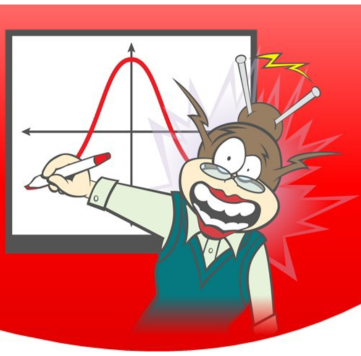
STUDENT REVISION SERIES



Complex Numbers Part 1

Question 1.

Let $z = cis\left(\frac{2\pi}{5}\right)$.

$Im(z^4)$ is

A. -0.95 B. 0.09 C. 0.31 D. 0.81

Question 21.

If $z = -a + ai$ where $a > 0$ then $Arg(z^3)$ is equal to:

A. $\frac{27\pi^3}{64}$ B. $\frac{\pi}{4}$ C. $-\frac{\pi}{4}$ D. $\frac{9\pi}{4}$ E. $\frac{3\pi}{4}$

Question 3

If $w^2 = 16cis(\frac{\pi}{3})$ then a possible value of w is:

A. $4cis(\frac{\pi}{6})$ B. $4cis(\frac{2\pi}{3})$ C. $8is(\frac{\pi}{6})$ D. $16cis(\frac{\pi}{6})$ E. $32cis(\frac{2\pi}{3})$

Question 4.

Express $\frac{\sqrt{10}}{2}(1 - i)$ in polar form.

A. $\sqrt{5}cis(\frac{\pi}{4})$ B. $\sqrt{5}cis(\frac{-\pi}{4})$ C. $-\sqrt{5}cis(\frac{-\pi}{4})$ D $cis(\frac{-\pi}{4})$ E. $\sqrt{10}cis(\frac{-\pi}{4})$

Question 5.

Convert $\sqrt{3}cis(-\frac{2\pi}{3})$ to Cartesian form

A. $\frac{-\sqrt{3}}{2} - \frac{3}{2}i$ B. $\frac{\sqrt{3}}{2} - \frac{3}{2}i$ C. $\frac{-\sqrt{3}}{2} + \frac{3}{2}i$ D. $\frac{-2\sqrt{3}}{25} - \frac{3}{5}i$ E $\frac{-\sqrt{3}}{5} - \frac{3}{5}i$

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Question 6.

Multiplying a non-zero complex number by $\frac{1+i}{1-i}$ results in a rotation about the origin on an argand diagram. What is the rotation?

A. Clockwise by $\frac{\pi}{4}$
B. Clockwise by $\frac{\pi}{2}$
C. Anticlockwise by $\frac{\pi}{4}$
D. Anticlockwise by $\frac{\pi}{2}$
E. Clockwise by π

Question 7.

Which of the following is the principal argument of $\frac{-4+4\sqrt{3}i}{-\sqrt{2}+\sqrt{2}i}$

A. $\frac{\pi}{12}$
B. $\frac{11\pi}{12}$
C. $\frac{-13\pi}{12}$
D. $\frac{13\pi}{12}$
E. $\frac{-\pi}{12}$

Question 8.

Given that $z_1 = 2 + 2i$ and $z_2 = p - 8i, p \in R$, find:

a) $z_1\overline{z_2}$ in terms of p

b) The value of p such that $z_1\overline{z_2}$ is purely imaginary

Question 9.

Let $z = 1 + i\sqrt{3}$

a) Express z in polar form

b) Show that z^9 is real

c) For what other values of n is z^n real.

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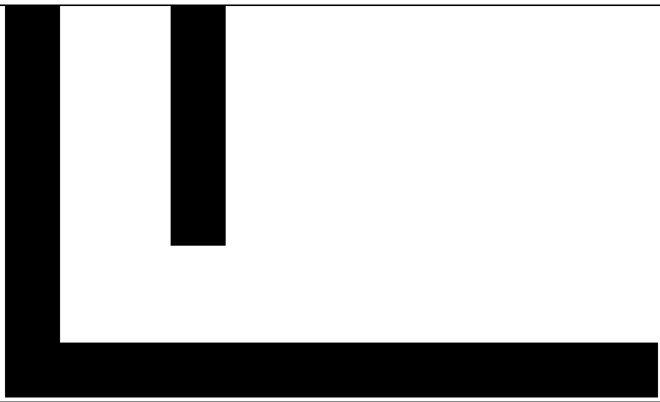
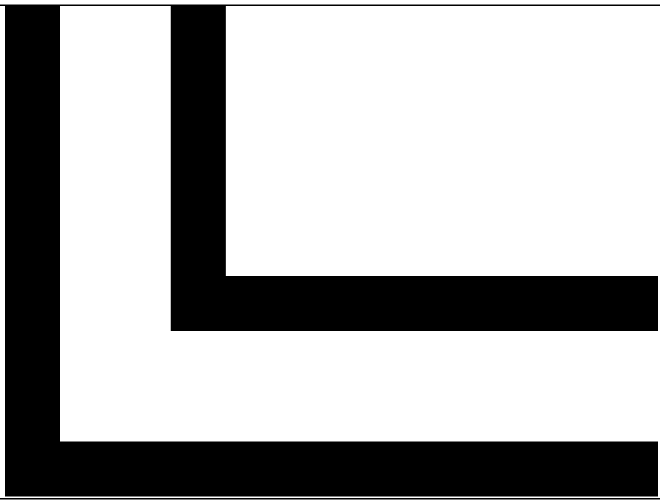
Answers

1. A	2. E	3. A	4. B	5. A	6. D	7. E

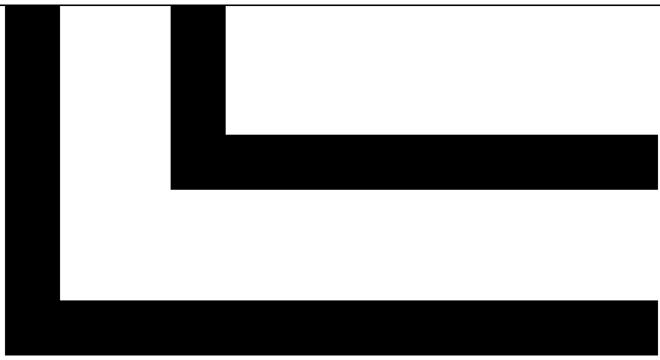
Question 1. Answer A

Enter in polar form as shown. Automatically converted to Cartesian form. Calulate z4 and read imaginary part -0.95 Or use the imaginary value function.	

Question 2. Answer **E**

$z = -a + ai$ $z = a(-1 + i)$ a>0 so angle determined by (-1+i) Express using π notation to march alternatives or change angle setting to degrees and convert to radians using π/180. Alternatively use reasoning and sketch – second quadrant – real & imaginary components equal in magniture therefore 3π/4	

Question 3. Answer A

Angle Settings in radians Enter value given as shown – expressed in rectangular form automatically Take sqrt wo find w. Change settings to degrees and convert to polar.	

Question 4. Answer B

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Question 5. Answer A

.	$\sqrt{3}\cdot e^{\frac{-2\cdot\pi}{3}\cdot i}$ $-0.866025-1.5\cdot i$

Question 6. Answer D

Geometrically, the effect of multiplying any complex number by the complex number $z = r\,\text{cis}\,\theta$ is to produce an anticlockwise turn through an angle θ about the origin. There anticlockwise rotation of $\frac{\pi}{2}$	

Question 7. Answer E

Enter expression as given Set to degrees Use angle function Answer in radians.	**Similar to Q6 above**

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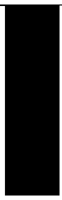
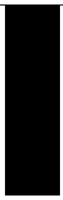
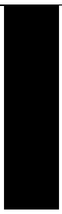
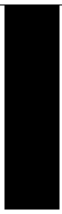
Question 8.

$z_1\overline{z_2} = (2+2i)(p+8i)$ $= ((2p-16)+(16+2p)i$ Require real part = 0 $2p-16=0$ $p=8$ Use calculator for checking if required	 

Question 9.

a) $z = 2cis(\frac{\pi}{3})$ b) $z^9 = 2^9 cis\left(\frac{9\pi}{3}\right)$ De Moivre's Th $z^9 = 512cis(3\pi)$ $z^9 = 512(cos(3\pi) + i\sin(3\pi))$ But $\sin(3\pi)=0$ so z^9 is real c) $z^n = 2^n cis\left(\frac{n\pi}{3}\right)$ $\sin(\theta) = 0$ for all multiples of pi therefore z^n will be real when n is a multiple of 3	  (

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