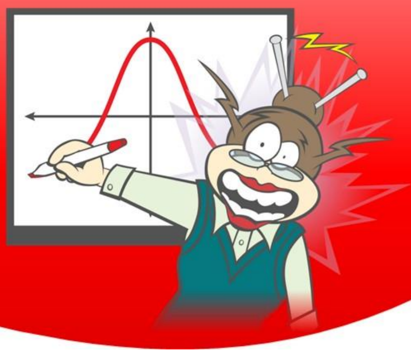

Vectors – Part 1

Each of the questions included here can be solved using the TI-Nspire.

Question: 1

Let $\boldsymbol{a} = 3\hat{\boldsymbol{i}} - 4\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}$ and $\boldsymbol{b} = -\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} - 2\hat{\boldsymbol{k}}$.

The magnitude of the vector $2\boldsymbol{a} - \boldsymbol{b}$ is

(A) $\sqrt{165}$

(B) $\sqrt{129}$

(C) $\sqrt{141}$

(D) $\sqrt{149}$

(E) $\sqrt{61}$

Question: 2

The cosine of the angle between $\boldsymbol{a} = \hat{\boldsymbol{i}} - 2\hat{\boldsymbol{k}}$ and $\boldsymbol{b} = 2\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} + 2\hat{\boldsymbol{k}}$, correct to two decimal places, is

(A) –0.38

(B) –0.30

(C) 0.30

(D) 0.38

(E) 0.89

Question: 3

Let $\boldsymbol{a} = 2\hat{\boldsymbol{i}} + 3\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}$ and $\boldsymbol{b} = 3\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}$.

$\boldsymbol{a} \times \boldsymbol{b}$ is equal to

(A) $\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}$

(B) $2\hat{\boldsymbol{i}} + 3\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}$

(C) $\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} - 5\hat{\boldsymbol{k}}$

(D) $2\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} - 5\hat{\boldsymbol{k}}$

(E) 13

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Question: 4

The line l is described by the vector equation: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 11 \end{pmatrix} + t\begin{pmatrix} 4 \\ 12 \\ -4 \end{pmatrix}$.

Which one of the following points lies on line l?

(A) $(0,5,9)$

(B) $(4,17,15)$

(C) $(4,5,7)$

(D) $(-4,-7,7)$

(E) $(2,11,9)$

Question: 5

Consider the points $P(2,-1,3)$, $Q(3,0,-2)$ and $R(2,y,z)$ where $y,z > 0$.

(a) Use a vector method to show that $\angle POQ = 90°$.

(b) Given that $\overrightarrow{OP}$, $\overrightarrow{OQ}$ and $\overrightarrow{OR}$ are mutually perpendicular, find the values of y and z.

Question: 6

Consider the points $O(0,0,0)$, $A(1,2,1)$ and $B(4,2,-1)$. Let P be the point on $\overrightarrow{OB}$ which is closest to A.

(a) Find the coordinates of P.

(b) Find the shortest distance between A and P. Give your answer correct to two decimal places.

Question: 7

Line l_1 has the vector equation $\boldsymbol{r}_1 = 2\hat{\boldsymbol{i}} - 2\hat{\boldsymbol{j}} + 5\hat{\boldsymbol{k}} + t\left(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}\right), t \in R$ and line l_2 has the vector equation $\boldsymbol{r}_1 = 2\hat{\boldsymbol{i}} + 4\hat{\boldsymbol{j}} + 7\hat{\boldsymbol{k}} + s\left(2\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}}\right), s \in R$.

(a) The lines l_1 and l_2 intersect at point P. Find the coordinates of P.

(b) Find the angle between l_1 and l_2. Give your answer correct to the nearest tenth of a degree.

Question: 8

The three planes $x - 3y - 2z = -9$, $2x - 5y + z = 3$ and $-3x + 6y + 2z = 8$ intersect at the point P.

Find the coordinates of P.

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Answers

Question: 1

$$2\boldsymbol{a}-\boldsymbol{b}=7\hat{\boldsymbol{i}}-10\hat{\boldsymbol{j}}+4\hat{\boldsymbol{k}}$$

$$|2\boldsymbol{a}-\boldsymbol{b}|=\sqrt{165}$$

Answer: A

Question: 2

$$\cos\theta=\frac{\left(\hat{\boldsymbol{i}}-2\hat{\boldsymbol{k}}\right)\cdot\left(2\hat{i}-\hat{j}+2\hat{\boldsymbol{k}}\right)}{\left|\hat{\boldsymbol{i}}-2\hat{\boldsymbol{k}}\right|\times\left|2\hat{i}-\hat{j}+2\hat{\boldsymbol{k}}\right|}=-0.298...$$

Answer: B

Question: 3

Answer: C

Question: 4

When $t=\frac{1}{2}$, $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\5\\11\end{pmatrix}+\begin{pmatrix}2\\6\\-2\end{pmatrix}$.

Answer: E

Question: 5

(a) $\overrightarrow{OP}\cdot\overrightarrow{OQ}=0\Rightarrow\overrightarrow{OP}\perp\overrightarrow{OQ}$ since $\overrightarrow{OP}$, $\overrightarrow{OQ}$ are non-zero and so $\angle POQ=90°$

(b) $\overrightarrow{OP}\times\overrightarrow{OQ}=2\hat{i}+13\hat{\boldsymbol{j}}+3\hat{\boldsymbol{k}}$ and so $y=13, z=3$ $(y,z>0)$

Question: 6

(a) $\overrightarrow{AP}=-\left(\hat{\boldsymbol{i}}+2\hat{\boldsymbol{j}}+\hat{\boldsymbol{k}}\right)+t\left(4\hat{\boldsymbol{i}}+2\hat{\boldsymbol{j}}-\hat{\boldsymbol{k}}\right)$

Solving $\overrightarrow{AP}\cdot\overrightarrow{OB}=0$ for t gives $t=\frac{1}{3}$.

Q is the point $\left(\frac{4}{3},\frac{2}{3},-\frac{1}{3}\right)$.

(b) When $t=\frac{1}{3}$, $\overrightarrow{AP}=\frac{1}{3}\hat{\boldsymbol{i}}-\frac{4}{3}\hat{\boldsymbol{j}}-\frac{4}{3}\hat{\boldsymbol{k}}$ and so $\left|\overrightarrow{AP}\right|=1.91$.

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Question: 7

(a) Solving 2 correct equations $2+t=2+2s$ and $-2-t=4+s$ for t and s gives $t=-4$ and $s=-2$.

The third equation is $5+t=7+3s$.

The coordinates of *P* are $(-2,2,1)$.

(b) $$\cos\theta=\frac{\left(\hat{\boldsymbol{i}}-\hat{\boldsymbol{j}}+\hat{\boldsymbol{k}}\right)\cdot\left(2\hat{\boldsymbol{i}}+\hat{\boldsymbol{j}}+3\hat{\boldsymbol{k}}\right)}{\left|\hat{\boldsymbol{i}}-\hat{\boldsymbol{j}}+\hat{\boldsymbol{k}}\right|\left|2\hat{\boldsymbol{i}}+\hat{\boldsymbol{j}}+3\hat{\boldsymbol{k}}\right|}=\frac{4}{\sqrt{42}}$$

$\theta=51.9°$

Question: 8

The coordinates of *P* are $(2,1,4)$.

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