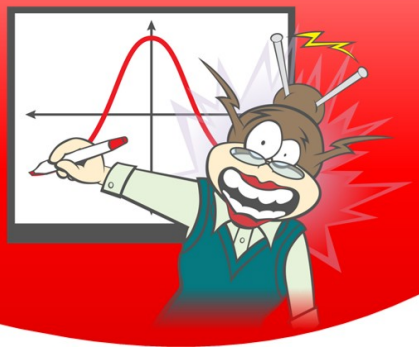

Recurrence models for linear growth & decay

Each of the questions included here can be solved using either the TI-Nspire CX or CX CAS.

Question 1

The first five terms of an arithmetic sequence are 72, 65, 58, 51, 44, …

a Express this sequence as a recurrence relation

b Find the value of V_{20}

Question 2

Calculate the first 5 terms for the linear recurrence relation with rule: $V_{n+1} = V_n + 17$, $V_0 = 73$

Question 3

An investment of $1500 is made attracting a simple interest rate of 3.5% p.a. The interest each year is added to balance of the investment account.

a Represent this investment as a linear recurrence relation

b What is the value of the investment after 7 years?

c How long does it take for the initial investment to double in value?

Question 4

A simple interest investment can be modelled on the following linear recurrence rule.
$V_{n+1} = V_n + 84$, $V_0 = 2000$. What is the associated annual rate of interest?

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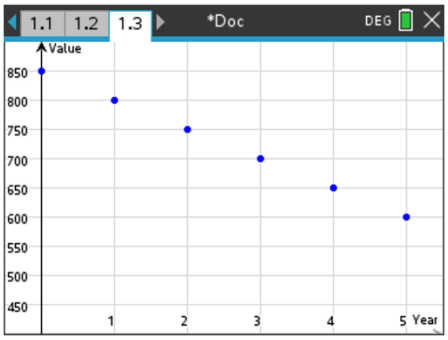
Question 5

The following plot displays the value of an asset as it depreciates in value (using flat rate depreciation method).



a What is the annual rate of depreciation?

b What is the expected value of this asset after 12 years?

Question 6

Ranjeev purchases a $10,000 coffee machine for his business which depreciates annually at a flat rate of 15% of the purchase price.

a By how much does Ranjeev's coffee machine depreciate each year?

b Express this flat rate depreciation as a linear recurrence relation

c Find the annual depreciated value of the machine over its first 5 years

d He intends to replace the machine when its depreciated value is $3000. After how many years will this occur?

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Question 7

A business purchases a photocopier for $139,000. It is devalued using a unit cost method of depreciation. After 3 years, it is valued at $80,000. In that time, it has been used to make and average of 320,000 copies per year.

a What is the unit cost (to the nearest cent)?

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b Express this depreciation as a linear recurrence relation.

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c What is the depreciated value after 5 years, based on the average usage pattern?

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d How many years will it take for the value to be less than $50,000?

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Answers

*Note: The file **RFM_linear.tns** (as shown in the webinar) can be used to help answer many of these questions.*

Question 1

a $V_{n+1} = V_n - 7$, $V_0 = 72$

b $Vn = V_0 - 7n = 72 - 7(20) = -68$

Question 2

73, 90, 107, 124, 141.

Question 3

a Interest $= r/100 \times V_0 = 3.5/100 \times 1500 = \$52.50$. So $V_{n+1} = V_n + 52.5$, $V_0 = 1500$

b $V_7 = 1500 + 7 \times 52.5 = \$1867.50$

c Solve $3000 = 1500 + 52.5n$ for n. Using calculator, gives $n = 28.5714$ (so 29 years)

Question 4

Interest = \$84, so $84 = r/100 \times 2000$. Solving gives $r = 4.2\%$ p.a.

Question 5

a From the graph, the value drops by \$50 per year, so $50 = r/100 \times \$850$. Solving for r gives $r = 5.88\%$ (to 2 d.p.)

b $V_{12} = 850 - 12 \times 50 = \$250$

Question 6

a Annual depreciation = $15\% \times \$10{,}000 = \$1500$

b $V_{n+1} = V_n - 1500$, $V_0 = 10{,}000$

c $V_5 = 10{,}000 - 5 \times 15{,}000 = \$2500$

d Solve $3000 = 10000 - n \times 1500$ for n. Answer is 4.667 years (so 5 years).

Question 7

a Unit cost = $(139{,}000 - 80{,}000)/(3 \times 320{,}000) = \$0.06$ per copy.

b $V_{n+1} = V_n - 0.06$, $V_0 = 139{,}000$

c In 5 years, usage would be $5 \times 320{,}000 = 1{,}600{,}000$. So Value = $139{,}000 - (1{,}600{,}000 \times 0.06) = \$43{,}000$

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