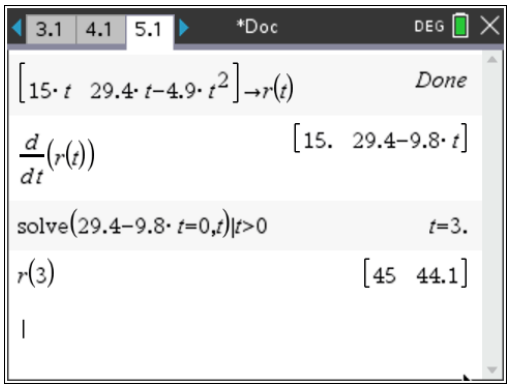
where $\underset{\sim}{i}$ is a unit vector in the east direction, $\underset{\sim}{j}$ is a unit vector in the north direction and $\underset{\sim}{k}$ is a unit vector vertically up. The origin *O* of the coordinate system is at ground level and displacement are measured in metres.

a) Find the velocity of the object at time $t = 4$.

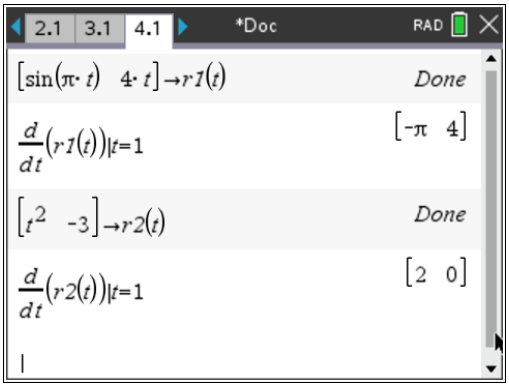
b) Find the angle between the path of the object and the ground after 4 seconds. Give your answer to the nearest degree.

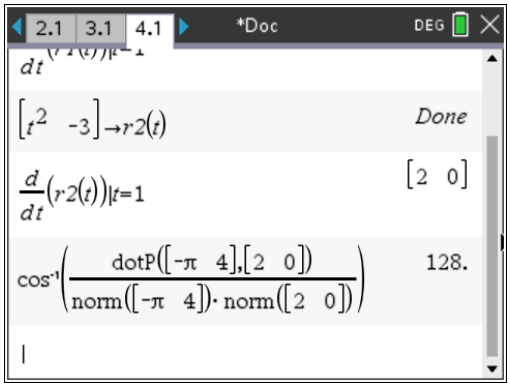
Answers

Question 4 **Option E**



Question 6 **Option E**





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Question 7

Enter the position vector on your CAS calculator and find the velocity vector.

$$\left[3\cdot\sin(t)+\sin(2\cdot t)\quad 3\cdot\cos(t)-\cos(2\cdot t)\quad t\right]\rightarrow r(t)$$

Done

$$\frac{d}{dt}\left(r(t)\right)$$

$$\left[2\cdot\cos(2\cdot t)+3\cdot\cos(t)\quad 2\cdot\sin(2\cdot t)-3\cdot\sin(t)\quad 1\right]$$

$$\left[2\cdot\cos(2\cdot t)+3\cdot\cos(t)\quad 2\cdot\sin(2\cdot t)-3\cdot\sin(t)\quad 1\right]\rightarrow v(t)$$

Done

Find the speed of the particle. Use tCollect to simplify the trigonometric expression inside the square root.

$$\text{norm}\left(v(t)\right)$$

$$\sqrt{2\cdot\left(6\cdot\cos(t)\cdot\cos(2\cdot t)-6\cdot\sin(t)\cdot\sin(2\cdot t)+7\right)}$$

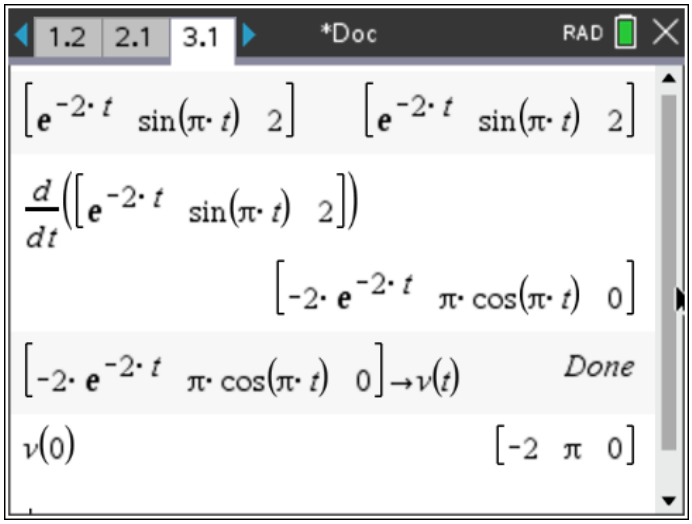
$$\text{tCollect}\left(\sqrt{2\cdot\left(6\cdot\cos(t)\cdot\cos(2\cdot t)-6\cdot\sin(t)\cdot\sin(2\cdot t)+7\right)}\right)$$

$$\sqrt{2\cdot\left(6\cdot\cos(3\cdot t)+7\right)}$$

$$\text{expand}\left(2\cdot\left(6\cdot\cos(3\cdot t)+7\right)\right)\qquad 12\cdot\cos(3\cdot t)+14$$

Answer: maximum speed $=\sqrt{26}$ and minimum speed $=\sqrt{2}$.

Question 8 Option D



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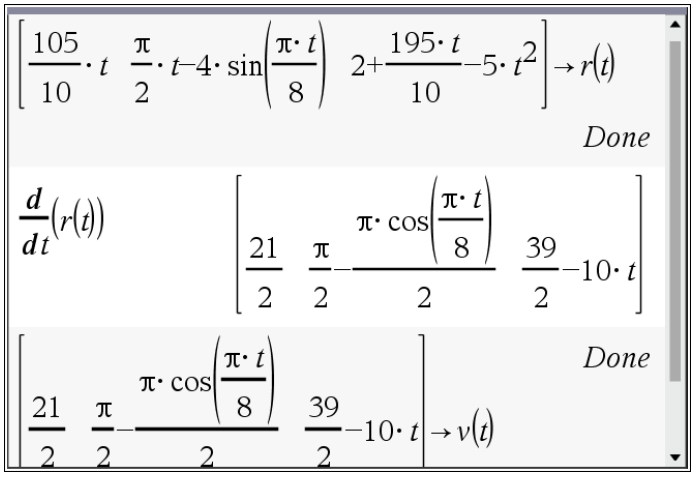
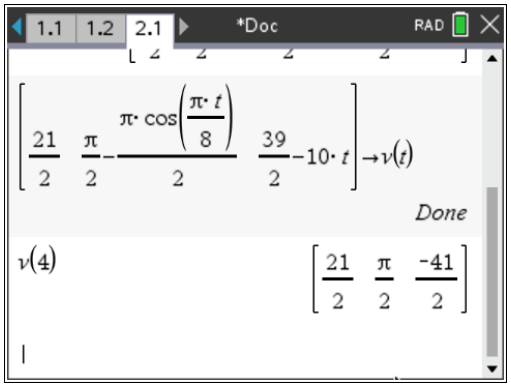
Question 9

a) Enter the position vector on you calculator and find the velocity vector.



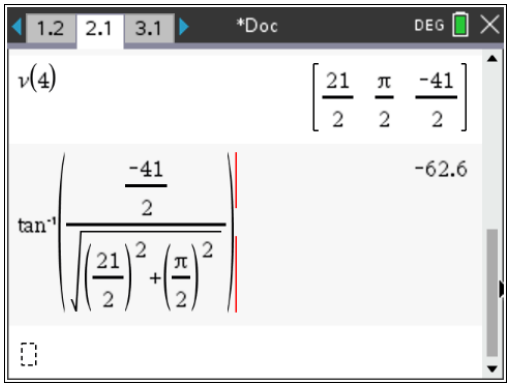
Note that entering fractions instead of decimals will give you exact values for the velocity vector.

Now, find velocity at time $t = 4$.



Answer: $\underset{\sim}{v}(4) = \frac{21}{2}\underset{\sim}{i} + \frac{\pi}{2}\underset{\sim}{j} - \frac{41}{2}\underset{\sim}{k}$

b) We need to find the angle between velocity vector and the horizontal. One way of doing so is to find the angle between the $\underset{\sim}{k}$ component and horizontal speed.



Answer: **63 degrees.**

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