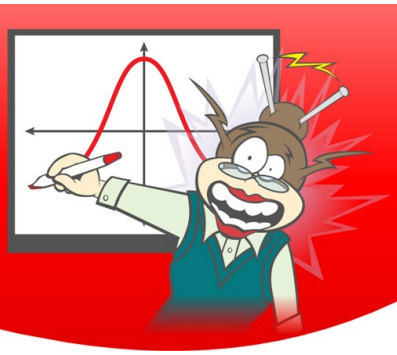

Complex Numbers Part 1

Question: 1.

If $P(z)$ is a polynomial of degree 4 with all its coefficients real with ai, $bi\left(a,b\in R\right)$ as two of its zeros, then the term that does not contain z is:

A. ab B. $a-b$ C. $a+b$ D. a^3b^3 E. a^2b^2

Question: 2.

If $P(z)=z^3+2z^2-6z+a$ and $P(1-i)=0$, then a is equal to:

A. 4 B. –6 C. 8 D. -8 E. 5

Question: 3.

The polynomial $P(z)$ has real coefficients, and $z=2-i$ is a root of $P(z)$.

Which quadratic polynomial must be a factor of $P(z)$?

A. z^2-4z+5
B. z^2+4z+5
C. z^2-4z+3
D. z^2+4z+3
E. z^2-4

Question: 4.

Let $z=x+yi$, where x and y are real numbers, which are not both zero. Which one of the following expressions does not necessarily represent a real number?

A. $z\overline{z}$ B. $z^{-1}z$ C. z^2 D. $\mathrm{Im}(z)$ E. $z+\overline{z}$

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Question: 5.

Solve the following equations over C:

a) $z^4 - 5z^2 - 6 = 0$ b) $z^3 - 8 = 0$

Question: 6.

Find the sum of the roots of $z^3 - z^2 + 3z + 5 = 0$ over C.

Question: 7.

The set of points in the complex plane described by $\{z : \mathrm{Im}\left(\frac{z+ai}{z+b}\right) = 0\}$ where $a, b \in R$

represents:

A a straight line.

B a circle.

C an ellipse.

D a hyperbola.

E a parabola.

Question: 8.

a) Find a polynomial function with real coefficients and with zeros 3, $1 - 2i$ and $2 + i$.
b) Find a quadratic function with roots 4 and $3 - i$.

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Question: 9.

The complex numbers z_1 and z_2 are given by

$$z_1 = p + 2i \text{ and } z_2 = 1 - 2i,\ p \in Z.$$

a) Find $\frac{z_1}{z_2}$ in the form $a + bi,\ a, b \in R$.

b) Given that $\left|\frac{z_1}{z_2}\right| = 13$, find the possible values of p.

Question: 10.

Given that p and q are real and that $1 + 2i$ is a root of the equation

$$z^2 + (p + 5i)z + q(2 - i) = 0$$

determine:

a) the values of p and q,

b) the other root of the equation.

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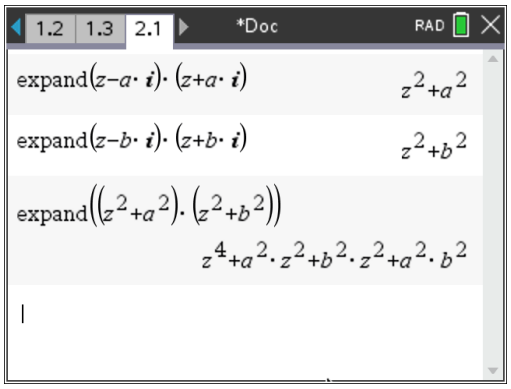
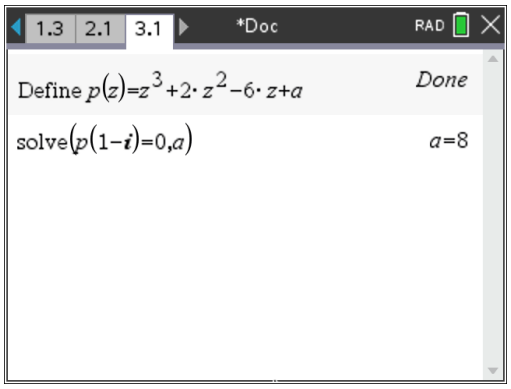
Answers

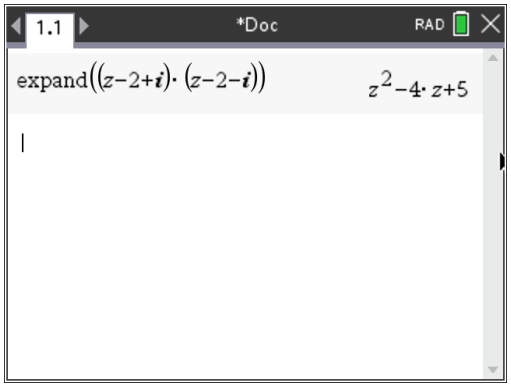
Question 1 Answer: E



Question 2 Answer: C



Question 3 Answer: A



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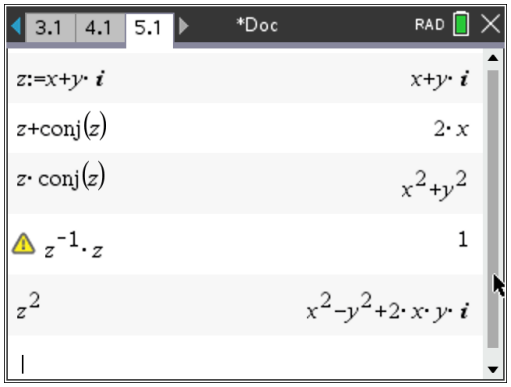
Question 4 Answer: C

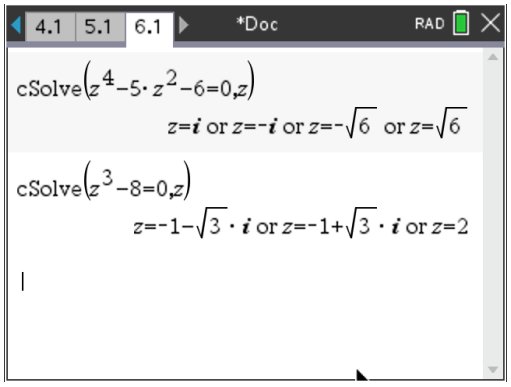


Question 5

Use Algebra Complex solve:

a) $z = \pm i, \pm\sqrt{6}$

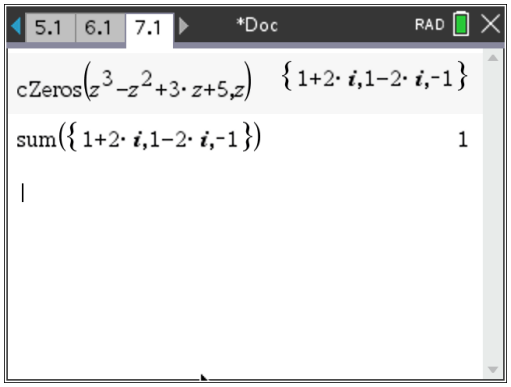
b) $z = -1 \pm \sqrt{3}i$ or $z = 2$



Question 6

Sum is 1.

Use cZeros and sum:



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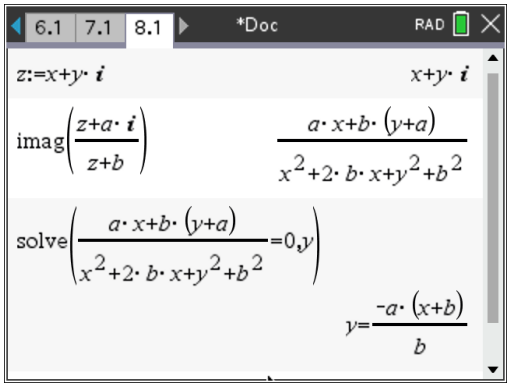
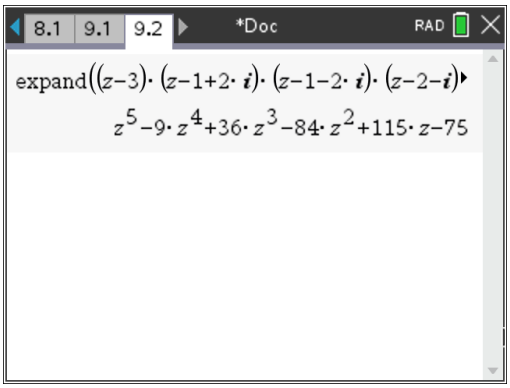
Question 7 Answer: A

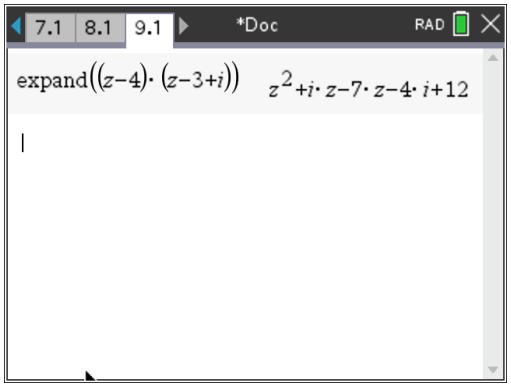


Question 8

a) $p(z) = z^5 - 9z^4 + 36z^3 - 84z^2 + 115z - 75$



b) $q(z) = z^2 + (-7+i)z + 12 - 4i$



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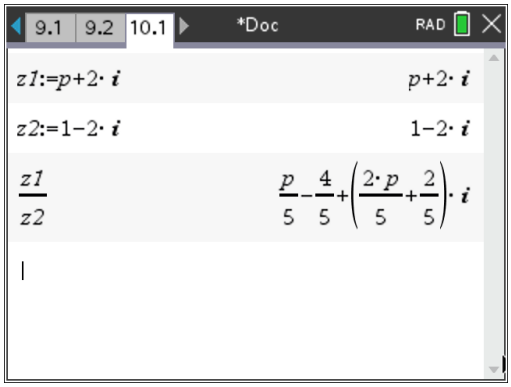
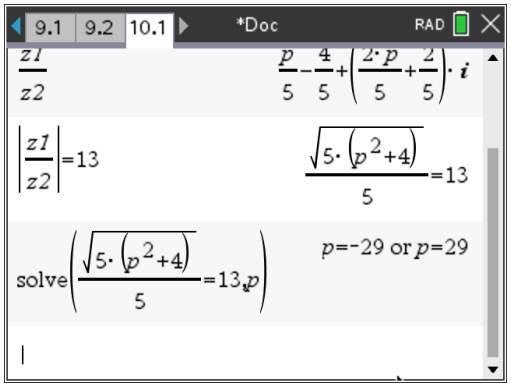
Question 9

a) $\dfrac{z_1}{z_2} = \dfrac{p-4}{5} + \dfrac{2p+2}{5}i$



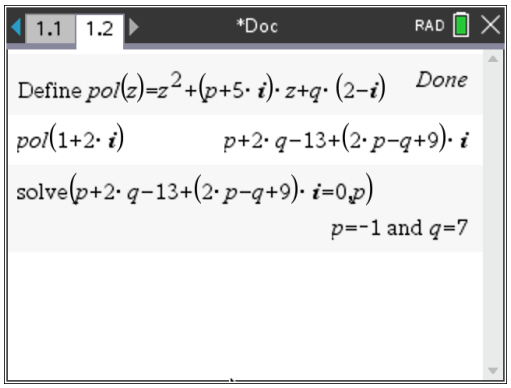
b) $p = -29$ or $p = 29$



Question 10

a) $p = -1, q = 7$ b) $-7 + i$

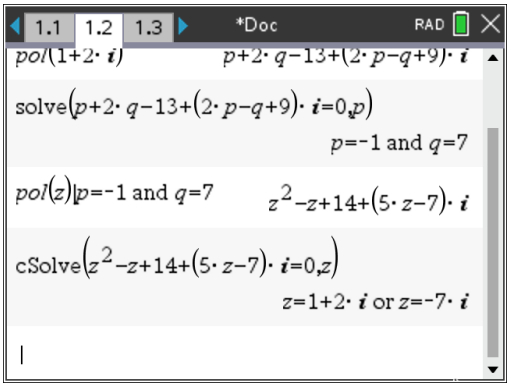
a) Define the polynomial. Note, cannot use $p(z)$ as we have p in the equation, so call it something else:



Find the value of the polynomial in $1+2i$.

As it is a root, then $p(z)=0$, and solve to obtain the values of the pronumerals.

b) Determine the polynomial for the obtained values of p and q and cSolve for zero to find the other root.



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