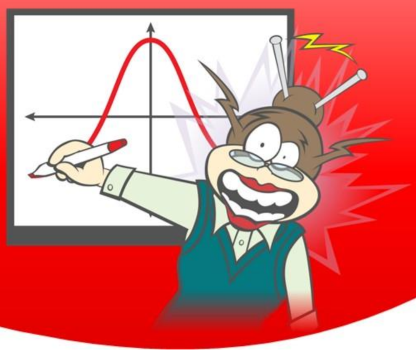
STUDENT REVISION SERIES



Vectors Part 1

Each of the questions included here can be solved using the TI-Nspire CX CAS.

Scan the QR code or use the link: https://bit.ly/Vectors-Part1

Question: 1.

The position vectors of points A and B respectively are $\underset{\sim}{a} = 3\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k}$ and $\underset{\sim}{b} = 5\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$. Find the perimeter P of $\triangle OAB$ in the form $\sqrt{a}\left(b+\sqrt{c}\right)$.

Question: 2.

Consider the vectors $\overrightarrow{OE} = 2\underset{\sim}{i} + 5\underset{\sim}{j} - 7\underset{\sim}{k}$ and $\overrightarrow{OF} = 10\underset{\sim}{i} + 14\underset{\sim}{j} + 4\underset{\sim}{k}$. The point G has a position vector such that $\overrightarrow{OG}$ bisects $\angle EOF$ and $\left|\overrightarrow{OG}\right| = \sqrt{218}$. Find the coordinates of G.

Question: 3.

Relative to an origin O, the points $S(1,2,-3)$, $T(5,3,-2)$, $U(6,7,-3)$ and $V(2,6,-4)$ form a quadrilateral $STUV$, with diagonal SU. Show that $\cos(\angle TSU) = \cos(\angle USV)$ and hence verify, using $\angle TSV$, that $\cos(2\theta) = 2\cos^2(\theta) - 1$.

Author: S Crouch

Question: 4.

Points A and B have position vectors $\overrightarrow{OA} = 2\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\overrightarrow{OB} = 2m\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ respectively, relative to O, where $m \in \mathbb{R}$. Given that $\cos(\angle AOB) = -\frac{2}{\sqrt{13}}$, find the area of $\triangle OAB$.

Question: 5.

Find a unit vector $\underset{\sim}{u} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$, $x > 0$, that is perpendicular to both $\underset{\sim}{v} = 3\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{w} = \underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$.

Question: 6.

Points $A(4,2,-1)$, $B(-1,5,2)$ and $C(3,-3,c)$ form a right-angled triangle, with the right angle at B. Find the value(s) of c.

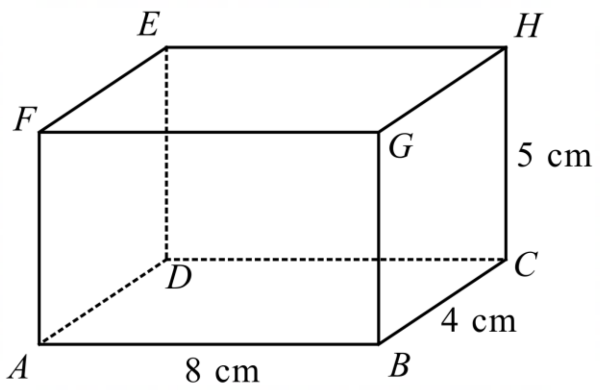
Author: S Crouch

Question: 7.

$\triangle KLM$ has two of its sides formed by the vectors $\overrightarrow{KL} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 5\underset{\sim}{k}$ and $\overrightarrow{KM} = 7\underset{\sim}{i} - 4\underset{\sim}{j} + 9\underset{\sim}{k}$. Find the length of the median from K.

Question: 8.

Consider the cuboid $ABCDEFGH$ below.

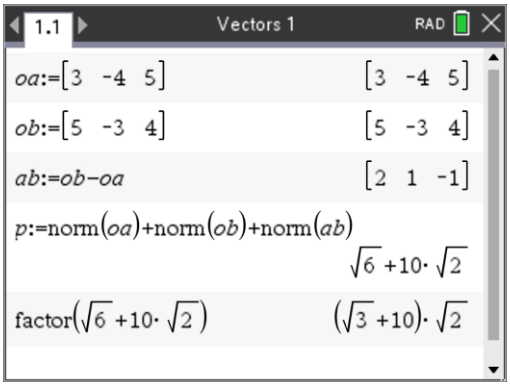


Use vector methods to find $\angle GAC$, correct to the nearest minute.

Author: S Crouch

Answers

Question 1 $P=\sqrt{2}\left(10+\sqrt{3}\right)$



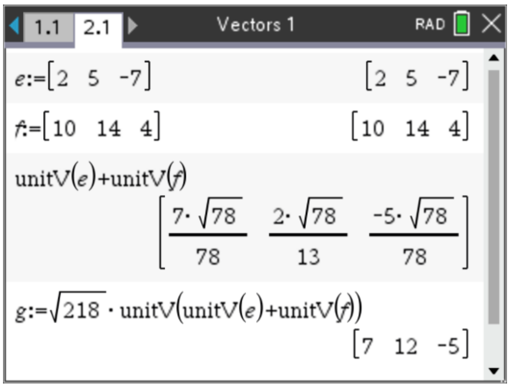
Define the vectors. ctrl [template key] can be used to get :=

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$

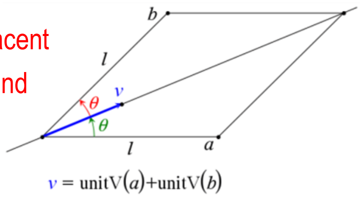
Magnitudes of vectors can be found using the norm command (menu 7 7 1).

The factor command is useful for numerical expressions too, not only algebraic.

Question 2 $G=\left(7,12,-5\right)$



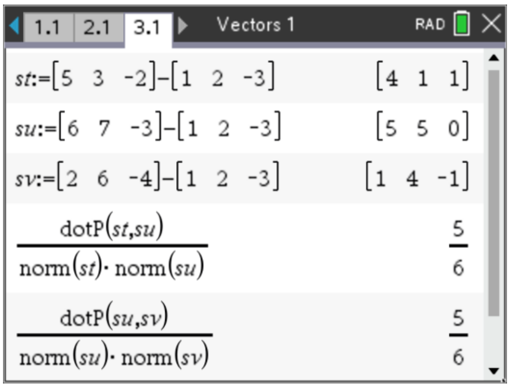
The diagonal of a rhombus bisects the adjacent sides. Therefore, the unit vectors of $\overrightarrow{OE}$ and $\overrightarrow{OF}$ construct adjacent sides of a unit rhombus, and their sum is the diagonal.

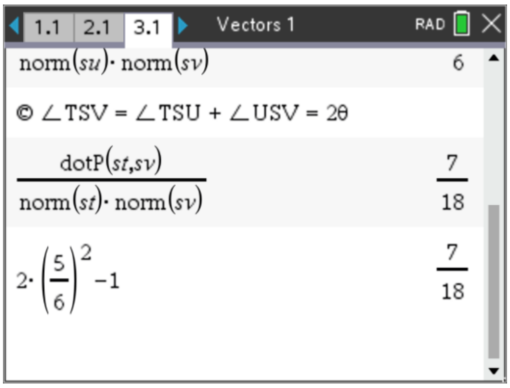


To find $\overrightarrow{OG}$, find the unit vector of the diagonal and multiply this by $\sqrt{218}$.

Since $\overrightarrow{OG}=7\underset{\sim}{i}+12\underset{\sim}{j}-5\underset{\sim}{k}$, G has coordinates $\left(7,12,-5\right)$.

Question 3 $\cos\left(\angle TSU\right)=\cos\left(\angle USV\right)=\frac{5}{6};\ \cos\left(\angle TSV\right)=\frac{7}{18}$





$\overrightarrow{ST}=\overrightarrow{OT}-\overrightarrow{OS}$, $\overrightarrow{SU}=\overrightarrow{OU}-\overrightarrow{OS}$, $\overrightarrow{SV}=\overrightarrow{OV}-\overrightarrow{OS}$

$\angle TSU$ is the angle between the tails of $\overrightarrow{ST}$ and $\overrightarrow{SU}$.

$\angle USV$ is the angle between the tails of $\overrightarrow{SU}$ and $\overrightarrow{SV}$.

Hence, $\cos\left(\angle TSU\right)=\frac{\overrightarrow{ST}\cdot\overrightarrow{SU}}{\left|\overrightarrow{ST}\right|\left|\overrightarrow{SU}\right|}$ and $\cos\left(\angle USV\right)=\frac{\overrightarrow{SU}\cdot\overrightarrow{SV}}{\left|\overrightarrow{SU}\right|\left|\overrightarrow{SV}\right|}$

Using dotP (menu 7 C 3): $\cos\left(\angle TSU\right)=\cos\left(\angle USV\right)=\frac{5}{6}$.

Since $\angle TSU=\angle USV=\theta$ and $\angle TSV=\angle TSU+\angle USV$, it can be seen that $\angle TSV=2\theta$. Finding $\cos\left(\angle TSV\right)$:

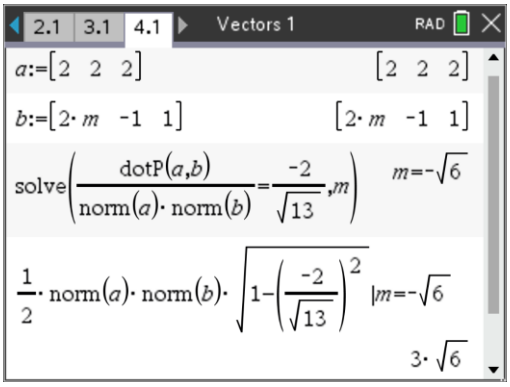
$$\cos\left(\angle TSV\right)=\frac{\overrightarrow{ST}\cdot\overrightarrow{SV}}{\left|\overrightarrow{ST}\right|\left|\overrightarrow{SV}\right|}=\frac{7}{18}$$

Verifying $\cos\left(2\theta\right)=2\cos^2\left(\theta\right)-1$:

$$\frac{7}{18}=2\left(\frac{5}{6}\right)^2-1\Rightarrow\frac{7}{18}=\frac{7}{18}$$ 👍

Author: S Crouch

Question 4 $A_{\triangle OAB} = 3\sqrt{6}$

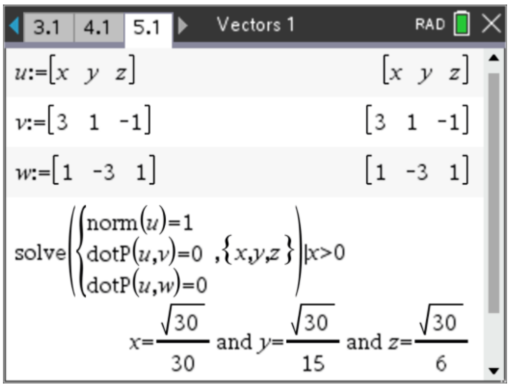


$\cos(\angle AOB) = \dfrac{\overrightarrow{OA}\cdot\overrightarrow{OB}}{\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|} = -\dfrac{2}{\sqrt{13}}$ (so $\angle AOB$ is obtuse)

$\sin(\theta) = \sqrt{1-\cos^2(\theta)}$

$A_{\triangle OAB} = \dfrac{1}{2}|OA||OB|\sin(\angle AOB) = 3\sqrt{6}$

Question 5 $\underset{\sim}{u} = \dfrac{\sqrt{30}}{30}\underset{\sim}{i} + \dfrac{\sqrt{30}}{15}\underset{\sim}{j} + \dfrac{\sqrt{30}}{6}\underset{\sim}{k} = \dfrac{1}{\sqrt{30}}\left(\underset{\sim}{i} + 2\underset{\sim}{j} + 5\underset{\sim}{k}\right)$



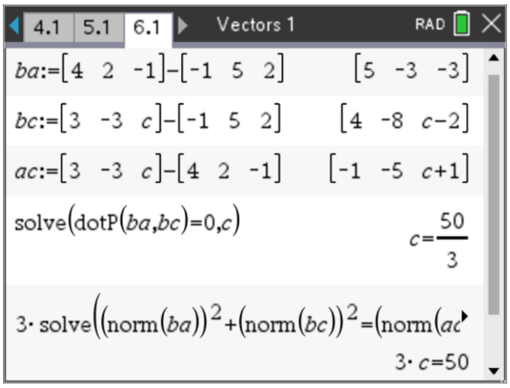
Let $\underset{\sim}{u} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$.

Solving a set of simultaneous equations can be done using menu 3 7 1.

$\left.\begin{array}{l} \underset{\sim}{u} \text{ is a unit vector, so } |\underset{\sim}{u}| = 1 \\ \underset{\sim}{u} \perp \underset{\sim}{v}, \text{ so } \underset{\sim}{u}\cdot\underset{\sim}{v} = 0 \\ \underset{\sim}{u} \perp \underset{\sim}{w}, \text{ so } \underset{\sim}{u}\cdot\underset{\sim}{w} = 0 \end{array}\right\} x = \dfrac{\sqrt{30}}{30}, y = \dfrac{\sqrt{30}}{15}, z = \dfrac{\sqrt{30}}{6}$

Ensure that the restriction that $x > 0$ is included in the input.

Question 6 $c = \dfrac{50}{3}$

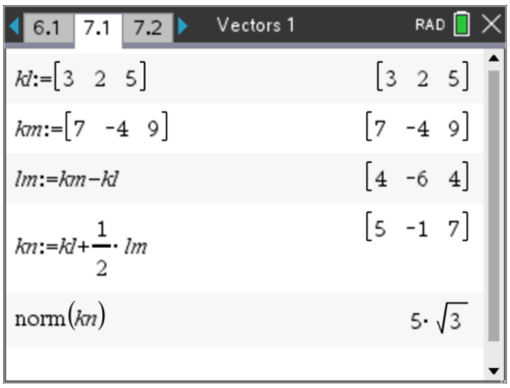


If there is a right angle at B , then $\overrightarrow{BA} \perp \overrightarrow{BC}$.

Finding $\overrightarrow{BA}\cdot\overrightarrow{BC}$, and setting this equal to zero gives $c = \dfrac{50}{3}$.

Alternatively, Pythagoras' Theorem can be used with the side lengths.

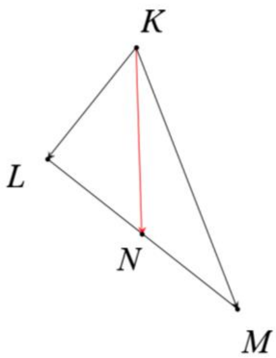
Question 7 $KN = 5\sqrt{3}$ where N is the midpoint of $\overrightarrow{LM}$.



$\triangle KLM$ is shown on the right, with N the midpoint of $\overline{LM}$.

Since N is the midpoint of $\overline{LM}$, it can be stated that $\overrightarrow{LN} = \dfrac{1}{2}\overrightarrow{LM}$.

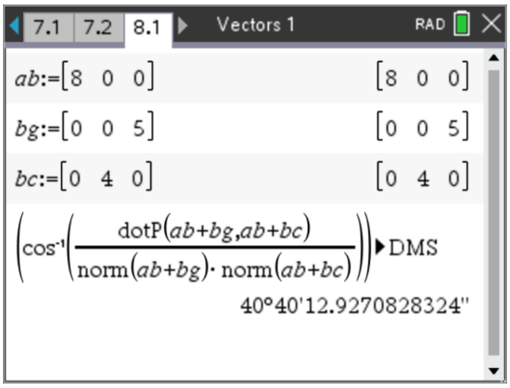
Therefore the median from K is $\overrightarrow{KN} = \overrightarrow{KL} + \overrightarrow{LN}$ and its length is $5\sqrt{3}$.



Author: S Crouch

Question 8

$\angle GAC = 40^\circ 40'$



Set A as the origin, relative to which all other vectors are stated.

$$\overrightarrow{AG} = \overrightarrow{AB} + \overrightarrow{BG} = 8\underset{\sim}{i} + 5\underset{\sim}{k}$$

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 8\underset{\sim}{i} + 4\underset{\sim}{j}$$

$$\cos(\angle GAC) = \frac{\overrightarrow{AG} \cdot \overrightarrow{AC}}{\left|\overrightarrow{AG}\right|\left|\overrightarrow{AC}\right|}$$

$$\angle GAC = 40^\circ 40'$$

Author: S Crouch