Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

FUNCTIONS, GRAPHS & ALGEBRA

Question 1 (Similar to Q.1 2019 MM NHT Exam 2. Max. domain questions: typically, 10-20% incorrect)

The **maximal domain** of the function with rule $g(x) = 2x + \log_e(1-x)$ is

A. R
B. $[1, \infty)$
C. $(-1, \infty)$
D. $(-\infty, 1)$
E. $(-\infty, -1]$

Question 2 (Similar to Q.4a. 2016 MM Sec. B Exam 2. Incorrect 48%.)

If $\frac{2x+1}{2-x}$ is expressed in the form $p + \frac{q}{2-x}$, where p and q are integers, then

A. $p = 2, q = 1$
B. $p = -1, q = 2$
C. $p = -5, q = 2$
D. $p = -2, q = 5$
E. $p = 2, q = -3$

Some questions on exams **prior to 2016** are not examinable in 2019. Check with your teacher which questions are not examinable before working through one of these exams.

Question 3 (Similar to Q.2 2015 MM Exam 2. Incorrect 50%.)

The **inverse** of $f : (-\infty, 3) \to R,\ f(x) = \frac{1}{\sqrt{3-x}}$ is

A. $f^{-1} : R \setminus \{0\} \to R,\ f^{-1}(x) = \frac{1}{3-x^2}$
B. $f^{-1} : R^+ \to R,\ f^{-1}(x) = x^2 - 3$
C. $f^{-1} : (3, \infty) \to R,\ f^{-1}(x) = 3 - x^2$
D. $f^{-1} : R^+ \to R,\ f^{-1}(x) = 3 - \frac{1}{x^2}$
E. $f^{-1} : (0, 1] \to R,\ f^{-1}(x) = 3 - \frac{1}{x^2}$

Variant of Q. 3. The inverse of $f:(-\infty,2)\to R,\ f(x)=\dfrac{1}{\sqrt{3-x}}$ is: (same A – E as in Q.3)

Functional equations

Question 4 (Similar to Q.11 2016 MM Exam 2. Incorrect 53%.)

The function f has the property $f(x)+f(y)=(x+y)f(xy)$ for all non-zero real numbers x and y. Which one of the following is a possible rule for the function?

A. $f(x)=2x$

B. $f(x)=x^2+2$

C. $f(x)=\dfrac{1}{2x}$

D. $f(x)=\dfrac{1}{x^2+2}$

E. $f(x)=\log_e(2x)$

Simultaneous equations

Question 5 (Similar to Q.8 2019 NHT* Exam 2 and concept of Q. 17 2012 MM Exam 2. Incorrect 57%.)

The equations $(m-2)x+3y=6$ and $2x+(m+2)y=m$ will have **no solution** when

A. $m=2$

B. $m=2$ or $m=-2$

C. $m\in R\setminus\{2\}$

D. $m\in R\setminus\{\sqrt{10}\}$

E. $m=\sqrt{10}$ or $m=-\sqrt{10}$

Solution of trigonometric equations

Question 6 (Similar to Q.6 2018 MM NHT* Exam 2)

The sum of the solutions to the equation $-3\sin(2x)=\sqrt{3}\cos(2x)$ for $x=[-\pi,\pi]$ is

A. $\dfrac{\pi}{6}$

B. $\dfrac{2\pi}{3}$

C. $\dfrac{5\pi}{6}$

D. $\dfrac{4\pi}{3}$

E. $\dfrac{13\pi}{3}$

*NHT is VCAA Northern Hemisphere Timetable exams. Visit https://bit.ly/2MePvsa

Variant of Q. 6. (Similar to Q. 12, 2017 MM Exam 2. Incorrect: 55%)

The sum of the solutions to the equation $-3\sin(2x)=\sqrt{3}\cos(2x)$ for $x=[-\pi,k]$ is $\frac{5\pi}{3}$.

The value of k could be (same A – E as Q. 6 above).

Transformations

Question 7 (Similar to Q.12 2016 MM Exam 2. Incorrect: 48%)

The graph of a function f is obtained from the graph of the function g with rule $g(x)=3-\sqrt{1-4x}$ by

- a reflection in the *x*-axis
- followed by a translation of 2 units to the right
- followed by a dilation from the *y*-axis by a factor of $\frac{1}{3}$.

Which one of the following is the rule for the function f ?

A. $f(x)=\sqrt{9-12x}-3$

B. $f(x)=3-\sqrt{12x-23}$

C. $f(x)=-3+\sqrt{-12x-23}$

D. $f(x)=3+\frac{\sqrt{12x-15}}{3}$

E. $f(x)=\frac{\sqrt{33-12x}}{3}-3$

CALCULUS

Stationary points

Question 8 (Similar to Q.5 2018 MM Exam 2. Incorrect: 33%)

Consider $g: R\setminus\{0\}\rightarrow R,\ g(x)=\frac{k}{x^2}-x$, where $k\in R$.

There is a stationary point on the graph of g when $x=-3$.

The value of k is

A. $-\frac{8}{3}$

B. $-\frac{27}{2}$

C. 2

D. $\frac{8}{3}$

E. $\frac{27}{2}$

Tangent line

Question 9 (Similar to Q.9 2018 MM Exam 2. Incorrect: 43%)

Consider the function $f:\left[\frac{3}{2},\infty\right)\to R,\ f(x)=x^2e^{-2x}$.

A tangent to the graph of f has a gradient of $-4e^{-4}$.

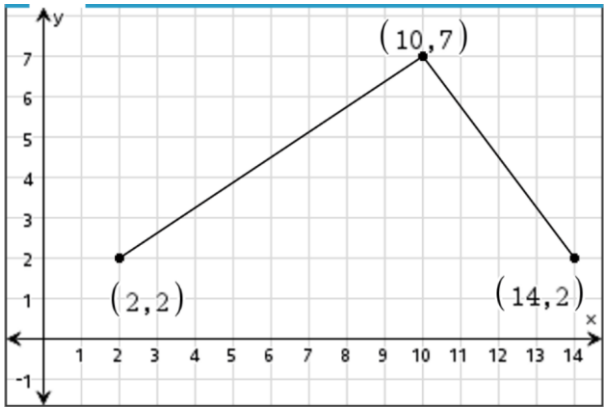
The tangent will cross the *x*-axis at

A. 0
B. 2
C. 3
D. $12e^{-4}$
E. $3e^{-4}$

TIP: Effective use of technology should include the ability to make sound judgements about the most efficient way to solve a particular problem. This might be mentally - 'by thinking', 'by hand', using technology, or a combination of these.

Average value – conceptual question (can be done 'mentally'. No formal calculation required)
Question 10 Similar to Q.20 2014 MM Exam 2 (Incorrect 56%)

The graph of a function $f:[2,14]\to R$ is shown below.



The average value of f over the interval $[2,14]$ is

A. $\frac{7}{2}$
B. $\frac{9}{2}$
C. 5
D. 8
E. 10

*Average **rate** of change of a function* and *Average **value** of a function* are frequently confused by students during exams. Be clear about which of these the question requires.

Average value calculation

Question 11 (Similar to Q.10 2012 MM Exam 2. Incorrect: 40%)

Consider the function $f:[-1,\infty)\to R,\ f(x)=\dfrac{(x-1)^2}{4}+2$.

The average **value** of f the over the interval $[-1,k]$ is $\dfrac{13}{3}$.

The value of k is

A. 1
B. 3
C. 4
D. 5
E. 7

Average rate of change (Graphically, this is the average gradient between endpoints of the interval)

Question 12 (Similar to Q.4 2016 MM Exam 2. Incorrect: 15%)

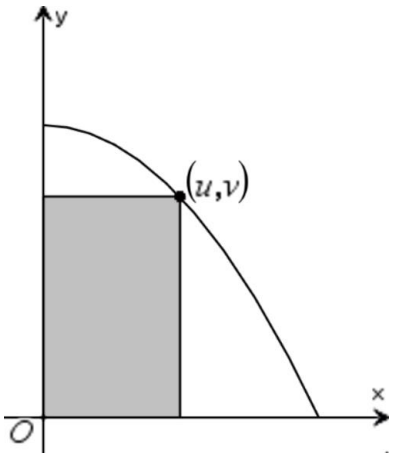
The **average rate of change** of the function $f:[-1,\infty)\to R,\ f(x)=\frac{1}{2}x^2+p\sqrt{x+1}$, between $x=0$ and $x=3$, is 1. The value of p is

A. 2
B. $\dfrac{1}{2}$
C. $-\dfrac{1}{2}$
D. $\dfrac{3}{2}$
E. $-\dfrac{3}{2}$

Maximum/minimum

Question 13 (Similar to Q.14 2016 MM Exam 2. Incorrect: 63%)

A rectangle is formed using part of the x- and y-axes and a point (u, v) on the parabola $y = \frac{36 - x^2}{6}$, where $u > 0$ and $v > 0$.



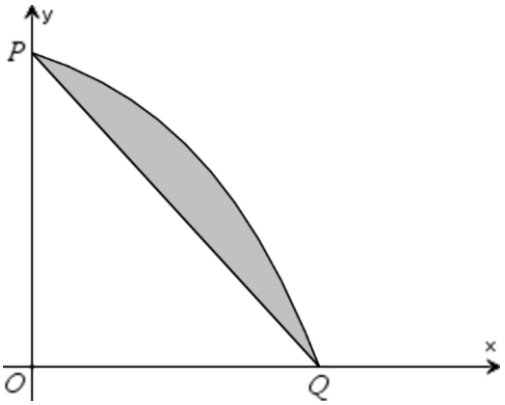
The maximum area of the rectangle is

A. 4
B. 8
C. $2\sqrt{3}$
D. $8\sqrt{3}$
E. $16\sqrt{3}$

Area between curves

Question 14 (Similar to Q.18 2019 NHT MM Exam 2)

The graph of the function with rule $f(x) = 8 - 2^x$ intersects the axes at the points *P* and *Q*, as shown below. Also shown on the graph is the line segment joining *P* and *Q*.



The area of the shaded region is

A. $\frac{15}{2} - \frac{7}{\log_e(2)}$

B. $\frac{7}{\log_e(2)} - \frac{15}{2}$

C. $\frac{27}{2} - \frac{7}{\log_e(2)}$

D. $\frac{7}{\log_e(2)} - \frac{27}{2}$

E. $\frac{69}{2} - \frac{7}{\log_e(2)}$

PROBABILITY & STATISTICS

Binomial Distribution

Question 15 (Similar to Q.18 2017 MM Exam 2. Incorrect 62%)

The discrete random variable, *X*, is binomially distributed with $X \sim \text{Bi}(n, p)$, $0 < p < 1$.

If the mean and standard deviation of this distribution are equal, then the smallest number of trials such that $p \leq 0.015$ is

A. 15
B. 39
C. 49
D. 66
E. 85

Continuous random variables - Normal Distribution / Probability density function

Question 16 (Similar to Q.11 2012 MM Exam 2. Incorrect 38%)

The lifespan of Britelite car headlights is normally distributed with mean 890 hours and standard deviation 175 hours.
The manufacturer claims that 35% of headlights have a lifespan of more than x hours.
The maximum possible value of x is closest to

A. 1065
B. 957
C. 925
D. 823
E. 343

Question 17 (Similar to Q.18 2016 MM Exam 2. Incorrect 38%)

The probability density function of the continuous random variable, X, is given by

$$f(x)=\begin{cases}\frac{1}{4}\sin\left(\frac{x}{4}\right) & 8\pi \le x \le 10\pi \\ 0 & \text{elsewhere}\end{cases}$$

The value of k such that $\Pr(X > k)=\frac{\sqrt{3}}{2}$ is

A. $\frac{17\pi}{2}$
B. $\frac{26\pi}{3}$
C. $\frac{53\pi}{6}$
D. $\frac{28\pi}{3}$
E. $\frac{19\pi}{2}$

Sample proportion $\hat{P}$

Question 18 (Similar to Q.17 2016 MM Exam 2. Incorrect 44%)

Assume that in a very large city 50% of people named on the electoral roll are female.
A sample of 20 names is selected at random from the electoral roll.

For samples of 20 names, $\hat{P}$ is the random variable of the distribution of sample proportions of females.

$\Pr(\hat{P} \le 0.3)$ is closest to

A. 0.0207
B. 0.0577
C. 0.8210
D. 0.9423
E. 0.9793

Standard deviation of $\hat{P}$

Question 19 (Similar to Q.19 2018 NHT MM Exam 2)

A jar contains 25 000 jellybean lollies. Less than half of the jellybeans are black.

Random samples of 400 jellybeans are taken from the jar. Each sample is obtained by sampling with replacement.

$\hat{P}$ is the random variable of the distribution of sample proportions of black jellybeans.

If the standard deviation of $\hat{P}$ is 0.018, then the number of black jellybeans in the jar is closest to

A. 450
B. 2 778
C. 3 825
D. 12 500
E. 21 175

Confidence Intervals

Question 20 (Similar concept to Q. 19 2019 NHT MM Exam 2)

A researcher surveyed a random sample of 640 voters in a large city and found that 416 supported a particular proposal. An approximate 90% confidence interval for the proportion of all voters in the city who support the proposal is closest to

A. (0.619, 0.681)
B. (0.613, 0.687)
C. (0.381, 0.319)
D. (0.387, 0.313)
E. (0.416, 0.640)