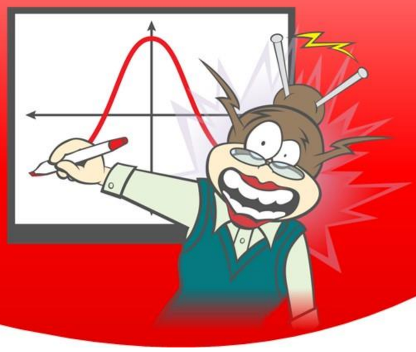


FURTHER CALCULUS & INTEGRATION

Each of the questions included here can be solved using TI-Nspire CX.

Question 1

Find the gradient of the curve $y = \sin(2x) - 1$ at (0,-1).S

Question 2

Find the equation of the normal to the curve $y = e^x + 2$ at $x = 0$.

Question 3

A population of bacteria after t hours is given by $P(t) = 5000e^{0.18t}$. Calculate the **rate** of increase of the population (to the nearest unit) at 15 minutes.

Question 4

A particle moves along the x-axis with position at time, t, given by $x(t) = -e^t \cos(t)$ for $0 \le t \le 2\pi$. Calculate each time, t, for which the particle is at rest. (**hint**: use maximum and minimum)

Author: M Hourigan

Question 5

The number of rabbits increases according to the model $n(t) = Ae^{bt}$, where t is time in years, $n(t)$ is the population size at time t, A is the initial size of the population and b is the relative rate of growth.

Rabbits were introduced to a small island 7 years ago. The current rabbit population on the island is estimated to be 3600, with a relative growth rate of 45% per year.

Determine when the population is increasing at a rate of 5000 rabbits per year.

Question 6

Rainwater is being collected in a water tank. The rate of change of volume, *V* litres, with respect to time, *t* seconds, is given by $\frac{dV}{dt} = \frac{2t^3}{3} + \frac{3t^2}{2} + t$. Determine the volume of water that is collected in the tank between *t=2* and *t=5.*

Question 7

Find the value of $\int_1^4 \left(2x - 3x^{\frac{1}{2}}\right) dx$

Question 8

Find the value of $\int_{-1}^{1} \frac{e^x + e^{-x}}{2}\, dx$

Question 9

A particle moves in a straight line. The velocity of the particle, *v* m/s, at time, *t* seconds, is given by $v=2t-3$ for $t \geq 0$. Find the particle's displacement after 4 seconds.

Question 10

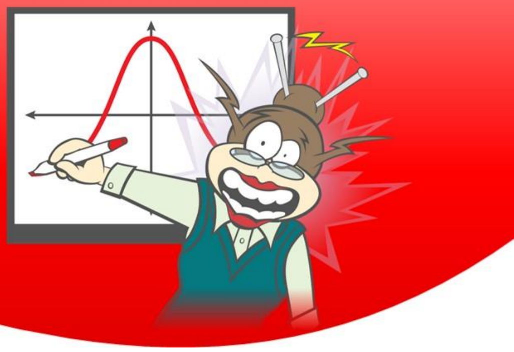
Heat escapes from a storage tank at the rate of $\frac{dH}{dt} = 1 + \frac{3}{4}\sin(\frac{\pi t}{60})$ kilojoules per day. If H(t) is the total accumulated heat loss at time *t* days, find the amount of heat lost in the first 150 days.

Questions used in this worksheet were sourced from/inspired by:

- https://www.qcaa.qld.edu.au/senior/senior-subjects/mathematics/mathematics-methods/assessment
- Mathematical Methods Units3 & 4 for Queensland, Cambridge University Press

Author: M Hourigan

STUDENT REVISION SERIES

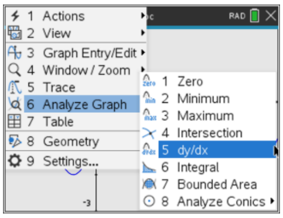


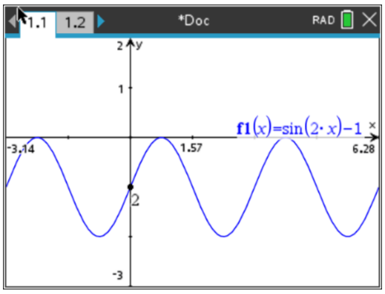
Mathematical Methods
Unit 3: FURTHER CALCULUS & INTEGRATION
SOLUTIONS

Question 1
Find the gradient of the curve $y = \sin(2x) - 1$ at (0,-1).

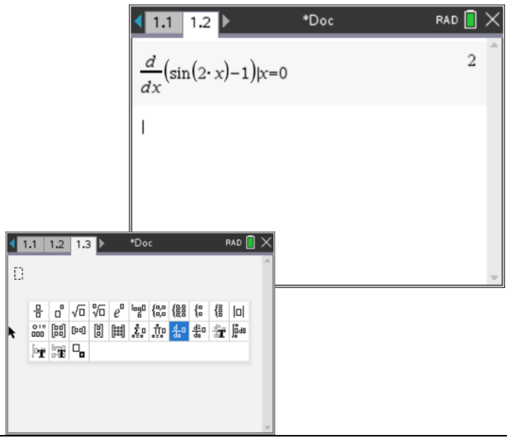
Gradient of graph is equal to the value of the derivative at the point (0,-1). Need to find derivative at point.
Gradient at (0,-1) = 2.

Option 1 – Graph Page:
Enter function
Menu - >Analyze Graph -> dy/dx
Place point at *x*=0





Option 2 – Calculator Page:
Use **template** key
Choose derivative, Enter function followed by condition *x*=0



Question 2
Find the equation of the normal to the curve $y = e^x + 2$ at $x = 0$.

Equation of normal $y = mx + c$

$m_n = \frac{-1}{m_t}$

$m_t = \frac{dy}{dx}\ at\ x = 0$

Graph Page:

$m_t = 1$

$m_n = \frac{-1}{m_t}$

$m_n = \frac{-1}{1}$

$m_n = -1$

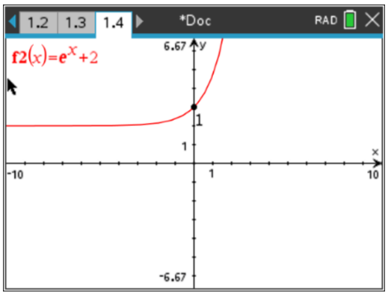
Point (0,2)

$y = mx + c$

$2 = -1 * 0 + c$

$2 = c$

Equation of normal is $y = -x + 2$



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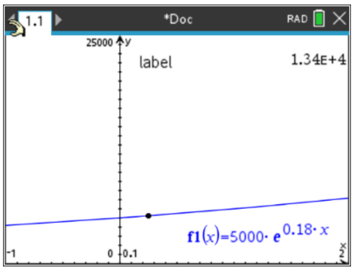
Question 3

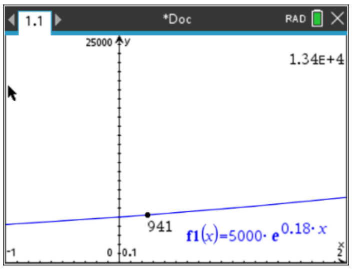
A population of bacteria after t hours is given by $P(t) = 5000e^{0.18t}$. Calculate the **rate** of increase of the population (to the nearest unit) at 15 minutes.

Rate of increase of population is the value of derivative at $t = 0.25$ (15min is ¼ of an hour)

Option 1 – Graph page:



Graph the function
Change the **window**



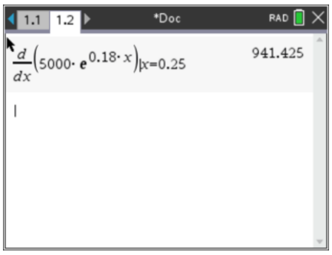
Menu->Analyze Graph->dy/dx
Place point at $x = 0.25$

Rate = 941

The rate of increase of the population is 941 bacteria/hour at 15min

Option 2 – Calculator page:

Use **template** key
Choose derivative, Enter function followed by condition x=0.25



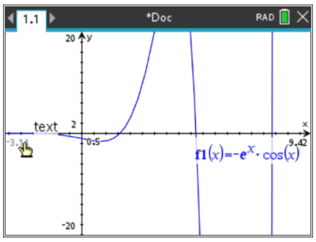
Rate = 941.425 given it is bacteria the solution is:

The rate of increase of the population is 941 bacteria/hour after 15min

Question 4

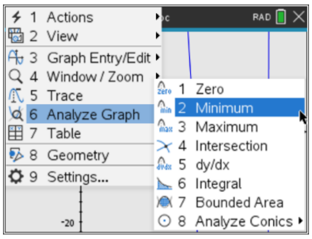
A particle moves along the x-axis with position at time t given by $x(t) = -e^t \cos(t)$ for $0 \le t \le 2\pi$. Calculate each time t for which the particle is at rest. (**hint**: use maximum and minimum)

Particle is at rest when the derivative = 0 (stationary point)



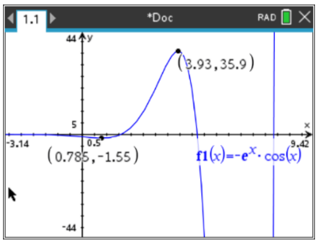
Graph the function
Change the **window** settings

Derivative = 0 at stationary points
Stationary points occur at maximums and minimums



Menu->6:**Analyze Graph**->2:**Minimum**
Identify boundaries

Menu->6:**Analyze Graph**->3:**Maximum**
Identify boundaries



The stationary points are at (0.785,-1.55) and (3.93,35.9)
The particle is at rest when t=0.785 and t=3.93

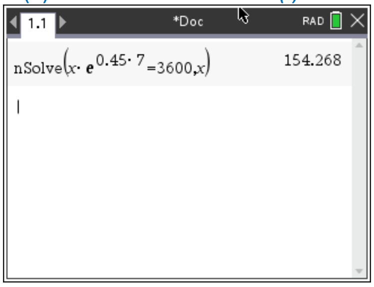
$t=\frac{\pi}{4}$ $t=\frac{5\pi}{4}$ (found by dividing x values by π)

Author: M Hourigan

Question 5

The number of rabbits increases according to the model $n(t) = Ae^{bt}$, where t is time in years, $n(t)$ is the population size at time t, A is the initial size of the population and b is the relative rate of growth.
Rabbits were introduced to a small island 7 years ago. The current rabbit population on the island is estimated to be 3600, with a relative growth rate of 45% per year.
Determine when the population is increasing at a rate of 5000 rabbits per year.

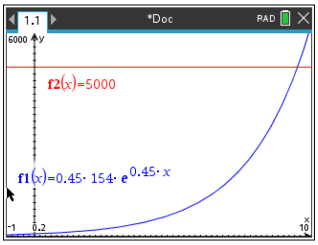
$n(t) = Ae^{bt}$ where t=7 $n(t)$ = 3600 and b=0.45 , find the value of A



Using **Calculator page**
Menu->3:**Algebra**->1:**Numerical Solve**
A is the initial size of the rabbit population so must be a whole number
A=154

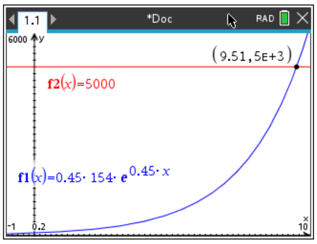
$n(t) = 154e^{0.45t}$
Determine when $n'(t) = 5000$
$n'(t) = 0.45 * 154e^{0.45t}$

Graph page



Enter derivative function
Enter f(x)=5000
Find **Point of Intersection**

Menu->6:**Analyze Graph**->4:**Intersection**

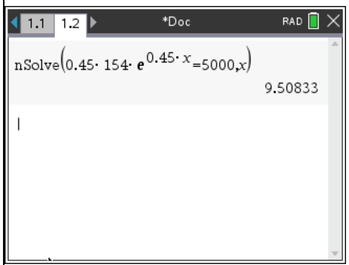


Intersection is at *t*=9.51

Therefore the population will be increasing at a rate of 5000 rabbits/year during the 10th year.

Calculator Page

Use **Numerical Solve**
Enter $0.45*154e^{0.45x}=5000$



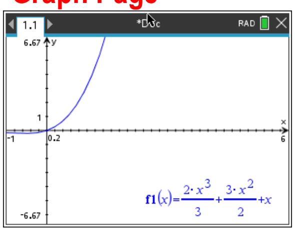
Solution is t=9.5
Therefore the population will be increasing at a rate of 5000 rabbits/year during the 10th year.

Question 6

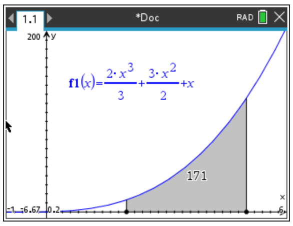
Rainwater is being collected in a water tank. The rate of change of volume, *V* litres, with respect to time, *t* seconds, is given by $\frac{dV}{dt} = \frac{2t^3}{3} + \frac{3t^2}{2} + t$. Determine the volume of water that is collected in the tank between *t=2* and *t=5.*

Integrate to find equation for Volume of water (*V*), then find V when t=2 and t=5, calculate the difference.
OR use Integral between t=2 and t=5

Graph Page

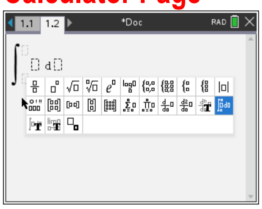


Graph function
Menu->6:**Analyze Graph**->6:**Integral**
Lowerboundary = 2
Upperboundary=5



Integral is 171
There would be 171L of water collected between 2 and 5 seconds.

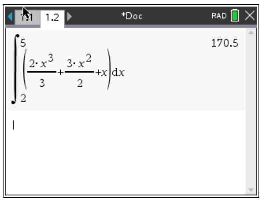
Calculator Page



Using **template** key

Choose **Integral**

Enter function with end points



Integral is 170.5
There would be approx. 171L of water collected.

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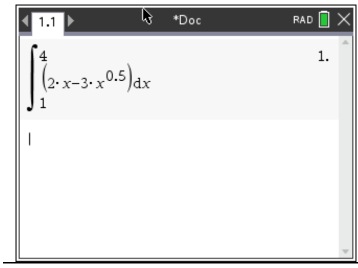
Question 7

Find the value of $\int_1^4 \left(2x - 3x^{\frac{1}{2}}\right) dx$

Calculator Page:



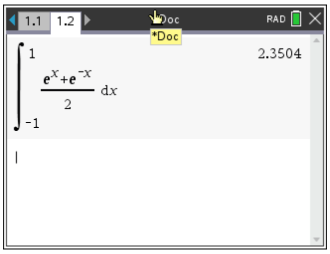
Use **template** key
Choose **Integral**
Enter integral

$$\int_1^4 \left(2x - 3x^{\frac{1}{2}}\right) dx = 1$$

Question 8

Find the value of $\int_{-1}^1 \frac{e^x+e^{-x}}{2}\, dx$

Calculator Page:



Use **template** key
Choose **Integral**
Enter integral

$$\int_{-1}^1 \left(\frac{e^x + e^{-x}}{2}\right) dx = 2.35$$

Question 9

A particle moves in a straight line. The velocity of the particle, *v* m/s, at time, *t* seconds, is given by *v*=*2t*−*3* for *t* ≥0. Find the particle's displacement after 4 seconds.

Displacement = $\int velocity$

Calculator Page:

Use **template** key
Choose **Integral**
Enter integral

$$\int_0^4 (2t - 3)dt = 4$$

The particle's displacement after 4 seconds is 4m

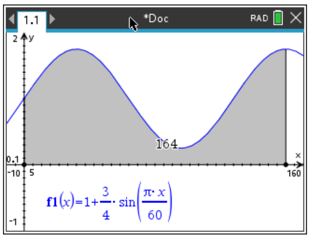
Question 10

Heat escapes from a storage tank at the rate of $\frac{dH}{dt} = 1 + \frac{3}{4}\sin(\frac{\pi t}{60})$ kilojoules per day. If H(t) is the total accumulated heat loss at time *t* days, find the amount of heat lost in the first 150 days.

Amount of heat lost in first 150days = $\int_0^{150} \frac{dH}{dt}\, dt$

Author: M Hourigan

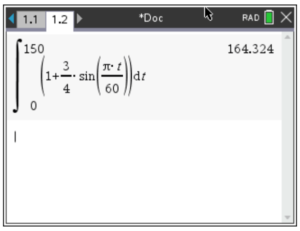
Graph Page:



Graph function
Menu->6:**Analyze Graph**->6:**Integral**
Lowerboundary = 0
Upperboundary=150

During the first 150 days, 164kJ of heat was lost.

Calculator Page:



Use **template** key
Choose **Integral**
Enter integral

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