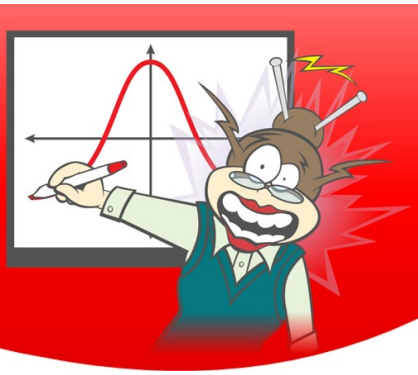
STUDENT REVISION SERIES



Vectors (2)

Each of the questions included here can be solved using the TI-Nspire.

Scan the QR code or use the link:

Question: 1

A curve C has parametric equations $x = 4\cos\left(\frac{2\pi t}{3}\right)$ and $y = 4\sin\left(\frac{2\pi t}{3}\right)$ for $0 \le t \le 3$.

The Cartesian equation of C is

(A) $x^2 - y^2 = 16$

(B) $x + y = 16$

(C) $x - y = 4$

(D) $x^2 + y^2 = 1$

(E) $x^2 + y^2 = 16$

Question: 2

The position vector of a particle at time t is given by $\boldsymbol{r}(t) = 4t\hat{\boldsymbol{i}} + \left(8t - t^2\right)\hat{\boldsymbol{j}}$, $0 \le t \le 3$.

The Cartesian equation of the particle's path is

(A) $y = 2x - \frac{x^2}{16}, x \ge 0$

(B) $y = 2x - \frac{x^2}{16}, 0 \le x \le 12$

(C) $y = 2x - \frac{x^2}{4}, 0 \le x \le 12$

(D) $y = 2x - \frac{x^2}{16}, 0 \le x \le \frac{3}{4}$

(E) $y = \frac{32}{x} - \frac{16}{x^2}, 0 < x \le 12$

Author:

Question: 3

A particle moves so that its position vector at time t is $\boldsymbol{r}(t) = 4\sin(2t)\hat{\boldsymbol{i}} - 3\cos(2t)\hat{\boldsymbol{j}}$, $t \geq 0$.

The initial velocity of the particle is

(A) $-3\hat{\boldsymbol{j}}$

(B) $-8\hat{\boldsymbol{i}}$

(C) $8\hat{\boldsymbol{i}}$

(D) $6\hat{\boldsymbol{j}}$

(E) $-6\hat{\boldsymbol{j}}$

Question: 4

The position vector of a projectile at time t seconds, relative to a point on level ground, is given by $\boldsymbol{r}(t) = 10t\hat{\boldsymbol{i}} + (19.6t - 4.9t^2)\hat{\boldsymbol{j}}$ for $t \geq 0$ where $\hat{\boldsymbol{i}}$ is horizontal and $\hat{\boldsymbol{j}}$ is vertically upwards. Distances are measured in metres.

The maximum height reached by the projectile is

(A) 4.9

(B) 10

(C) 19.6

(D) 20

(E) 40

Question: 5

The parametric equations of a curve C are $x = 2\sec(t)$ and $y = 3\tan(t)$, where $-\pi \leq t \leq \pi, t \neq \pm\frac{\pi}{2}$.

(a) Find the Cartesian equation of C.

(b) State the domain and range of C.

(c) Sketch the graph of C.

Question: 6

The position vector of a particle at time t is given by $\boldsymbol{r}(t) = \cos^2(t)\hat{\boldsymbol{i}} + 4\sin^2(t)\hat{\boldsymbol{j}}$, $t \geq 0$.

(a) Find the Cartesian equation of the path of the particle.

(b) Sketch the path of the particle indicating any points of intersection with the coordinate axes.

(c) State the position of the particle at $t = 0$.

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Question: 7

Two particles, *A* and *B*, commence motion at time $t = 0$. At time *t* seconds, their respective position vectors, $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$, are given by $\boldsymbol{r}_A(t) = (6 - t)\hat{\boldsymbol{i}} + 2t\hat{\boldsymbol{j}}$ and $\boldsymbol{r}_B(t) = (2t - 5)\hat{\boldsymbol{i}} + 2t\hat{\boldsymbol{j}}$, $t \geq 0$.

(a) Show that *A* and *B* collide and find the time of collision.

(b) Find the coordinates of the point of collision.

Question: 8

The position vector of a particle at time t seconds is given by $\boldsymbol{r}(t) = \left(t - 2e^t\right)\hat{\boldsymbol{i}} + \left(3\cos(t) - 2t\right)\hat{\boldsymbol{j}}$, $0 \leq t \leq \pi$.

All distances are measured in metres.

(a) Find the particle's initial velocity.

(b) Find the particle's speed at $t = \dfrac{\pi}{2}$. Give your answer correct to two decimal places.

(c) Find the magnitude of the particle's acceleration at $t = \dfrac{\pi}{2}$. Give your answer correct to two decimal places.

Author:

Answers

Question: 1

Squaring each equation and adding gives $x^2 + y^2 = 16$.

Answer: E

Question: 2

Substituting $t = \frac{x}{4}$ into $y = 8t - t^2$ gives $y = 2x - \frac{x^2}{16}$.

$0 \le \frac{x}{4} \le 3$ and so $0 \le x \le 12$

Answer: B

Question: 3

$\boldsymbol{r}'(t) = 8\cos(2t)\hat{\boldsymbol{i}} + 6\sin(2t)\hat{\boldsymbol{j}}$ and $\boldsymbol{r}'(0) = 8\hat{\boldsymbol{i}}$

Answer: C

Question: 4

The maximum height occurs when the $\hat{\boldsymbol{j}}$ component of $\boldsymbol{r}'(t)$ is zero.

Solving $19.6 - 9.8t = 0$ for t gives $t = 2$.

$\boldsymbol{r}(2) = 20\hat{\boldsymbol{i}} + 19.6\hat{\boldsymbol{j}}$ so the maximum height is 19.6 metres.

Answer: C

Author:

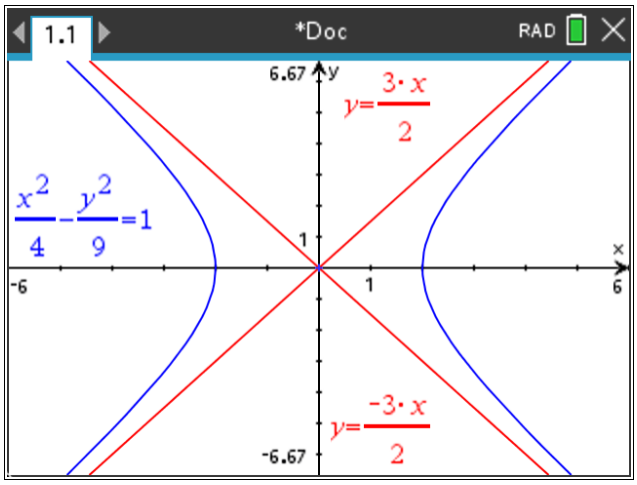
Question: 5

(a) Use of $1+\tan^2(t)=\sec^2(t)$ gives $\frac{x^2}{4}-\frac{y^2}{9}=1$.

(b) The domain of C is the range of $x=2\sec(t)$ which is $(-\infty,-2]\cup[2,\infty)$.

The range of C is the range of $y=3\tan(t)$ which is R.

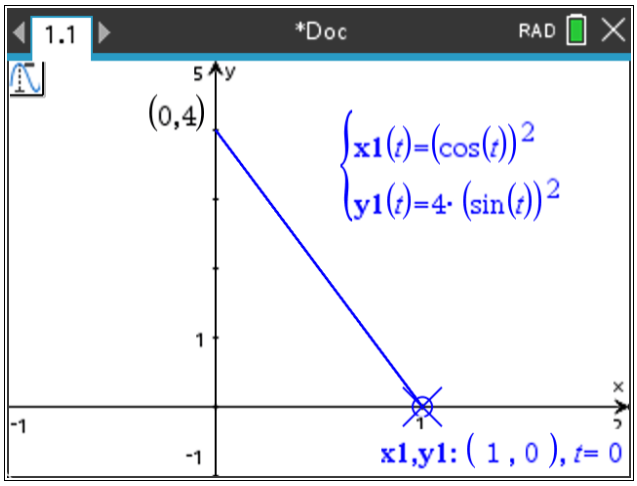
(c) The graph is a hyperbola centred at the origin. The asymptotes have equations $y=\pm\frac{3x}{2}$.



Question: 6

(a) $y=4-4x$ for $0\le x\le 1$

(b) The path is a straight line joining $(0,4)$ and $(1,0)$



(c) $(1,0)$ at $t=0$

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Question: 7

(a) $\boldsymbol{r}_A(t) = \boldsymbol{r}_B(t)$ occurs at $t = \dfrac{11}{3}$

(b) point of collision is $\left(\dfrac{7}{3}, \dfrac{22}{3}\right)$

Question: 8

(a) $\boldsymbol{r}'(t) = \left(1 - 2e^t\right)\hat{\boldsymbol{i}} - \left(3\sin(t) + 2\right)\hat{\boldsymbol{j}}$ and $\boldsymbol{r}'(0) = -\hat{\boldsymbol{i}} - 2\hat{\boldsymbol{j}}$

(b) $\boldsymbol{r}'\left(\dfrac{\pi}{2}\right) = \left(1 - 2e^{\frac{\pi}{2}}\right)\hat{\boldsymbol{i}} - 5\hat{\boldsymbol{j}}$ and $\left|\boldsymbol{r}'\left(\dfrac{\pi}{2}\right)\right| = 9.97$ (m/s)

(c) $\boldsymbol{r}''\left(\dfrac{\pi}{2}\right) = -2e^{\frac{\pi}{2}}\hat{\boldsymbol{i}}$ and $\left|\boldsymbol{r}''\left(\dfrac{\pi}{2}\right)\right| = 9.62$ (m/s^2)

Author: