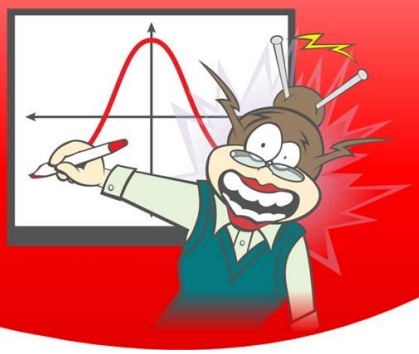


AREAS UNDER & BETWEEN CURVES

Each of the questions included here can be solved using TI-Nspire CX.

Question 1

Find the area of the region enclosed by the graphs of $y = x\sqrt{x+1}$ and $y = 2x$.

Question 2

Find the area enclosed by $f(x) = 24 - 2x - 2x^2$ and the *x*-axis.

Question 3

Find the area enclosed by the graph of $f(x) = e^{5x} - 2\sin(4x)$, the *x*-axis and the end points $x = -1$ and $x = 1$.

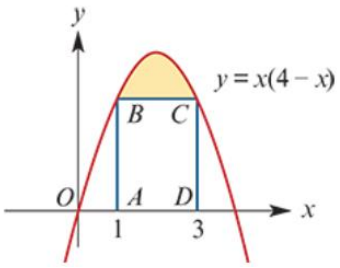
Question 4

Determine the signed area and the physical enclosed by the graph of $f(x) = (x-2)(x-4)(x+1)^2$ and the *x*-axis.

Author: M Hourigan

Question 5

For the graph shown at the right, find the area of the shaded region.



Question 6

For the function $y = \ln(5x + e) - 1$, determine the area under the curve between $x = 0\ and\ x = 29$. Then decide where a vertical line should be placed to divide this area exactly in half?

Question 7

Use the trapezoidal rule with *n*=5 to approximate the area under the curve $f(x) = \frac{1}{x}$ from $x = 2\ to\ x = 3$.

Question 8

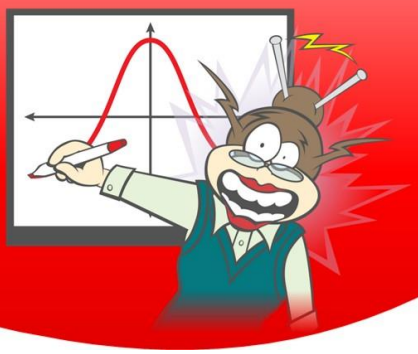
The size of a bacterial colony on an agar plate increases with time according to the formula: $G(t) = \frac{15}{t^2+1}$ where *G(t)* is the increase in the area covered (cm^2) after *t* hours. Using the trapezoidal rule, find the area covered after 5 hours.

Author: M Hourigan

Questions used in this worksheet were sourced from/inspired by:

- https://www.qcaa.qld.edu.au/senior/senior-subjects/mathematics/mathematics-methods/assessment
- Mathematical Methods Units3 & 4 for Queensland, Cambridge University Press

Author: M Hourigan



Mathematical Methods

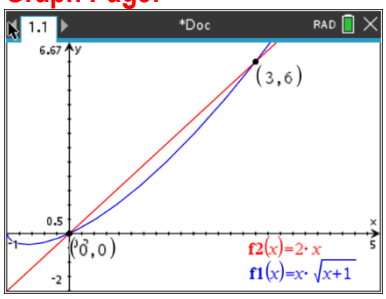
Unit 3: AREAS UNDER & BETWEEN CURVES

SOLUTIONS

Question 1

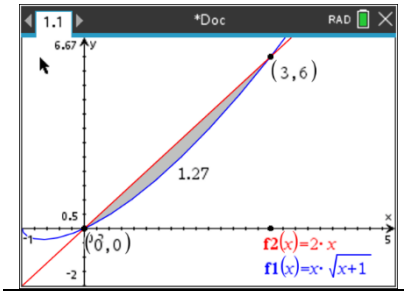
Find the area of the region enclosed by the graphs of $y = x\sqrt{x+1}$ *and* $y = 2x$

Graph Page:



Enter the 2 functions

Identify points of intersection
Menu->6:**Analyze Graph**->4:**Intersection**



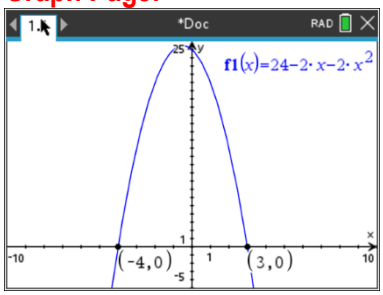
Identify the Bounded Region
Menu->6:**Analyze Graph**->7:**Bounded Area**
Lower Boundary=0
Upper Boundary = 3

Enclosed area = 1.27 units^2

Question 2

Find the area enclosed by $f(x) = 24 - 2x - 2x^2$ and the *x*-axis.

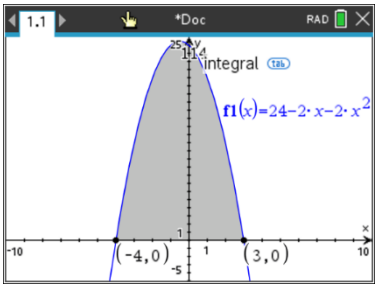
Graph Page:



Enter the function
Find the *x*-intercepts (zeros)
Menu->6:**Analyze Graph**->1:**Zeros**

Select Integral from Menu
Menu->6:**Analyze Graph**-
Use *x*-intercepts as the lower
Lower Boundary = -4
Upper Boundary = 3

Area = 114 units^2



>6:**Integral**
and upper boundary

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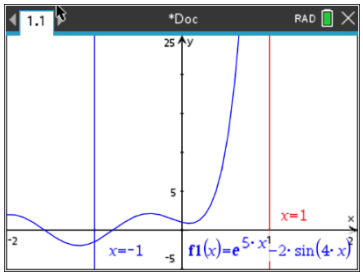
Question 3

Find the area enclosed by the graph of $f(x) = e^{5x} - 2\sin(4x)$, the *x*-axis and the end points $x = -1 \; and \; x = 1$.

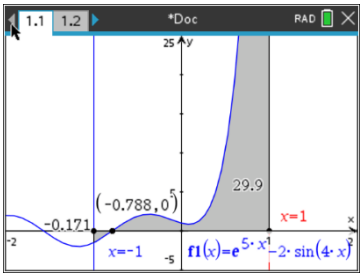
Graph Page:



Graph the function
(Also graphed are the relations *x*=-1 and *x*=1 – not required but helpful)

Area is both above and below the *x*-axis, area will be calculated in 2 parts.
Identify *x*-intercept

Menu->6:Analyze Graph-
x-intercept = -0.788



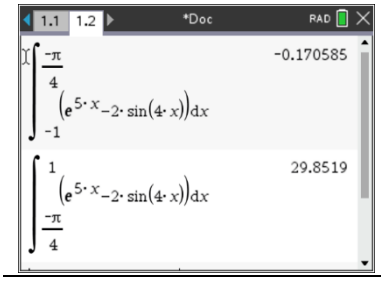
>1:**Zeros**
(approx. $\frac{-\pi}{4}$)

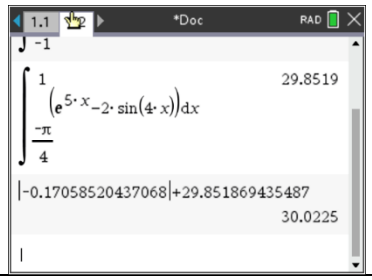
Find Area Under each part of
Menu->6:Analyze Graph-

curve
>6:**Integral**

Enclosed area = |-0.171|+29.9
= 30.071

Calculator Page:



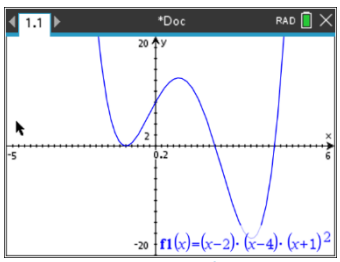


Enclosed area = 30.0225

Question 4

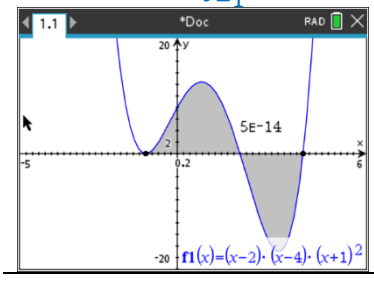
Determine the signed area and the physical enclosed by the graph of $f(x) = (x-2)(x-4)(x+1)^2$ and the *x*-axis

Graph Page:



Graph the function
As the function is in factorised form the *x*-intercepts are -1, 2, 4

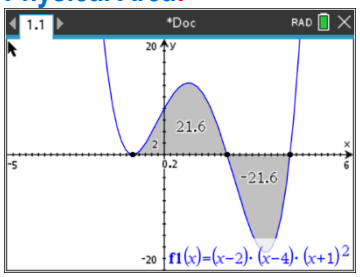
Signed Area = $\int_{-1}^{4} f(x)dx$



Menu->6:Analyze Graph-
>6:Integral
Lower Boundary = -1
Upper Boundary = 4

Area = $5*10^{-14}$
Approx. 0 units^2

Physical Area:



Calculate area in 2 parts

Area = 21.6+|-21.6|
= 43.2units^2

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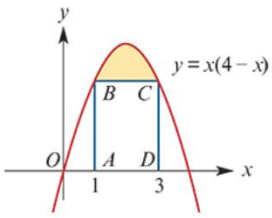
Question 5

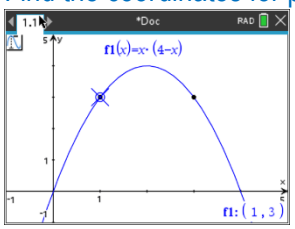
For the graph shown at the right, find the area of the shaded region



Graph Page:

Graph Function

Find the coordinates for points B (x=1) and C (x=3)



Menu->5:**Trace**

x=1

point = (1,3)

Therefore line BC is at y=3

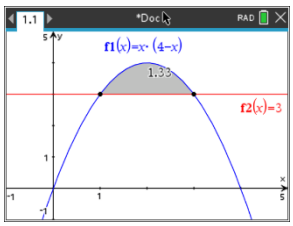
Graph y=3

Use Area Between Curves

Menu->6:**Analyze** **Graph**->7:**Bounded Area**

Lower Boundary = 1

Upper Boundary = 3

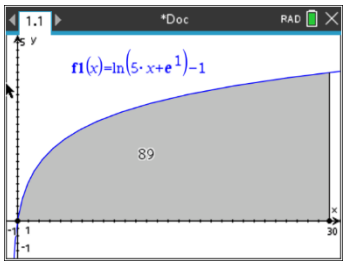


Shaded area = $1\frac{1}{3}$ units^2

Question 6

For the function $y = \ln(5x + e) - 1$, determine the area under the curve between $x = 0$ and $x = 29$. Then decide where a vertical line should be placed to divide this area exactly in half?

Graph Page:



Graph the function

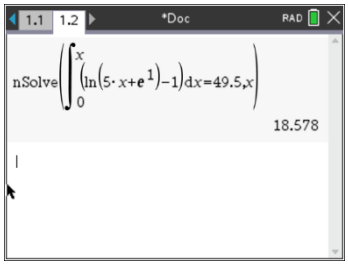
Find the Area Under Curve between *x*=0 and *x*=29

Menu->6:**Analyze Graph**->6:**Integral**

Lower Boundary = 0

Upper Boundary = 29

Enclosed Area = 89units^2

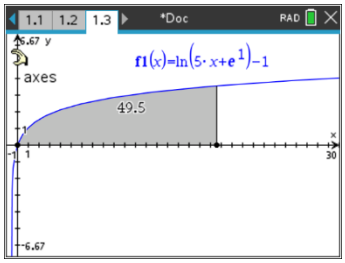


Half of Enclosed Area = $\frac{89}{2}$

= 49.5units^2

Use **Calculator Page & Numerical Solve:**

Check on **Graph** page by finding Integral between *x*=0 and *x*=18.578



The vertical line should be placed at *x*=18.578 to halve the original area.

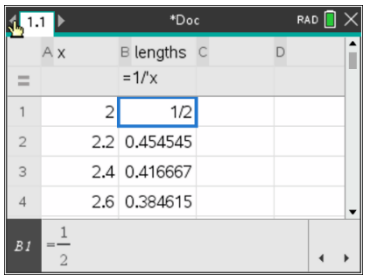
Author: M Hourigan

Question 7

Use the trapezoidal rule with *n*=5 to approximate the area under the curve $f(x) = \frac{1}{x}$ from $x = 2\ to\ x = 3$

With 5 trapezoids between *x*=2 and *x*=3 the width of each trapezoid = $\frac{1}{5}$ or 0.2

Lists & Spreadsheets Page:



Find the lengths of the parallel sides of each trapezoid using lists and spreadsheets page

Area = $\frac{b-a}{2n}[f(x1) + 2f(x2) + 2(fx3) + 2f(x4) + f(x5)]$

= $\frac{1}{10}\left[\frac{1}{2} + 2 * \frac{1}{2.2} + 2 * \frac{1}{2.4} + 2 * \frac{1}{2.6} + 2 * \frac{1}{2.8} + \frac{1}{3}\right]$

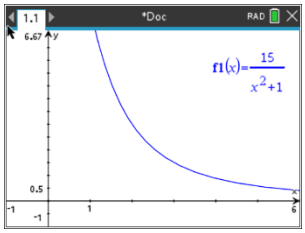
=0.4114units^2

Can check approximation using graph page or calculator page $\int_2^3 \frac{1}{x}\,dx$

Question 8

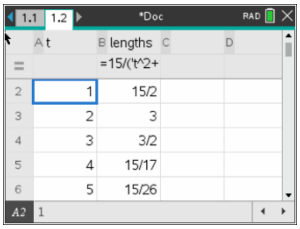
The size of a bacterial colony on an agar plate increases with time according to the formula: $G(t) = \frac{15}{t^2+1}$ where *G(t)* is the increase in the area covered (cm^2) after *t* hours. Using the trapezoidal rule, find the area covered after 5 hours.

Graphs Page: To view the area



Lists & Spreadsheets Page:

Find the lengths of the parallel sides of trapezoids – let *n*=5 so width of each trapezoid is 1cm.



Area = $\frac{b-a}{2n}[f(x1) + 2f(x2) + 2(fx3) + 2f(x4) + 2f(x5) + f(x6)]$

= $\frac{5}{2*5}\left[15 + 2 * 7.5 + 2 * 3 + 2 * 1.5 + 2 * \frac{15}{17} + \frac{15}{26}\right]$

= 20.67cm^2

The bacterial colony will cover approx. 20.67cm^2 after 5 hours.

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