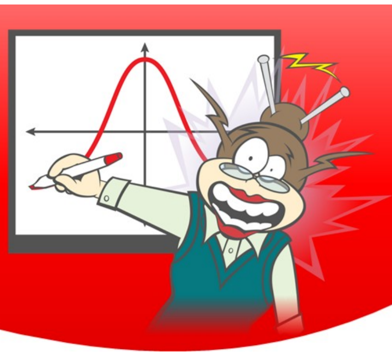

Complex Numbers Part 2

Question: 1.

If $z = a\,cis\left(\frac{\pi}{b}\right)$, $a, b \neq 0$, then $(\bar{z})^{-1}$ is equal to:

A. $\frac{1}{a}cis\left(\frac{\pi}{b}\right)$

B. $\frac{1}{a}cis\left(-\frac{\pi}{b}\right)$

C. $\frac{1}{a}cis\left(\frac{b}{\pi}\right)$

D. $\frac{1}{a}cis\left(-\frac{b}{\pi}\right)$

E. $\bar{a}\,cis\left(\frac{b}{\pi}\right)$

Question: 2.

If $z = -a + ai$ where $a > 0$ then $\mathrm{Arg}\left(z^3\right)$ is equal to:

A $\frac{27\pi^3}{64}$

B $\frac{\pi}{4}$

C $-\frac{\pi}{4}$

D $\frac{9\pi}{4}$

E $\frac{3\pi}{4}$

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Question: 3.

If $w^2 = 16cis\left(\frac{\pi}{3}\right)$ then a possible value of w is:

A. $4cis\left(\frac{\pi}{6}\right)$ B. $4cis\left(\frac{2\pi}{3}\right)$ C. $8cis\left(\frac{\pi}{6}\right)$ D. $16cis\left(\frac{\pi}{6}\right)$ E. $32cis\left(\frac{2\pi}{3}\right)$

Question: 4.

In the complex plane, the point $2-i$ lies on the graph of the relation

A. $\text{Arg}(z) = \frac{\pi}{6}$

B. $|z| = |z+1|$

C. $2\,\text{Re}(z) = \text{Im}(z)$

D. $|z-2| = 1$

E. $(\bar{z})^2 = 2z$

Question: 5.

Sets of points in the complex plane are defined by

$$S = \{z : |z+1-2i| = 5\} \text{ and } T = \{z : \text{Re}(z) + 2\,\text{Im}(z) = 8\}.$$

Find the coordinates of the points of intersection between S and T.

Question: 6.

Find cube roots of $-27i$. Give your answer in the form $a+bi$, $a,b \in R$.

Question: 7.

Find the values of n for which: $(\sqrt{3}+i)^n - (\sqrt{3}-i)^n = 0$.

Question: 8.

Given that $z = (b+i)^2$, $b \in R^+$, find the value of b when $\text{Arg}(z) = \frac{\pi}{6}$.

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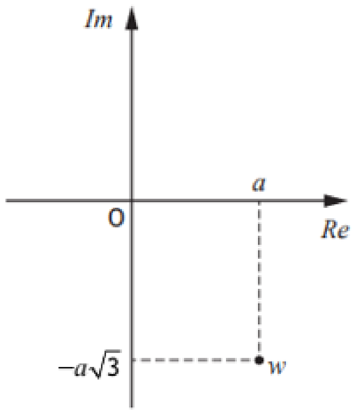
Question: 9.

Given that $|z| = 2\sqrt{5}$, find the complex number z that satisfies the equation

$$\frac{25}{z} - \frac{15}{\overline{z}} = 1 - 8i$$

Question: 10.

The complex number w has been plotted on an Argand diagram, as shown below.



where $a > 0$.

a) Express w in Cartesian form and in polar form.

The complex number z_1 is a root of $z^3 = w$, where $z_1 = k\,cis\left(\frac{\pi}{m}\right)$ for $k, m \in Z$.

Given that $a = 4$,

b) determine the values of k and m,
c) find the remaining roots.

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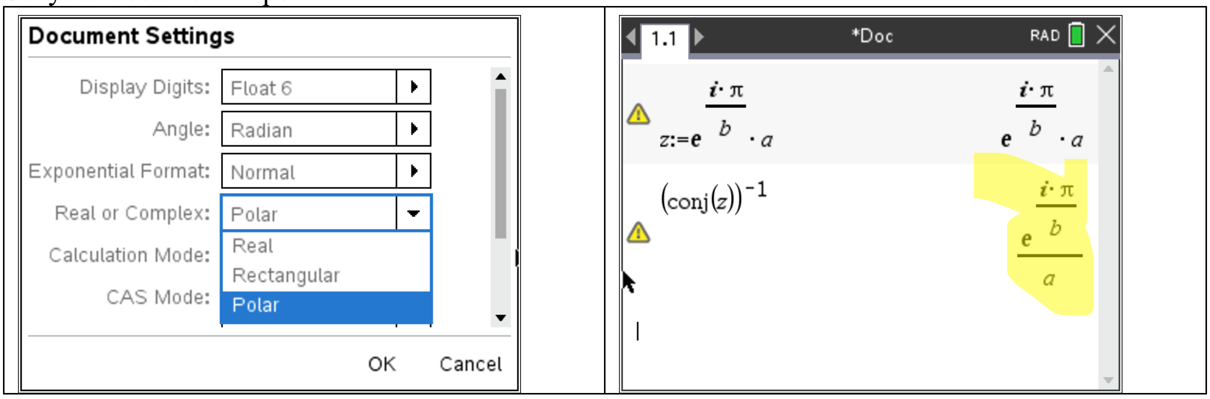
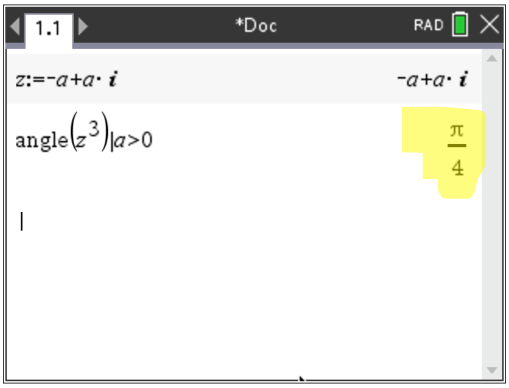
Answers

Question 1 Answer: A

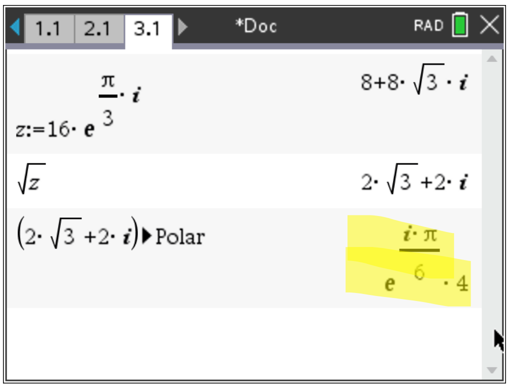
Set your document in polar



Question 2 Answer: B



Question 3 Answer: A



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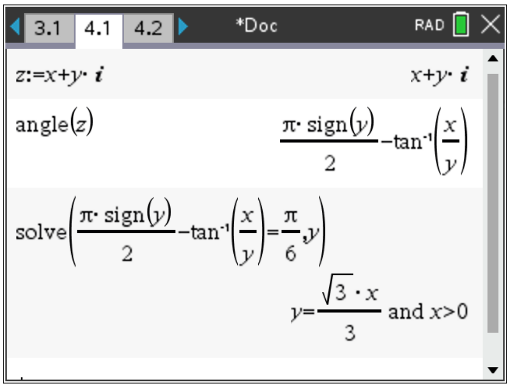
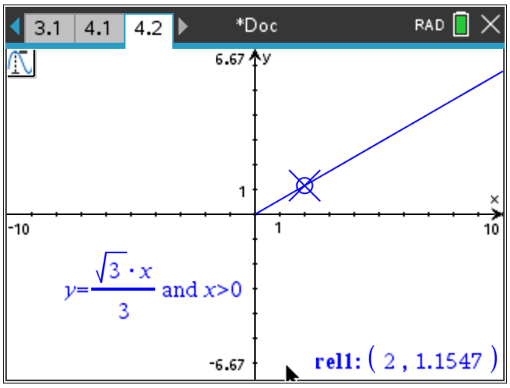
Question 4 **Answer: D**

Method 1

Draw a ray in answer A

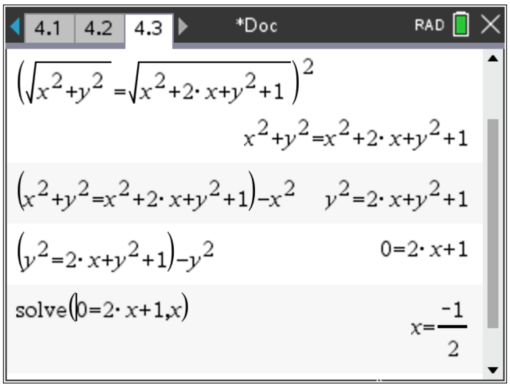


Draw in Relations and Trace, enter x=2



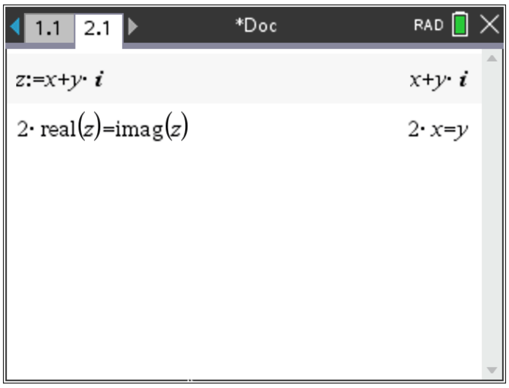
A is incorrect

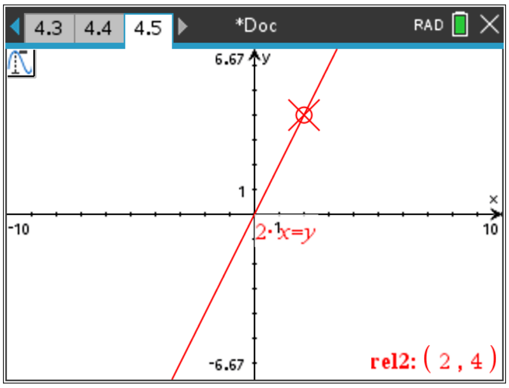
Check B (perpendicular bisector)



B is incorrect

Check C





C is incorrect

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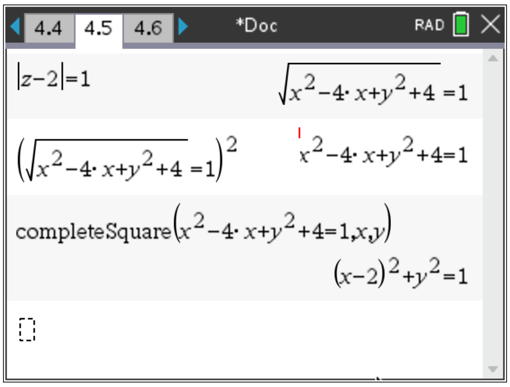
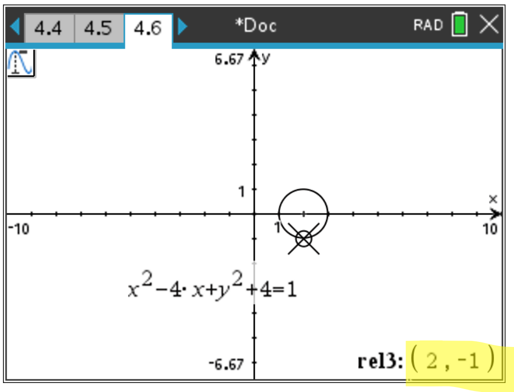
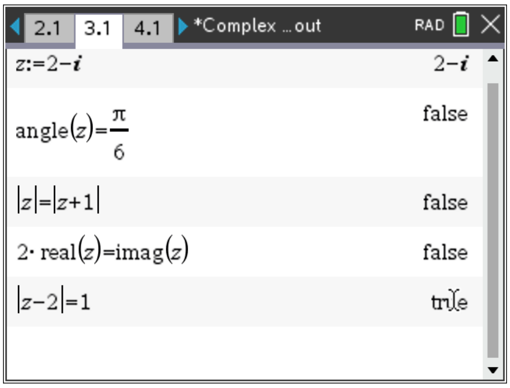
Check D





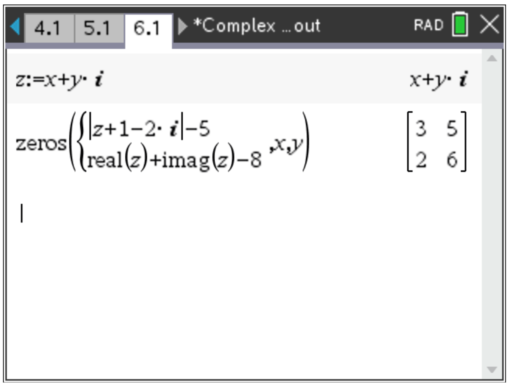
D is correct

Method 2



Question 5

$(2,6)$ and $(3,5)$



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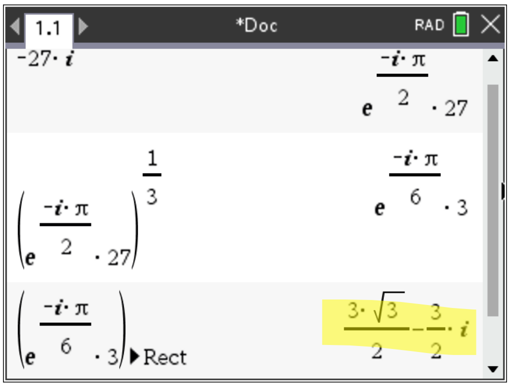
Question 6

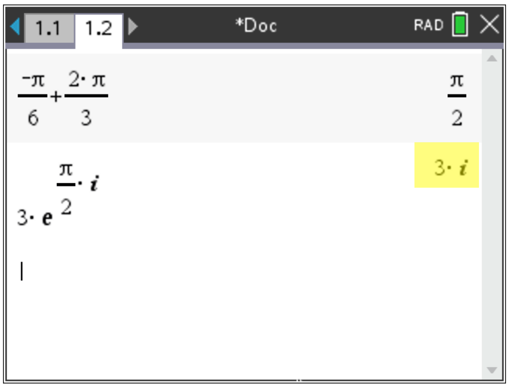
$$\frac{3\sqrt{3}}{2}-\frac{3}{2}i,\ 3i,\ -\frac{3\sqrt{3}}{2}-\frac{3}{2}i$$

Find the first cube root. Start in polar form and convert to Rectangular

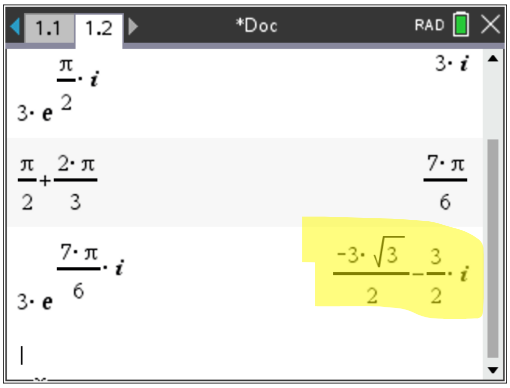


Find the other two roots in polar form and convert to cartesian.

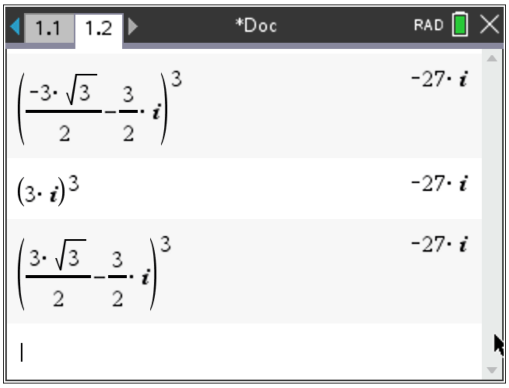
Second root:



Third root:

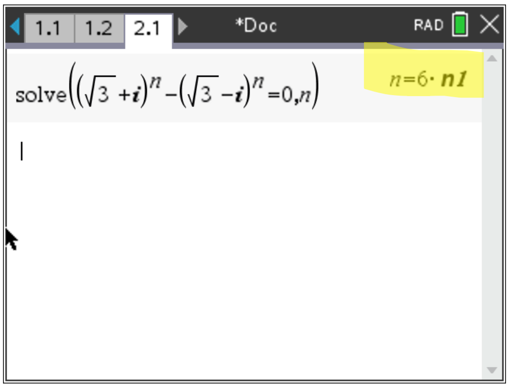


Check:

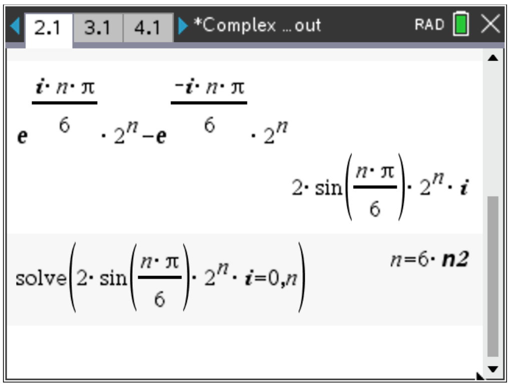


Question 7

$n = 6k,\ k \in Z.$

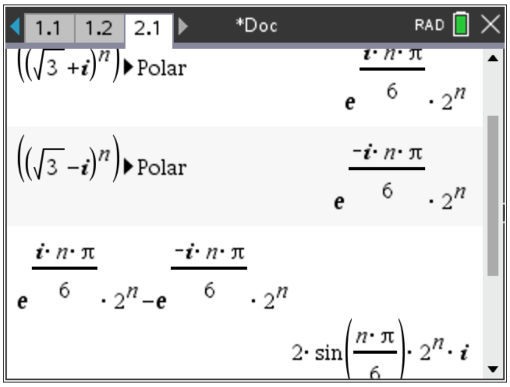


Deduce that it only has an imaginary part, real parts cancel out. Therefore, the equation is zero when



which gives us the same answer as a general solution to a trig equation.

Alternatively:
Convert to polar first, subtract to see the result.



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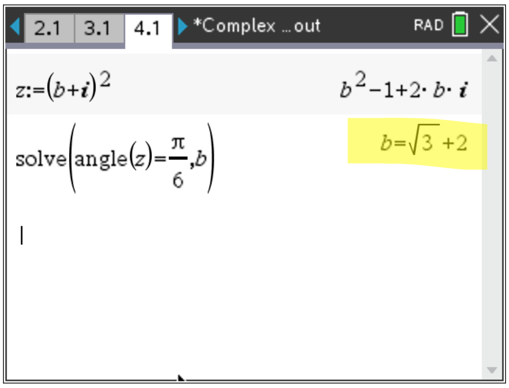
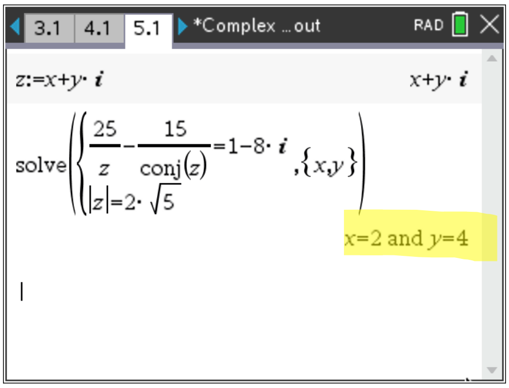
Question 8

$b = 2 + \sqrt{3}$



Question 9

$z = 2 + 4i$



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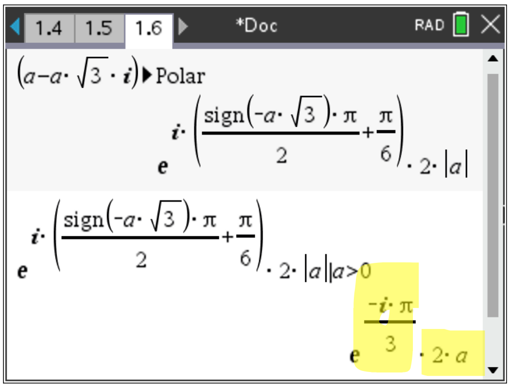
Question 10

a) $w = a - a\sqrt{3}\,i;\ \ w = 2a\,cis\left(-\frac{\pi}{3}\right)$.

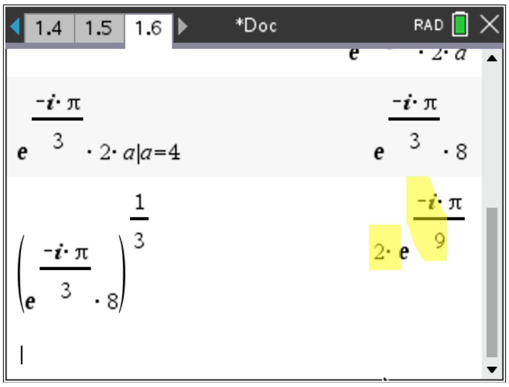
b) $k = 2,\ m = -9$

c) $z_2 = 2\,cis\left(\frac{5\pi}{9}\right),\ z_3 = 2\,cis\left(-\frac{7\pi}{9}\right)$

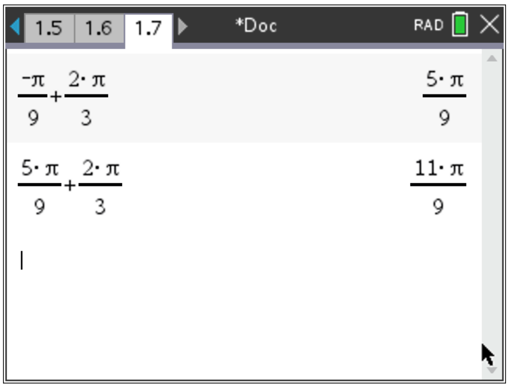
a) Convert to polar with condition $a>0$.



b) With CAS in polar setting, set $a=4$ and then raise the obtained answer to the power of 1/3.



c) The remaining roots are evenly spread over the circle with radius 2 every $\frac{2\pi}{3}$:



and express with a principal argument.

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