First Principles

7 8 9 10 **11** 12

TI-30XPlus MathPrint™ | Worksheet | 15 min

Calculator Skills:

- Define functions: $f(x)$ and $g(x)$
- Store variables
- Generate a table of values.

Formula:

$$f'(x) = \lim_{d \to 0} \frac{f(x+d) - f(x)}{d}$$

Question: 1.

For each of the following, use first principles to determine the approximate the gradient for the given value of *d* at the corresponding value for *x*.

i) $f(x) = x^2 + 5x + 6$ where *d* = 0.1 and *x* = 2

ii) $f(x) = x^2 - 4x + 3$ where *d* = 0.01 and *x* = 0

iii) $f(x) = (x+3)(x+1)(x-1)$ where *d* = 0.001 and *x* = 1

Question: 2.

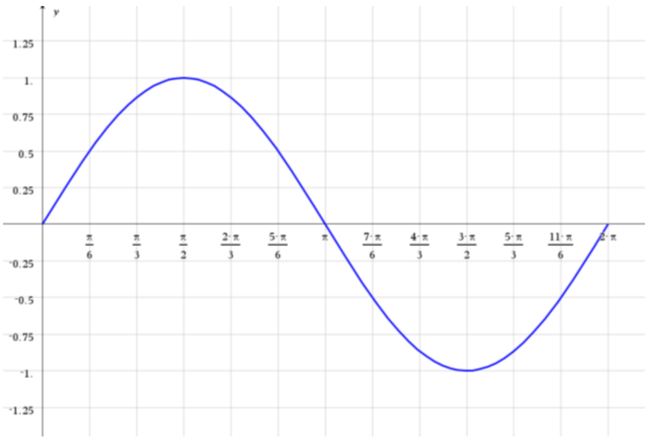
Use first principles to find the gradient of the function $f(x) = \frac{1}{x-1}$ where it crosses the *y* axis.

Question: 3.

Use first principles to find the gradient of the function $f(x) = x^2 - 1$ where it crosses the *x* axis.

Question: 4.

A graph of $f(x) = \sin(x)$ over the domain $0 \le x \le 2\pi$ is shown opposite.
Generate a table of values for the gradient (from first principles) starting at 0 in steps of π/6 and d = 0.0001. Make sure your calculator is in RADIAN mode. Graph the results.
What is the equation for the gradient function?



Extension

The TI-30XPlus MathPrint is not an algebraic calculator at all, however you can use regression to work out some equations. Let $f(x) = x^3 - 2x^2 + 7x - 1$, the gradient function is a quadratic. Use the lists feature in the calculator to determine the gradient of $f(x)$ for at least 3 different *x* values. Use quadratic regression to determine the corresponding equation for the gradient function.

Answers on Page 2

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Question: 1.

i) 9.1

ii) -3.99

iii) 8.006001

Question: 2.

Graph crosses the y axis when x = 0. The approximate gradient (d=0.001) is -1.001 which appears to be approaching -1.

Question: 3.

Graph crosses the x axis in to locations: x = -1 and x = 1. The approximate gradient (d=0.001):

When x = -1 the gradient is -2

When x = 1 the gradient is 2 [The graph is symmetrical so the result should not be surprising.]

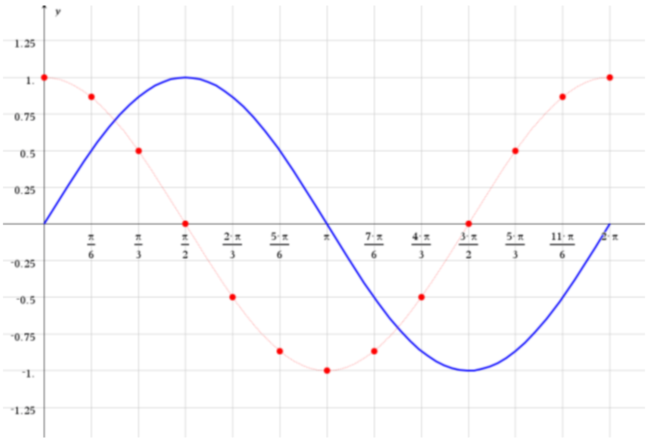
Question: 4.

Plotting the points on the graph reveals that the result appears to be a cosine curve.
Note: The calculator MUST be in radians!

Comment: The Taylor polynomial for sin(x) where x is measured in radians is given by:

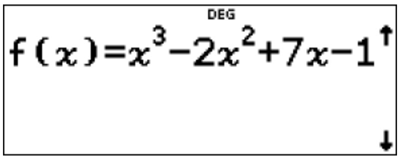
$$s(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!}...$$

The 'power rule' for differentiation can be used to determine the rule for the derivative of sin(x).



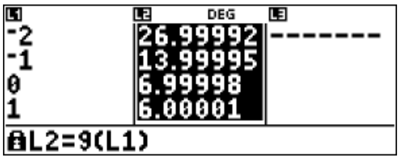
Extension:

Define the function in f(x) and make sure g(x) is expressed as the gradient from first principles. Set d = 0.00001 (small).

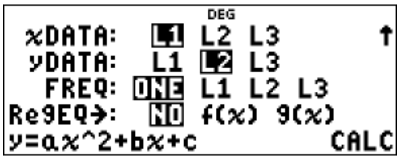


Enter some x values in List 1 and use g(x) [Formula] to generate list 2.

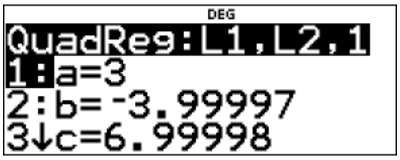
Note that only three x values are required to perform quadratic regression.



Use Quadratic regression on List 1 and List 2. Don't store the equation as it will over-write you current function definitions. The regression analysis will return the coefficients: a, b & c.



The gradient of the function: $f(x) = x^3 - 2x^2 + 7x - 1$ is therefore: $f'(x) = 3x^2 - 4x + 7$. Notice how the exponent for each term changes and the corresponding coefficients.



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