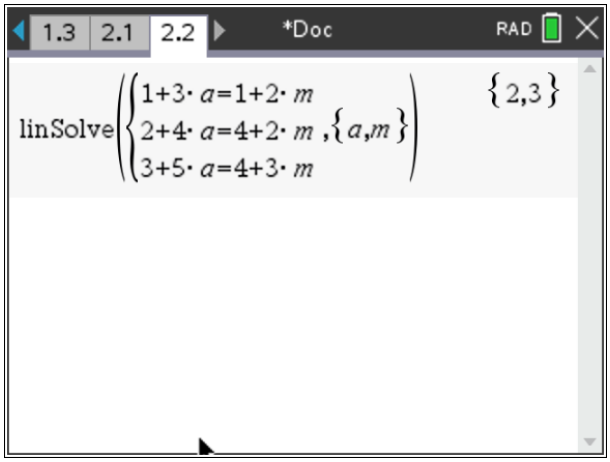
Lines and planes with CAS.

Intersecting lines in 3D

Find the point of intersection of the following lines given in vector form:

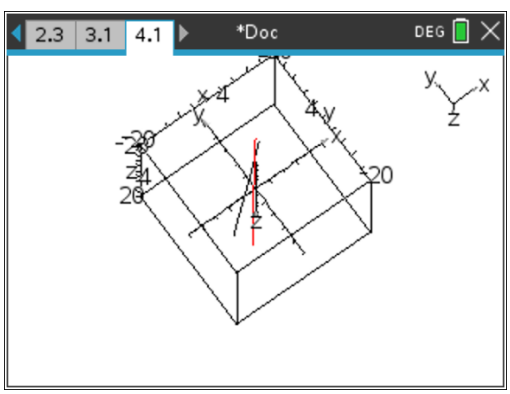
$$\underset{\sim}{r}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \text{ and } \underset{\sim}{r}_2 = \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}.$$



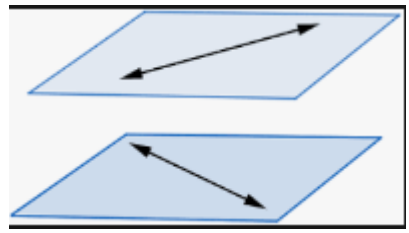
$\lambda = 2,\ \mu = 3.$

The two lines intersect in the point $(7,10,13)$.

We can draw the two lines in 3D on CAS Graph Entry Parametric.



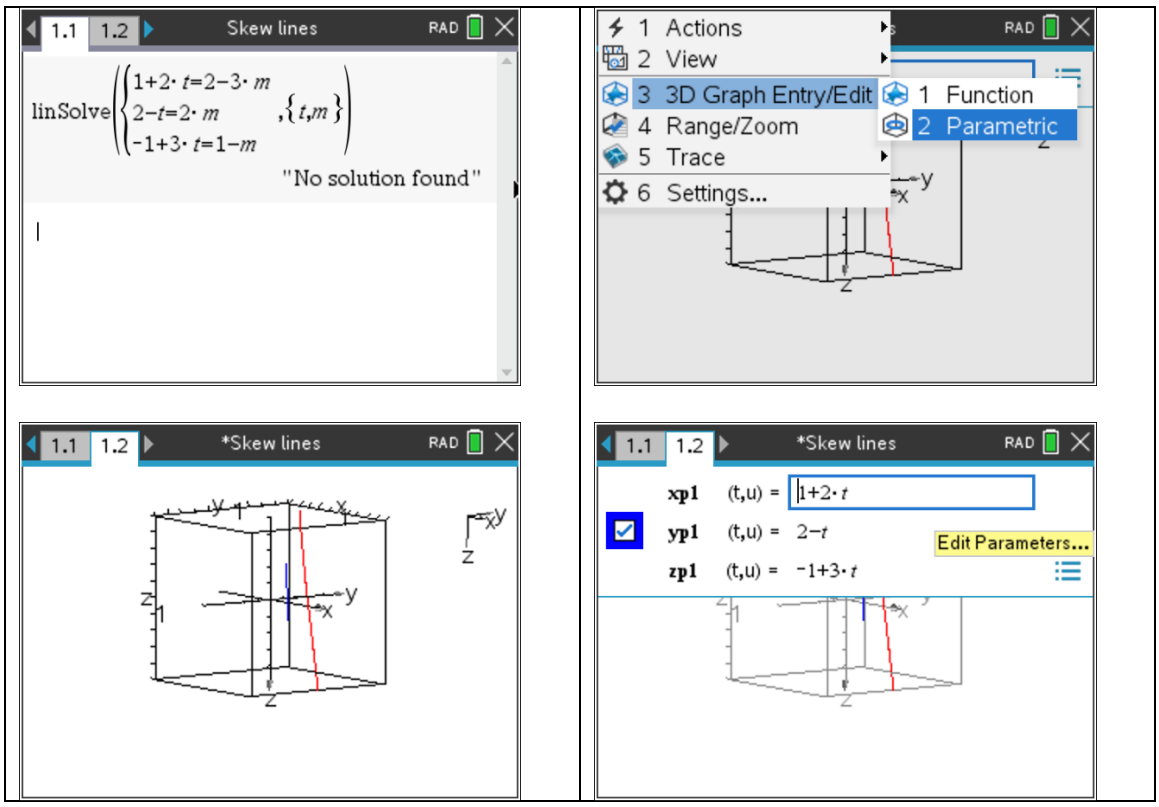
Skew lines investigation



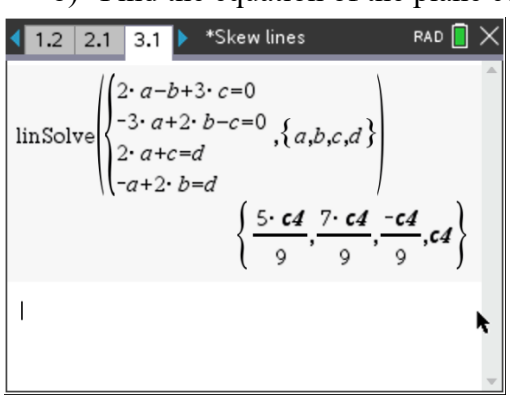
1. Consider the two lines:

$$L_1: \quad x = 1 + 2t \qquad L_2: \quad x = 2 - 3m$$
$$y = 2 - t \qquad\qquad y = 2m$$
$$z = -1 + 2t \qquad\qquad z = 1 - m$$

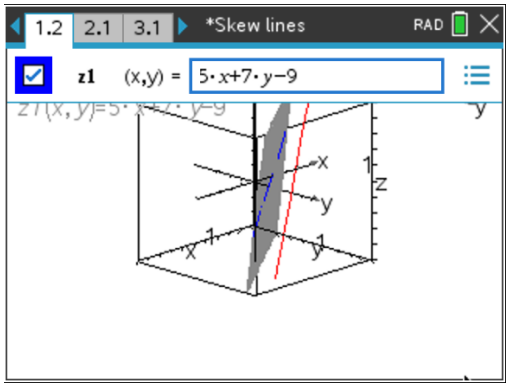
a) Show that these two lines do not intersect.

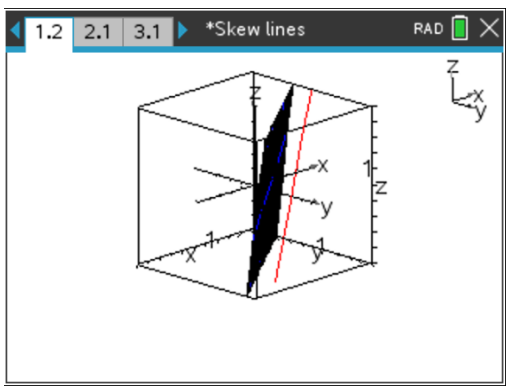


b) Find the equation of the plane containing L_2 which is parallel to L_1.



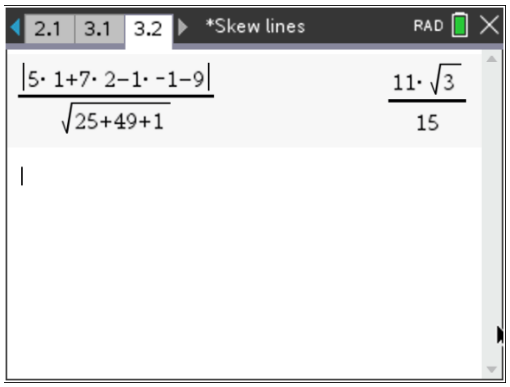
$$\frac{5}{9}x+\frac{7}{9}y-\frac{1}{9}z=1$$
$$5x+7y-z=9$$
Check with CAS by entering in Function 3D entry:



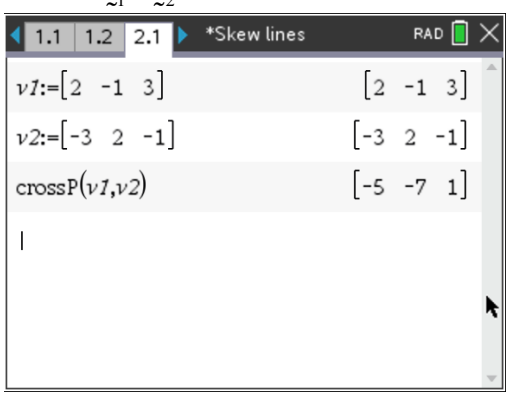


c) Taking any specific point on L_1, find its distance from this plane.
Use the formula for the distance from the point to the plane:

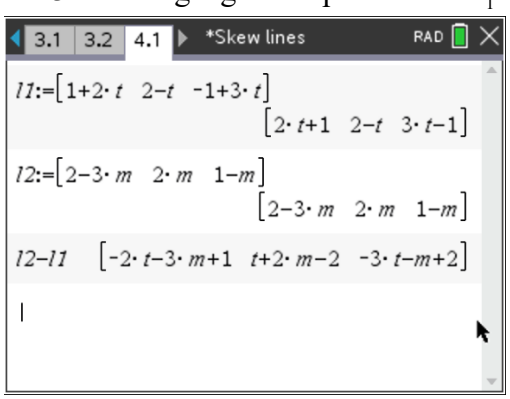
$$d=\frac{|ax_0+by_0+cz_0-d|}{\sqrt{a^2+b^2+c^2}}$$



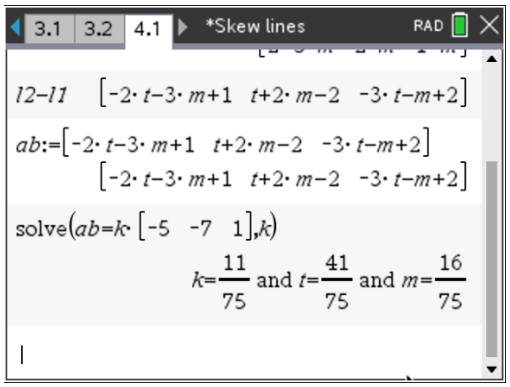
2. Given that $\underset{\sim}{v}_1$ and $\underset{\sim}{v}_2$ are direction vectors for L_1 and L_2, find the vector product $\underset{\sim}{v}_1 \times \underset{\sim}{v}_2$.



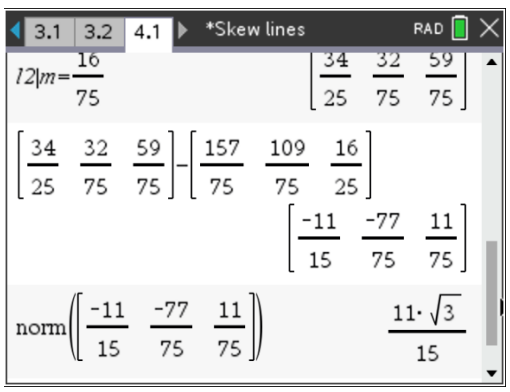
3. Taking a general point A on L_1 and general point B on L_2, find the vector $\overrightarrow{AB}$.



4. Find points A and B such that $\overrightarrow{AB}$ is parallel to $\underset{\sim}{v}_1 \times \underset{\sim}{v}_2$.



5. Find $\left|\overrightarrow{AB}\right|$. Compare the result with your answer to part 1 above. Comment on the result.



Same result. Therefore, this is another way to find the distance between two skew lines.