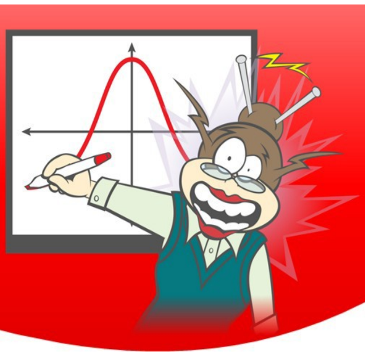
STUDENT REVISION SERIES



Matrices Part 1

Question 1.

Represent the following as a matrix equation and solve using matrix operations

$$2x + y - 2z = -1$$
$$3x - 3y - z = 5$$
$$x - 2y + 3z = 6$$

Question 2.

Find a solution for x,y,z using the linsolve function.

$$3x - 4y + 2z = -15$$
$$2x + 4y + z = 16$$
$$2x + 3y + 5z = 20$$

Question 3.

Transform the following set of equations into row reduced echelon form and write down the solution set.

$$2x - y + 3z = 17$$
$$-5x + 4y - 2z = -46$$
$$2y + 5z = -7$$

Question 4.

Show that the following set of equations has no solution.

$$x + y + z = 2$$
$$y - 3z = 1$$
$$2x + y + 5z = 0$$

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Question 5.

Show that the following system does not have a unique solution.

$$x + y + z = 7$$
$$3x - 2y - z = 4$$
$$x + 6y + 5z = 24$$

Question 6.

Use Gaussian elimination (ie row operations) to solve the following system of equations.

$$2x - y + 3z = 17$$
$$-5x + 4y - 2z = -46$$
$$2y + 5z = -7$$

Question 7

Determine if the following systems have unique solutions, infinite solutions or no solutions. Justify your response.

A)

$$2x + 3y - 6z = 1$$
$$-4x - 6y + 12z = -2$$
$$x + 2y + 5z = 10$$

B)

$$x + y + z = 14$$
$$2y + 3z = -14$$
$$-16y - 24z = -112$$

C)

$$x + y + z = 0$$
$$2x - y + 3z = 0$$
$$x - z = 0$$

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Answer and solutions

Question 1.

Create matrix M and C as shown

$$M.X = C$$

Where X is a column matrix containing the variable x,y,z

Check $det(M) \neq 0$ therefore unique solution exists

$$M^{-1}M.X = M^{-1}C$$
$$X = M^{-1}C$$

Solution

$$x = 1$$
$$y = -1$$
$$z = 1$$

Question 2.

Question 3.

Create the augmented matrix

Apply the rref() command to the augmented matrix

x=4, y=-6, z=1

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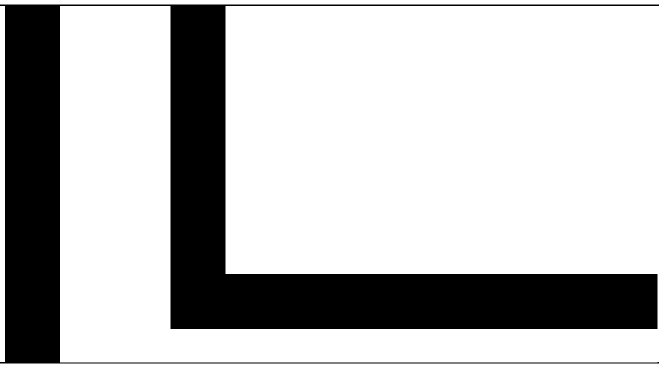
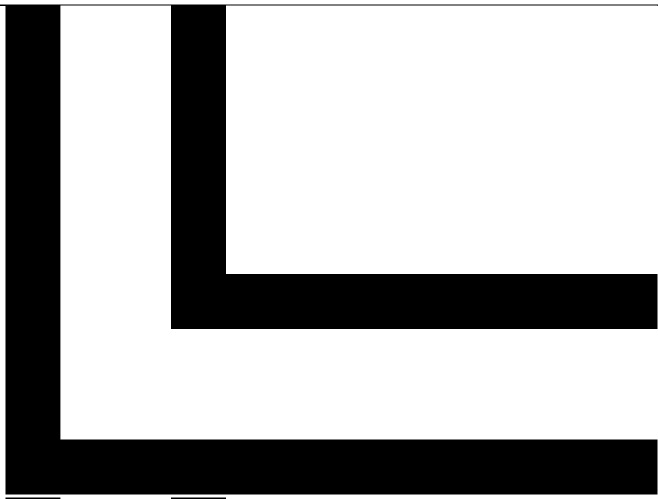
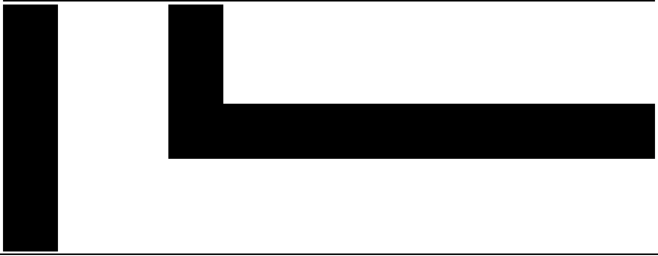
Use ***rref()*** or ***linsolve*** function Bottom row is inconsistent (0=1) therefore no unique solution.	

Question 5.

Bottom row is an identity therefore infinite number of solutions. z = λ, $y+\frac{4}{5}z=\frac{17}{5}$ $y=\frac{17}{5}-\frac{4}{5}z$ $y=\frac{17-4z}{5}$ $y=\frac{17-4\lambda}{5}$ Similarly $x=\frac{18-\lambda}{5}$ Alternative linsolve()	 

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Question 6.

$$\begin{bmatrix} 2 & -1 & 3 & 17 \\ -5 & 4 & -2 & -46 \\ 0 & 2 & 5 & -7 \end{bmatrix} \quad \begin{bmatrix} 2 & -1 & 3 & 17 \\ -5 & 4 & -2 & -46 \\ 0 & 2 & 5 & -7 \end{bmatrix}$$

$$\mathrm{mRow}\left(0.5, \begin{bmatrix} 2 & -1 & 3 & 17 \\ -5 & 4 & -2 & -46 \\ 0 & 2 & 5 & -7 \end{bmatrix}, 1\right) \quad \begin{bmatrix} 1. & -0.5 & 1.5 & 8.5 \\ -5 & 4 & -2 & -46 \\ 0 & 2 & 5 & -7 \end{bmatrix}$$

$$\mathrm{mRowAdd}\left(5, \begin{bmatrix} 1. & -0.5 & 1.5 & 8.5 \\ -5 & 4 & -2 & -46 \\ 0 & 2 & 5 & -7 \end{bmatrix}, 1, 2\right) \quad \begin{bmatrix} 1. & -0.5 & 1.5 & 8.5 \\ 0. & 1.5 & 5.5 & -3.5 \\ 0 & 2 & 5 & -7 \end{bmatrix}$$

$$\mathrm{mRow}\left(\frac{2}{3}, \begin{bmatrix} 1. & -0.5 & 1.5 & 8.5 \\ 0. & 1.5 & 5.5 & -3.5 \\ 0 & 2 & 5 & -7 \end{bmatrix}, 2\right) \quad \begin{bmatrix} 1. & -0.5 & 1.5 & 8.5 \\ 0. & 1. & 3.66667 & -2.33333 \\ 0 & 2 & 5 & -7 \end{bmatrix}$$

$$\mathrm{mRowAdd}\left(-2, \begin{bmatrix} 1. & -0.5 & 1.5 & 8.5 \\ 0. & 1. & 3.6666666666667 & -2.3333333333333 \\ 0 & 2 & 5 & -7 \end{bmatrix}, 2, 3\right)$$

$$\begin{bmatrix} 1. & -0.5 & 1.5 & 8.5 \\ 0. & 1. & 3.66667 & -2.33333 \\ 0. & 0. & -2.33333 & -2.33333 \end{bmatrix}$$

$$\mathrm{mRow}\left(\frac{-3}{7}, \begin{bmatrix} 1. & -0.5 & 1.5 & 8.5 \\ 0. & 1. & 3.6666666666667 & -2.3333333333333 \\ 0. & 0. & -2.3333333333334 & -2.3333333333334 \end{bmatrix}, 3\right)$$

$$\begin{bmatrix} 1. & -0.5 & 1.5 & 8.5 \\ 0. & 1. & 3.66667 & -2.33333 \\ 0. & 0. & 1. & 1. \end{bmatrix}$$

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$$\text{mRowAdd}\left(\frac{-11}{3},\begin{bmatrix}1. & -0.5 & 1.5 & 8.5\\ 0. & 1. & 3.6666666666667 & -2.3333333333333\\ 0. & 0. & 1. & 1.\end{bmatrix},3,2\right) \quad \begin{bmatrix}1. & -0.5 & 1.5 & 8.5\\ 0. & 1. & 0. & -6.\\ 0. & 0. & 1. & 1.\end{bmatrix}$$

$$\text{mRowAdd}\left(-1.5,\begin{bmatrix}1. & -0.5 & 1.5 & 8.5\\ 0. & 1. & 0. & -6.\\ 0. & 0. & 1. & 1.\end{bmatrix},3,1\right) \quad \begin{bmatrix}1. & -0.5 & 0. & 7.\\ 0. & 1. & 0. & -6.\\ 0. & 0. & 1. & 1.\end{bmatrix}$$

$$\text{mRowAdd}\left(0.5,\begin{bmatrix}1. & -0.5 & 0. & 7.\\ 0. & 1. & 0. & -6.\\ 0. & 0. & 1. & 1.\end{bmatrix},2,1\right) \quad \begin{bmatrix}1. & 0. & 0. & 4.\\ 0. & 1. & 0. & -6.\\ 0. & 0. & 1. & 1.\end{bmatrix}$$

Question 7A.

Infinite Solutions	$\text{linSolve}\left(\begin{cases}2\cdot x+3\cdot y-6\cdot z=1\\ -4\cdot x-6\cdot y+12\cdot z=-2\\ x+2\cdot y+5\cdot z=10\end{cases},\{x,y,z\}\right)$ $\{27\cdot \boldsymbol{c1}-28,19-16\cdot \boldsymbol{c1},\boldsymbol{c1}\}$

Question 7B.

No Solution	$\text{linSolve}\left(\begin{cases}x+y+z=14\\ 2\cdot y+3\cdot z=-14\\ -16\cdot y-24\cdot z=-112\end{cases},\{x,y,z\}\right)$ "No solution found"

Question 7C.

Unique Solution	$\text{linSolve}\left(\begin{cases}x+y+z=0\\ 2\cdot x-y+3\cdot z=0\\ x-z=0\end{cases},\{x,y,z\}\right)$ $\{0,0,0\}$

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