Normal Distribution

TI-30XPlus
MathPrint™

Worksheet

15 min

Calculator Skills:

- Normal (pdf) and Inverse Normal distributions

Music Algorithm:

In 2018, 75% of the music industry's total revenue came from streaming services. Most services use an algorithm that pays artists on per play basis. There is a finite amount of time that anyone can listen to music, shorter tracks mean more songs and therefore more royalties! Has the music industry responded? From 2013 to 2018, the average track length of the top 100 hits reduced by 20 seconds! Is this just coincidence or part of a natural variation in song duration?
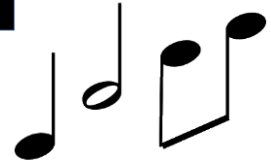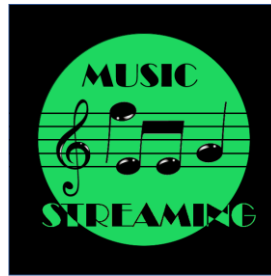
> Song duration is given in minutes and seconds. Convert each song duration into seconds before attempting to calculate any probability. Make sure your mean and standard deviation are also given in seconds.

Question: 1.

In 2013 the mean length of a top 100 music hit was 3 minutes and 50 seconds. Song lengths of this era were approximately normally distributed with a standard deviation of 30 seconds.

i) Songs such as Ho Hey (2:40); Demons (2:56) and Treasure (2:58) were less than 3 minutes in duration. What proportion of songs from 2013 were likely to be less than 3 minutes in duration?

ii) Songs such as: Blurred Lines (4:23); Wake Me Up (4:07); I Will Wait (4:35); Just Give Me a Reason (4:02) and A Thousand Years (4:40) were all longer than 4 minutes. What proportion of songs from 2013 were likely to be longer than 4:00minutes?

iii) In 2013 Justin Timberlake had a hit with "Mirrors" (8:03). What proportion of songs in 2013 would you expect to be longer in duration?

Question: 2.

In 2018 the mean length of a top 100 music hit was 3 minutes and 30 seconds. Song lengths of this era were approximately normally distributed with a standard deviation of 18 seconds.

i) Songs such as Meant to Be (2:43); Let Me Go (2:53) and Eastside (2:52) were less than 3 minutes in duration. What proportion of songs from 2018 were likely to be less than 3 minutes in duration?

ii) An artist writing a song in 2018 wants to ensure their next song is not classified as "too long". A song is considered "too long" if it is in the top 10%. What is the maximum length of the song the artist should produce?

iii) An artist decides to do a cover of a 2013 song. In 2013 the song was not considered too long, just falling in to the top 30% of song durations. Will this song be considered "too long" (top 10%) in 2018?

Answers on Page 2

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TEXAS INSTRUMENTS

Question: 1.

(i)   Convert the time of 3:50 into seconds: 3 x 60 + 50 = 230 seconds.

Use normalcdf and enter the values for the mean: 230 seconds and standard deviation: 30 seconds.

```
          DEG                        DEG                          DEG
Normalcdf          ↑    Normalcdf          ↑    Normalcdf                  ↑
mean=mu=230             LOWERBnd=0              VALUE=0.0477903303597
si9ma=30■              UPPERBnd=180
                                                STORE: ■ x y z t a b c d
                    ↓              CALC         SOLVE AGAIN           QUIT
```

Answer: The probability of a song being less than 3 minutes is relatively small ≈ 0.048.

Applying this probability to the top 100, indicates we might see approximately 5 songs of such duration.

> ⚠ The lower-bound used here is 0 seconds. This has practical and theoretical significance. Practically you can't have a song less than 0 seconds, however the Normal distribution is continuous and therefore from a theoretical perspective extends to $-\infty$. A time of 0 seconds in this case is more than 7 standard deviations from the mean: μ. The likelihood of an event more than 7 standard deviations from the mean is of the order: 1×10^{-12}.

(ii)   Convert the time of 4 minutes into seconds: 4 x 60 = 240 seconds. Selecting "Solve Again" is an efficient step in this question since the mean and standard deviation (sigma) are unchanged. Given the mean is 230 seconds and the standard deviation is 30 we are not even one standard deviation from the mean and therefore expect a significant proportion of songs (not more than 50%!) with such length. **Answer**: 0.3694

```
          DEG                        DEG                          DEG
Normalcdf          ↑    Normalcdf          ↑    Normalcdf                  ↑
mean=mu=230             LOWERBnd=240            VALUE=0.3694414036837
si9ma=30■              UPPERBnd=600
                                                STORE: ■ x y z t a b c d
                    ↓              CALC         SOLVE AGAIN           QUIT
```

(iii)   Convert the time of 8 minutes and 3 seconds into seconds: 8 x 60 + 3 = 483 seconds. The question asks for a song of equal or greater length. **Answer**: 1.7×10^{-17}.

```
          DEG                        DEG                          DEG
Normalcdf          ↑    Normalcdf          ↑    Normalcdf                  ↑
mean=mu=230             LOWERBnd=483            VALUE=1.700110896857E⁻17
si9ma=30■              UPPERBnd=600
                                                STORE: ■ x y z t a b c d
                    ↓              CALC         SOLVE AGAIN           QUIT
```

Question: 2.

(i)   Given the average song length has reduced by 20 seconds, it would be reasonable to think there is more likelihood of a song being less than 3 minutes, however the standard deviation has also reduced.

```
          DEG                        DEG                          DEG
Normalcdf          ↑    Normalcdf          ↑    Normalcdf                  ↑
mean=mu=210             LOWERBnd=0              VALUE=0.0477903303597
si9ma=18■              UPPERBnd=180
                                                STORE: ■ x y z t a b c d
                    ↓              CALC         SOLVE AGAIN           QUIT
```

Answer: The probability is the same ≈ 0.048. This is because the song duration is the same number of standard deviations below the mean (Z score): Z_1 = 1.667; Z_2 = 1.667.

To help understand how this can be possible (visually), study the two graphs shown opposite. The scales are the same. The shaded area (proportions) are also the same. This occurs when the song duration is 180 seconds. This would not be true for other times such as 210 seconds.
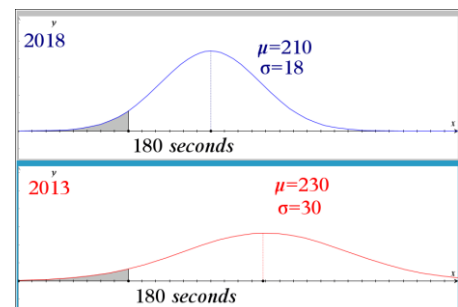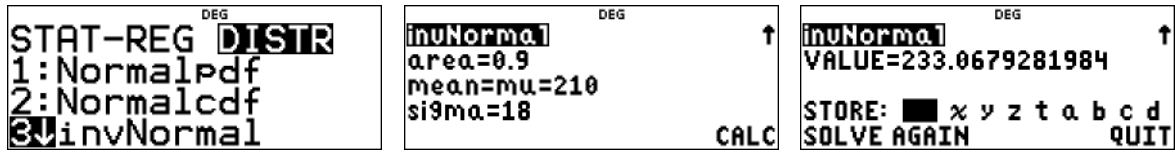
Figure 1: TI-Nspire™ Graphing Calculator Screen

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TEXAS INSTRUMENTS

(ii) In this question we are provided with information about the proportion and need to work backwards to find the time, to achieve this, use the inverse Normal distribution. The Inverse Normal distribution will always provide the region (area) to the left, therefore use an area of 0.90 to determine the corresponding time associated with the top 10% (0.10)

To help understand, consider the graph opposite displaying the region calculated using the Inverse-Normal distribution.

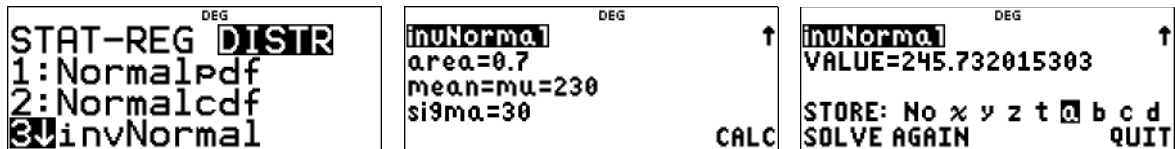**Answer**: 233 seconds or 3 minutes and 53 seconds.

(iii) This question provides us with an initial proportion (Top 30%), therefore, use inverse normal (0.7) with the corresponding mean and standard deviation for songs from 2013.
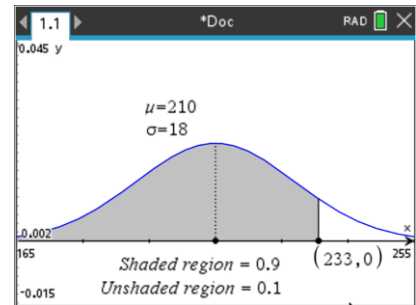
Figure 2: TI-Nspire Graphing Calculator Screen

This result tells us that the minimum length for the song in 2013 would have been 245.7 seconds. Songs longer than 245.7 seconds would be in the top 30%. The question states that the 2013 song was 'just' inside this range, therefore close to 245.7 seconds.

This time needs to be used for the 2018 data to locate the proportion of songs that would exceed this duration. An efficient step here is to store the variable in a memory location so it can be retrieved for the next calculation. The screen shots below use "Recall" variable.
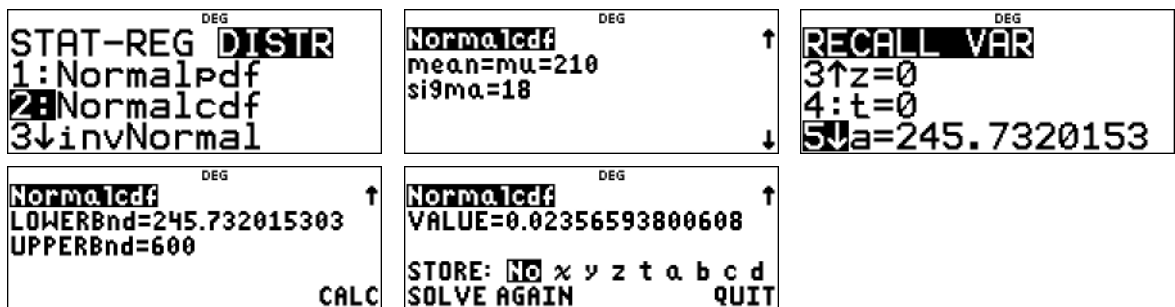
Answer: The cover song would now be considered in the top 2.357% of songs (duration).

The two graphs shown opposite illustrate how a 245 second duration song in 2013 was relatively close to the mean compared with the same song in 2018.

The songs in 2018 on average were 20 seconds shorter but also had a smaller variation. The standard deviation in 2013 was larger resulting a greater variation in song duration.
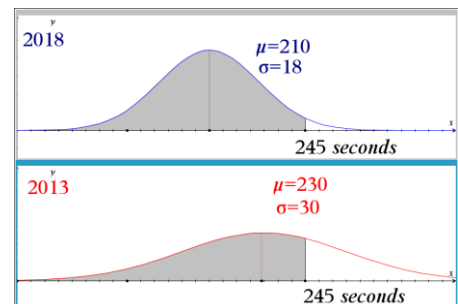

Figure 3: TI-Nspire Graphing Calculator Screen

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