STUDENT REVISION SERIES

Complex Numbers Part 1

Question: 1.

If P(z) is a polynomial of degree 4 with all its coefficients real with ai, $bi(a, b \in R)$ as two of its zeros, then the term that does not contain z is:

A. ab B. a-b C. a+b D. $a^{3}b^{3}$ E. $a^{2}b^{2}$

Question: 2.

If
$$P(z) = z^3 + 2z^2 - 6z + a$$
 and $P(1-i) = 0$, then *a* is equal to:
A. 4 B. -6 C. 8 D. -8 E. 5

Question: 3.

The polynomial P(z) has real coefficients, and z = 2 - i is a root of P(z).

Which quadratic polynomial must be a factor of P(z)?

A. $z^2 - 4z + 5$ B. $z^2 + 4z + 5$ C. $z^2 - 4z + 3$ D. $z^2 + 4z + 3$ E. $z^2 - 4$

Question: 4.

Let z = x + yi, where x and y are real numbers, which are not both zero. Which one of the following expressions does not necessarily represent a real number?

A. $z\overline{z}$ B. $z^{-1}z$ C. z^2 D. Im(z) E. $z + \overline{z}$

Question: 5.

Solve the following equations over C:

a) $z^4 - 5z^2 - 6 = 0$ b) $z^3 - 8 = 0$

Question: 6.

Find the sum of the roots of $z^3 - z^2 + 3z + 5 = 0$ over C.

Question: 7.

The set of points in the complex plane described by $\{z : \operatorname{Im}\left(\frac{z+ai}{z+b}\right) = 0\}$ where $a, b \in R$

represents:

- A a straight line.
- **B** a circle.
- C an ellipse.
- **D** a hyperbola.
- E a parabola.

Question: 8.

- a) Find a polynomial function with real coefficients and with zeros 3, 1 2i and 2 + i.
- b) Find a quadratic function with roots 4 and 3-i.

Question: 9.

The complex numbers z_1 and z_2 are given by

$$z_1 = p + 2i$$
 and $z_2 = 1 - 2i$, $p \in Z$.

a) Find
$$\frac{z_1}{z_2}$$
 in the form $a+bi$, $a,b \in R$.

b) Given that $\left| \frac{z_1}{z_2} \right| = 13$, find the possible values of *p*.

Question: 10.

Given that p and q are real and that 1+2i is a root of the equation

$$z^{2} + (p+5i)z + q(2-i) = 0$$

determine:

- a) the values of p and q,
- b) the other root of the equation.

Answers



Question 2 Answer: C



Question 3 Answer: A



Question 4 Answer: C

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z:=x+y∙ i				х+ <i>у</i> • <i>i</i>	
z+conj(z)				2• x	I
z• conj(z)				$x^{2}+y^{2}$	l
▲ z ⁻¹ ·z				1	l
z ²				$x^2 - y^2 + 2 \cdot x \cdot y \cdot i$	k
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Question 5

Use Algebra Complex solve:

a)
$$z = \pm i, \pm \sqrt{6}$$

$$4.1 5.1 6.1 \rightarrow Doc \qquad RAD \rightarrow C$$

$$cSolve(z^{4}-5 \cdot z^{2}-6=0,z)$$

$$z=i \text{ or } z=-i \text{ or } z=-\sqrt{6} \text{ or } z=\sqrt{6}$$

$$cSolve(z^{3}-8=0,z)$$

$$z=-1-\sqrt{3} \cdot i \text{ or } z=-1+\sqrt{3} \cdot i \text{ or } z=2$$

$$|$$

Question 6

Sum is 1.

Use cZeros and sum:



b) $z = -1 \pm \sqrt{3}i$ or z = 2

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Question 7 Answer: A



Question 8

a) $p(z) = z^5 - 9z^4 + 36z^3 - 84z^2 + 115z - 75$



b) $q(z) = z^2 + (-7+i)z + 12 - 4i$

◀ 7.1	8.1	9.1	•	*Doc			RAD 📘	×
expar	nd((z-	4)• (z	-3+i))) z	² +i•z-7	• <i>z</i> -4	• <i>i</i> +12	
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Author: B Graham

Question 9

a)
$$\frac{z_1}{z_2} = \frac{p-4}{5} + \frac{2p+2}{5}i$$

 ● 9.1 9.2 10.1 ▶ 	*Doc	RAD 📘	×
z1:=p+2· i		p +2∙ <i>i</i>	
z2:=1-2· i		1-2· <i>i</i>	
$\frac{z1}{z2}$	$\frac{p}{5} - \frac{4}{5}$	$\left(\frac{2 \cdot p}{5} + \frac{2}{5}\right) \cdot i$	
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b) p = -29 or p = 29

9.1 9.2 10.1 ▶ *Do	c RAD 🚺 🗙
$\frac{z_1}{z_2}$	$\frac{p}{5} - \frac{4}{5} + \left(\frac{2 \cdot p}{5} + \frac{2}{5}\right) \cdot i$
$\left \frac{z1}{z2}\right $ =13	$\frac{\sqrt{5 \cdot \left(p^2 + 4\right)}}{5} = 13$
solve $\left(\frac{\sqrt{5 \cdot \left(p^2 + 4\right)}}{5} = 13 p\right)$	<i>p</i> =-29 or <i>p</i> =29
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Question 10

a)
$$p = -1, q = 7$$
 b) $-7 + i$

a) Define the polynomial. Note, cannot use p(z) as we have p in the equation, so call it something else:



Find the value of the polynomial in 1+2i.

As it is a root, then p(z)=0, and solve to obtain the values of the pronumerals. b) Determine the polynomial for the obtained values of *p* and *q* and cSolve for zero to find the other root.

