

Translations

Teacher Notes and Answers

7 8 9 10 11 12



Introduction

In 1637 René Descartes published a revolutionary book *La Géométrie* where he brought together two areas of mathematics; Geometry and Algebra in an attempt to 'unify' mathematics. The "Cartesian" plane, named after René Descartes, is an integral component of every high school mathematics course. Transformations of 'shapes' can be explored geometrically or over-laid on the Cartesian plane so as to help students understanding when transformations are treated algebraically in senior levels.

The purpose of this instructional resource is to assist teachers in building their own TI-Nspire files which can be used as either a demonstration tool or the focus of a student investigation.

Instructions – Creating the environment

Create a New TI-Nspire file and insert a Graph Application.

The Graph Application includes integrated geometric functionality.

A grid is to be added to the background, this provides for discrete movement of points (if required) and integer co-ordinates for easier pattern identification.

[Menu] > View > Grid > Dot Grid

The dot grid adjusts automatically to match the scale on the axis.

To encourage students to use the correct language and referencing, it is appropriate to label geometric objects; it is also worthwhile ensuring that angles will be measured in degrees. A specific Graphs and Geometry Settings menu can customise this application:

[Menu] > Settings

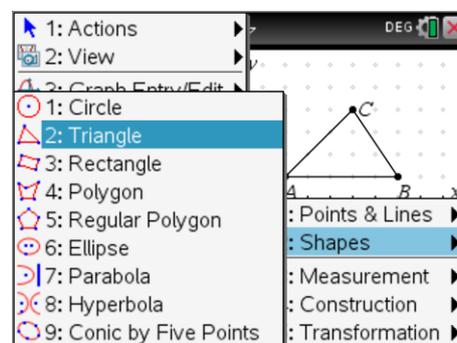
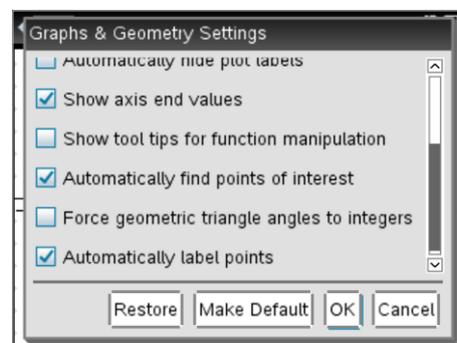
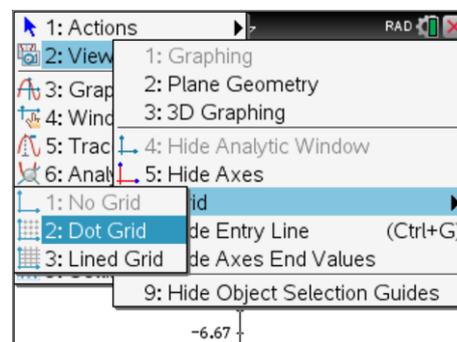
Change the Graph angle to degrees, scroll down and change the default setting to 'Automatically label points'.

A triangle is to be drawn on the Cartesian plane.

[Menu] > Geometry > Shapes > Triangle

As you move the mouse around the screen you will notice it is possible to place the vertices 'on' the grid. Place all three vertices on the grid to form a triangle in the first quadrant. Remember that the triangle will be translated, so don't make it too big!

Press [esc] to release the shape tool then grab any one of the triangle's vertices and move it around. You will notice that the vertices jump from grid point to grid point.



A translation involves a shape and a vector. The vector tool is also located in the Geometry menu.

[Menu] > Geometry > Points and Lines > Vector

Place the tail of the vector at the origin and the head on a grid point, one that is not shared already by the triangle.

This vector will represent the size and direction of the translation.

As the Vector is a geometric object the head and tail are also labelled, for the purposes of this activity students may prefer to 'hide' these labels and the point created at the head of the vector. The hide option is available through both the Actions and contextual menu.

The final step in this construction is to create the transformation.

[Menu] > Geometry > Transformation > Translation

Once the Translation tool is active (Top left icon on the screen) select the Vector followed by the triangle. Make sure the transformed shape has a solid outline before pressing [esc]. If you cannot see your translated triangle it could be that your translation vector is too big.

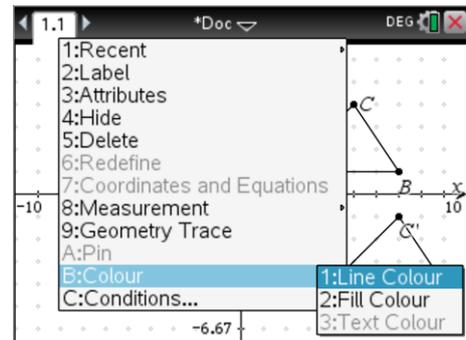
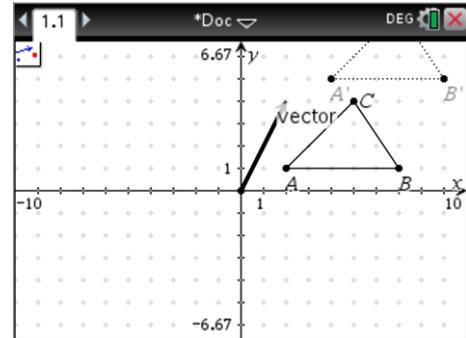
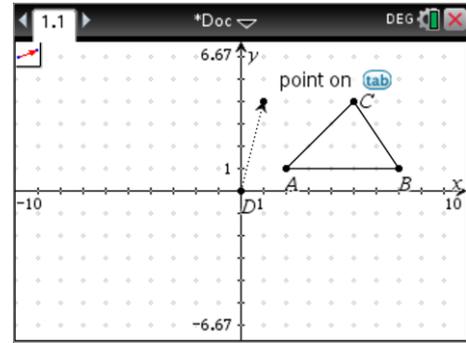
Note that the software is intelligent and flexible enough so that if the triangle is selected first followed by the vector, the same translation will occur.

It is possible to change the colour of the translated shape to make it easier to identify. The contextual menu is the easiest way to change an object's properties. Place the mouse (cursor) over the top of the triangle. The word 'triangle' will be displayed. Then press:

[Ctrl] + [Menu] (Equivalent to a right mouse click)

Select Colour, followed by line colour.

The same approach can be used to change the colour of the vertices.



Grab the head of the vector and move it around the screen, observe the location of the translated object. Try changing the location and form of the triangle to see the impact on the translated object.

Things to explore:

- Move the translation vector to create a translation parallel to the x axis. Observe how the translated triangle moves when the translation vector is kept parallel to the x axis.
 - How could you describe the translated triangle's movement in relation to the x axis?
 - How could you describe the translated triangle's movement in relation to the y axis?
- Explore other translations and consider the following:
 - What geometrical properties are conserved (invariant) under a translation?
 - What geometrical properties are changed under a translation?

Instructions – Making the environment more informative

The last dot point (previous page) prompts consideration of the geometrical properties of a shape under a translation. Side lengths, perimeter, area and angles can all be measured quickly and easily.

[Menu] > Geometry > Measurement > Length

If you click on the triangle it will automatically measure the perimeter. To measure a single side length, click on the respective vertices.

Repeat the above process to measure the corresponding side length on the translated triangle.

Angles can be measured in two ways:

Angle: Measures the smallest angle, (acute or obtuse) regardless of whether points are selected as $\angle ABC$ or $\angle CBA$.

Directed Angle: Measures the angle in a counter-clockwise direction, so if $\angle ABC = 30^\circ$ then $\angle CBA = 330^\circ$

Measure the internal angles of each triangle.

As the screen becomes more populated with information and measurements it is worth moving the measurements so that they can be compared directly. Measurements can be dragged away from the item they are measuring, however with a mix of measurements on the screen it is worthwhile connecting each measurement with its label.

Move a measurement to the far left of the screen then press:

[Ctrl] + [Menu].

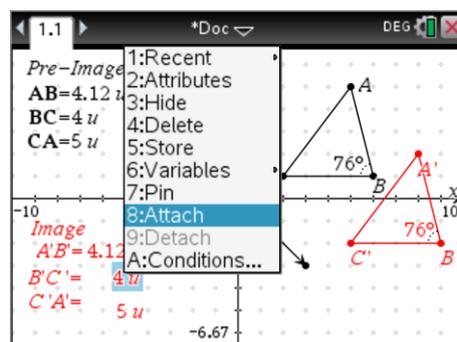
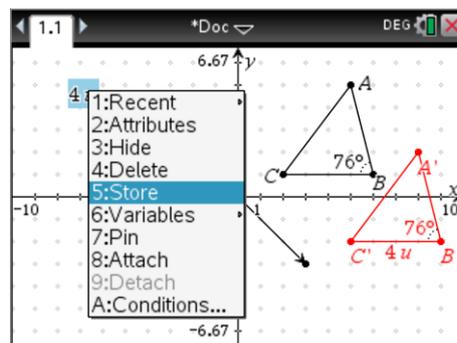
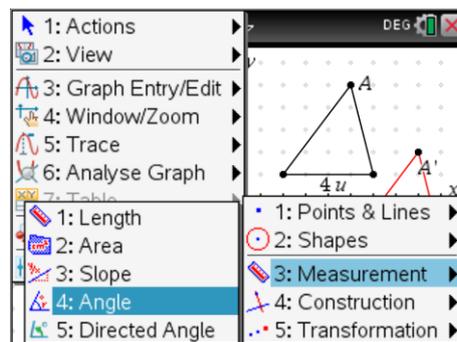
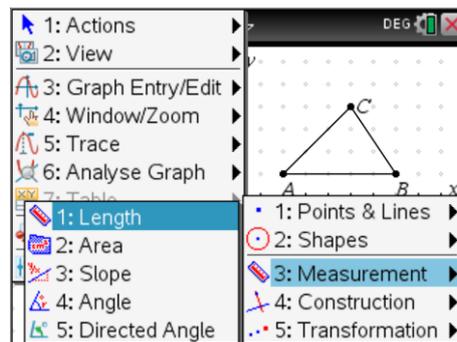
As mentioned previously, this is equivalent to a 'right mouse' click. Use the Store option and label each variable appropriately.

Note: It is not possible to store a variable as $A'B'$, see below for solution to this problem.

To create labels for sides such as $A'B'$, use the insert text option, then select the measurement, from the contextual menu select: "Attach".

Click on the text handle where you want to attach the value.

Repeat this for the entire page so that all the measurements for the triangle are neatly aligned at the side of the page.



Calculator Tip!



When several objects are located within a relatively small region, use the TAB key to alternate between the objects. Text is displayed to indicate which object will be selected.

Creating Focus – How to hide / show details on demand

As more and more information appears on the screen, the ability to hide or show things on-demand is a very powerful tool. Too much information is distracting and discourages students from reading the information, furthermore, as information appears and disappears, student attention is automatically drawn to the detail as it is appears.

[Menu] > Actions > Insert Slider

Match the settings shown opposite and also:

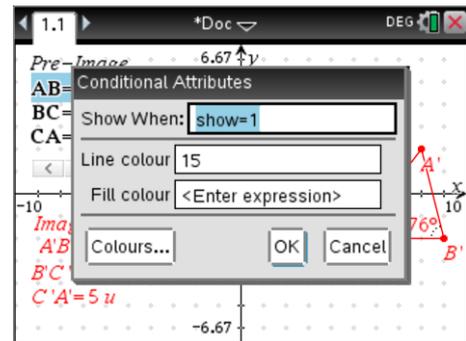
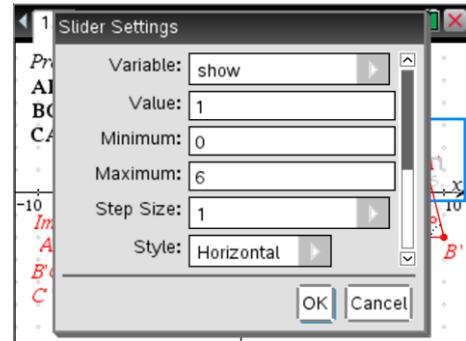
- Select 'Minimised'
- De-select 'Show scale'

Select the first measurement and label for side AB and press:

[Ctrl] + [Menu] > Conditions

The value of the slider can be used to hide or show objects. For the setting shown opposite the measurement for side AB will only appear on the screen when the slider value (show) is equal to one.

Repeat these conditions for all the side length measurements so when the slider is equal to one, all the side length measurements will be displayed.



Calculator Tip!



The following questions encourage 'exploration' of your current construction. You may want to keep an original version. This can be achieved in several ways such as saving and then saving under a new name. Another way is to copy and paste the current application into a New Problem. The first page of a document can also be saved as a Widget. The Widget can then be 'inserted' into any new document and used over and over again!

Before proceeding with any of the explorations (below), save your document!

Practice & Exploration

1. Measure the angles in the pre-image and image triangles, label them, place them to the side and make them appear when 'show = 2' and disappear for all other values of 'show'.
Note: The angles can be placed in the same location as the side lengths if both sets of measurements do not appear at the same time.
2. Measure the area of the pre-image and image triangles, label them, place them to the side and make them appear when 'show = 3' and disappear for all other values of 'show'.
3. Connect the respective vertices A to A', B to B' and C to C' using light grey dotted lines. Make these lines appear when 'show = 4'.
Note: The purpose of these lines is to show that each point is moved by the same vector.

4. Measure the coordinates of the vertices on the pre-image and image. Make the coordinates appear consecutively A and A' when show = 5, B and B' when show = 6 and C and C' when show = 7. For added effect the dotted line segments from the previous task could also be revealed consecutively, this can be achieved by editing their current conditional statements to include an "OR" option.
Example: Show When: show = 3 or show = 5.
5. Add another vector that is attached to two points anywhere on the grid (not too far apart!). Construct another translation on the pre-image triangle: $\triangle ABC$. Adjust the new vector so that it creates a translation that is the same as the first. What do you notice about the two vectors? What concepts can this help students understand well before they encounter 'vectors' in senior mathematics and physics classes?
6. Explore what happens if you apply a translation to an image (A second translation to the original triangle). Repeat the translations in the reverse order and see if this effects the location of the final image. Once again, a great opportunity to introduce basic vector concepts before they are taught in senior mathematics and physics classes. Consider also how this idea could be extended to students considering the order of other types of transformations and the applicable algebraic concepts in senior mathematics classes.
7. Insert a new problem and use the shapes tool to construct a 'polygon'. Create a translational vector and see what happens when the polygon is translated.