

## Vectors Part 1

Each of the questions included here can be solved using the TI-Nspire CX CAS.

Scan the QR code or use the link: <https://bit.ly/Vectors-Part1>

### Question: 1.

The position vectors of points  $A$  and  $B$  respectively are  $\underline{a} = 3\underline{i} - 4\underline{j} + 5\underline{k}$  and  $\underline{b} = 5\underline{i} - 3\underline{j} + 4\underline{k}$ . Find the perimeter  $P$  of  $\triangle OAB$  in the form  $\sqrt{a}(b + \sqrt{c})$ .

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### Question: 2.

Consider the vectors  $\overrightarrow{OE} = 2\underline{i} + 5\underline{j} - 7\underline{k}$  and  $\overrightarrow{OF} = 10\underline{i} + 14\underline{j} + 4\underline{k}$ . The point  $G$  has a position vector such that  $\overrightarrow{OG}$  bisects  $\angle EOF$  and  $|\overrightarrow{OG}| = \sqrt{218}$ . Find the coordinates of  $G$ .

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### Question: 3.

Relative to an origin  $O$ , the points  $S(1, 2, -3)$ ,  $T(5, 3, -2)$ ,  $U(6, 7, -3)$  and  $V(2, 6, -4)$  form a quadrilateral  $STUV$ , with diagonal  $SU$ . Show that  $\cos(\angle TSU) = \cos(\angle USV)$  and hence verify, using  $\angle TSV$ , that  $\cos(2\theta) = 2\cos^2(\theta) - 1$ .

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**Question: 4.**

Points  $A$  and  $B$  have position vectors  $\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$  and  $\overrightarrow{OB} = 2m\mathbf{i} - \mathbf{j} + \mathbf{k}$  respectively, relative to  $O$ , where  $m \in \mathbb{R}$ . Given that  $\cos(\angle AOB) = -\frac{2}{\sqrt{13}}$ , find the area of  $\triangle OAB$ .

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**Question: 5.**

Find a unit vector  $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ ,  $x > 0$ , that is perpendicular to both  $\mathbf{v} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$  and  $\mathbf{w} = \mathbf{i} - 3\mathbf{j} + \mathbf{k}$ .

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**Question: 6.**

Points  $A(4, 2, -1)$ ,  $B(-1, 5, 2)$  and  $C(3, -3, c)$  form a right-angled triangle, with the right angle at  $B$ . Find the value(s) of  $c$ .

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**Question: 7.**

$\triangle KLM$  has two of its sides formed by the vectors  $\overrightarrow{KL} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$  and  $\overrightarrow{KM} = 7\mathbf{i} - 4\mathbf{j} + 9\mathbf{k}$ . Find the length of the median from  $K$ .

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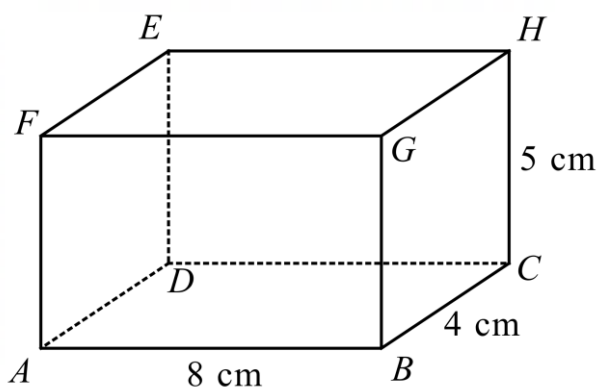
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**Question: 8.**

Consider the cuboid  $ABCDEFGH$  below.



Use vector methods to find  $\angle GAC$ , correct to the nearest minute.

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## Answers

Question 1  $P = \sqrt{2}(10 + \sqrt{3})$

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1.1 Vectors 1 RAD
oa:=[3 -4 5] [3 -4 5]
ob:=[5 -3 4] [5 -3 4]
ab:=ob-oa [2 1 -1]
p:=norm(oa)+norm(ob)+norm(ab)
[6 + 10*sqrt(2)]
factor(sqrt(6) + 10*sqrt(2)) (sqrt(3) + 10)*sqrt(2)
    
```

Define the vectors. `ctrl` `inf` can be used to get :=

$$\overline{AB} = \overline{OB} - \overline{OA}$$

Magnitudes of vectors can be found using the norm command (`menu` `7` `7` `1`).

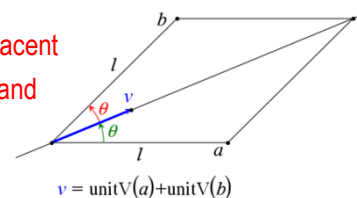
The factor command is useful for numerical expressions too, not only algebraic.

Question 2  $G = (7, 12, -5)$

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1.1 2.1 Vectors 1 RAD
e:=[2 5 -7] [2 5 -7]
f:=[10 14 4] [10 14 4]
unitV(e)+unitV(f)
[7*sqrt(78)/78 2*sqrt(78)/13 -5*sqrt(78)/78]
g:=sqrt(218) * unitV(unitV(e)+unitV(f))
[7 12 -5]
    
```

The diagonal of a rhombus bisects the adjacent sides. Therefore, the unit vectors of  $\overline{OE}$  and  $\overline{OF}$  construct adjacent sides of a unit rhombus, and their sum is the diagonal.



To find  $\overline{OG}$ , find the unit vector of the diagonal and multiply this by  $\sqrt{218}$ . Since  $\overline{OG} = 7\mathbf{i} + 12\mathbf{j} - 5\mathbf{k}$ ,  $G$  has coordinates  $(7, 12, -5)$ .

Question 3  $\cos(\angle TSU) = \cos(\angle USV) = \frac{5}{6}$ ;  $\cos(\angle TSV) = \frac{7}{18}$

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1.1 2.1 3.1 Vectors 1 RAD
st:=[5 3 -2]-[1 2 -3] [4 1 1]
su:=[6 7 -3]-[1 2 -3] [5 5 0]
sv:=[2 6 -4]-[1 2 -3] [1 4 -1]
dotP(st,su) 5
norm(st)*norm(su) 6
dotP(su,sv) 5
norm(su)*norm(sv) 6
    
```

$$\overline{ST} = \overline{OT} - \overline{OS}, \overline{SU} = \overline{OU} - \overline{OS}, \overline{SV} = \overline{OV} - \overline{OS}$$

$\angle TSU$  is the angle between the tails of  $\overline{ST}$  and  $\overline{SU}$ .

$\angle USV$  is the angle between the tails of  $\overline{SU}$  and  $\overline{SV}$ .

$$\text{Hence, } \cos(\angle TSU) = \frac{\overline{ST} \cdot \overline{SU}}{|\overline{ST}| |\overline{SU}|} \text{ and } \cos(\angle USV) = \frac{\overline{SU} \cdot \overline{SV}}{|\overline{SU}| |\overline{SV}|}$$

$$\text{Using dotP (menu 7 C 3): } \cos(\angle TSU) = \cos(\angle USV) = \frac{5}{6}.$$

```

1.1 2.1 3.1 Vectors 1 RAD
norm(st)*norm(sv) 6
© ∠ TSV = ∠ TSU + ∠ USV = 2θ
dotP(st,sv) 7
norm(st)*norm(sv) 18
2 * (5/6)^2 - 1 7/18
    
```

Since  $\angle TSU = \angle USV = \theta$  and  $\angle TSV = \angle TSU + \angle USV$ , it can be seen that  $\angle TSV = 2\theta$ . Finding  $\cos(\angle TSV)$ :

$$\cos(\angle TSV) = \frac{\overline{ST} \cdot \overline{SV}}{|\overline{ST}| |\overline{SV}|} = \frac{7}{18}$$

Verifying  $\cos(2\theta) = 2\cos^2(\theta) - 1$ :

$$\frac{7}{18} = 2\left(\frac{5}{6}\right)^2 - 1 \Rightarrow \frac{7}{18} = \frac{7}{18} \quad \text{👍}$$

Question 4

$$A_{\triangle OAB} = 3\sqrt{6}$$

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2.1 3.1 4.1 Vectors 1 RAD
a:=[2 2 2] [2 2 2]
b:=[2·m -1 1] [2·m -1 1]
solve(dotP(a,b)/norm(a)·norm(b)=-2/√13,m) m=-√6
1/2·norm(a)·norm(b)·√(1-(-2/√13)²) |m=-√6
3·√6
    
```

$$\cos(\angle AOB) = \frac{\overline{OA} \cdot \overline{OB}}{|\overline{OA}| |\overline{OB}|} = -\frac{2}{\sqrt{13}} \text{ (so } \angle AOB \text{ is obtuse)}$$

$$\sin(\theta) = \sqrt{1 - \cos^2(\theta)}$$

$$A_{\triangle OAB} = \frac{1}{2} |\overline{OA}| |\overline{OB}| \sin(\angle AOB) = 3\sqrt{6}$$

Question 5

$$\underline{u} = \frac{\sqrt{30}}{30} \underline{i} + \frac{\sqrt{30}}{15} \underline{j} + \frac{\sqrt{30}}{6} \underline{k} = \frac{1}{\sqrt{30}} (\underline{i} + 2\underline{j} + 5\underline{k})$$

```

3.1 4.1 5.1 Vectors 1 RAD
u:=[x y z] [x y z]
v:=[3 1 -1] [3 1 -1]
w:=[1 -3 1] [1 -3 1]
solve(norm(u)=1, dotP(u,v)=0, {x,y,z}|x>0, dotP(u,w)=0)
x=√30/30 and y=√30/15 and z=√30/6
    
```

Let  $\underline{u} = x\underline{i} + y\underline{j} + z\underline{k}$ .

Solving a set of simultaneous equations can be done using `menu 3 7 1`.

$$\left. \begin{array}{l} \underline{u} \text{ is a unit vector, so } |\underline{u}| = 1 \\ \underline{u} \perp \underline{v}, \text{ so } \underline{u} \cdot \underline{v} = 0 \\ \underline{u} \perp \underline{w}, \text{ so } \underline{u} \cdot \underline{w} = 0 \end{array} \right\} x = \frac{\sqrt{30}}{30}, y = \frac{\sqrt{30}}{15}, z = \frac{\sqrt{30}}{6}$$

Ensure that the restriction that  $x > 0$  is included in the input.

Question 6

$$c = \frac{50}{3}$$

```

4.1 5.1 6.1 Vectors 1 RAD
ba:=[4 2 -1]-[-1 5 2] [5 -3 -3]
bc:=[3 -3 c]-[-1 5 2] [4 -8 c-2]
ac:=[3 -3 c]-[4 2 -1] [-1 -5 c+1]
solve(dotP(ba,bc)=0,c) c=50/3
3·solve((norm(ba))²+(norm(bc))²=(norm(ac))²)
3·c=50
    
```

If there is a right angle at  $B$ , then  $\overline{BA} \perp \overline{BC}$ .

Finding  $\overline{BA} \cdot \overline{BC}$ , and setting this equal to zero gives  $c = \frac{50}{3}$ .

Alternatively, Pythagoras' Theorem can be used with the side lengths.

Question 7

$$KN = 5\sqrt{3} \text{ where } N \text{ is the midpoint of } \overline{LM}.$$

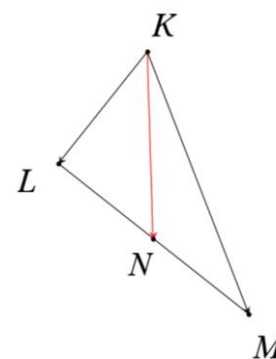
```

6.1 7.1 7.2 Vectors 1 RAD
kl:=[3 2 5] [3 2 5]
km:=[7 -4 9] [7 -4 9]
lm:=km-kl [4 -6 4]
kn:=kl+1/2·lm [5 -1 7]
norm(kn) 5·√3
    
```

$\triangle KLM$  is shown on the right, with  $N$  the midpoint of  $\overline{LM}$ .

Since  $N$  is the midpoint of  $\overline{LM}$ , it can be stated that  $\overline{LN} = \frac{1}{2} \overline{LM}$ .

Therefore the median from  $K$  is  $\overline{KN} = \overline{KL} + \overline{LN}$  and its length is  $5\sqrt{3}$ .



Question 8

$$\angle GAC = 40^\circ 40'$$

```

7.1 7.2 8.1 Vectors 1 RAD
ab:=[8 0 0] [8 0 0]
bg:=[0 0 5] [0 0 5]
bc:=[0 4 0] [0 4 0]
(cos^1(dotP(ab+bg,ab+bc)
(norm(ab+bg)*norm(ab+bc))))DMS
40°40'12.9270828324"
    
```

Set  $A$  as the origin, relative to which all other vectors are stated.

$$\vec{AG} = \vec{AB} + \vec{BG} = 8\vec{i} + 5\vec{k}$$

$$\vec{AC} = \vec{AB} + \vec{BC} = 8\vec{i} + 4\vec{j}$$

$$\cos(\angle GAC) = \frac{\vec{AG} \cdot \vec{AC}}{|\vec{AG}| |\vec{AC}|}$$

$$\angle GAC = 40^\circ 40'$$