



## Increasing & Decreasing Functions

Each of the questions included here can be solved using either the TI-nspire CX or CX CAS.

Scan the QR code or use the link: <http://bit.ly/IncreasingDecreasing>

### Question: 1.

Determine the region for which  $f(x) = x^2 - 6x + 8$  is an increasing function.

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### Question: 2.

Show that  $g(x) = x^3 + x$  is an increasing function.

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### Question: 3.

If  $f(x) = x^3 + bx^2 + 15x + 4$  is an increasing function, determine the range of values for  $b$ .

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### Question: 4.

The number of hours of darkness (night time) in Melbourne can be approximated by the function:

$$f(x) = 2.57 \sin\left(\frac{2\pi}{365}(x+100)\right) + 12.1$$

Where  $x$  represents the number of days since the start of the year, determine the interval (approximate dates) for when the 'nights' are getting shorter.


## Answers

### Question 1

This can be done by completing the square or by calculus.  $x^2 - 6x + 8 = (x - 3)^2 - 1$ . This locates the turning point (3, -1), so the function is increasing over the region  $[3, \infty)$

### Question 2

This quickest way to show that  $g(x) = x^3 + x$  is to determine the derivative:  $g'(x) = 3x^2 + 1$ . As per the video, since  $x^2 \geq 0$  then it follows that  $3x^2 \geq 0$  and therefore  $3x^2 + 1 > 0$ , so our function  $g(x)$  is increasing for all  $x \in \mathbb{R}$

### Question 3

#### TI-Nspire CX CAS shown

[Line 1]: Again the quickest solution is to use calculus.

[Line 2]: We expect a quadratic result and know from the video that we are looking for a quadratic that is always positive, so completing the square is one option.

[Line 3]: Since  $3\left(x + \frac{b}{3}\right)^2 \geq 0$  then we require  $-\frac{b^2 - 45}{3} \geq 0$

The screenshot shows a TI-Nspire CX CAS interface. The top bar displays '1.1 1.2 \*Doc DEG'. The main window contains the following text:  
 $\frac{d}{dx}(g(x))$   $3 \cdot x^2 + 2 \cdot b \cdot x + 15$   
 $\text{completeSquare}(3 \cdot x^2 + 2 \cdot b \cdot x + 15, x)$   
 $3 \cdot \left(x + \frac{b}{3}\right)^2 - \frac{b^2 - 45}{3}$   
 $\text{solve}\left(\frac{b^2 - 45}{3} < 0, b\right)$   $-3 \cdot \sqrt{5} < b < 3 \cdot \sqrt{5}$

### Question 4

#### TI-nspire CX CAS shown

We can use the derivative or our knowledge of trigonometric transformations. [Make sure the calculator is set in radian mode]

When using the solve command, place domain restrictions so that the dates may be obtained within a 12 month period.

The solutions: 173 and 356 represent when the gradient is zero. A quick check of either the graph or by substitution into the derivative reveals that the region we want is:  $173 \leq x \leq 356$ .

You can use a calendar to determine when these dates occur or the "days between dates" function on the calculator. (DMY = Day, Month, Year)

Whilst this function cannot be used in the Solve command, a rough estimate will help determine the actual dates. An estimate of 20<sup>th</sup> June reveals that it is day 171, so 22<sup>nd</sup> June will be 173 days.

Note that the values have been truncated 173.75 became 173. In practical terms the days either side of 22<sup>nd</sup> June have almost the same number of night time hours.

The screenshot shows a TI-Nspire CX CAS interface. The top bar displays '1.1 1.2 \*Doc RAD'. The main window contains the following text:  
 $\frac{d}{dx}\left(2.57 \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (x+100)\right) + 12.1\right)$   
 $-0.044241 \cdot \sin(0.017214 \cdot x + 0.150624)$   
 $\text{solve}(-0.044240510245073 \cdot \sin(0.01721420 \cdot x + 0.150624), x)$   
 $x = 173.75 \text{ or } x = 356.25$   
 $\text{dbd\_DMY}(\{1, 1, 20\}, \{20, 6, 20\})$  171  
 $\text{dbd\_DMY}(\{1, 1, 20\}, \{22, 6, 20\})$  173  
 $\text{dbd\_DMY}(\{1, 1, 20\}, \{24, 6, 20\})$  175