# Approximate Areas

7 8 9 10 **11** **12**

TI-30XPlus MathPrint™ | Worksheet | 15 min

## Calculator Skills:

- Define function: $f(x)$
- Store variables
- Generate and sum lists

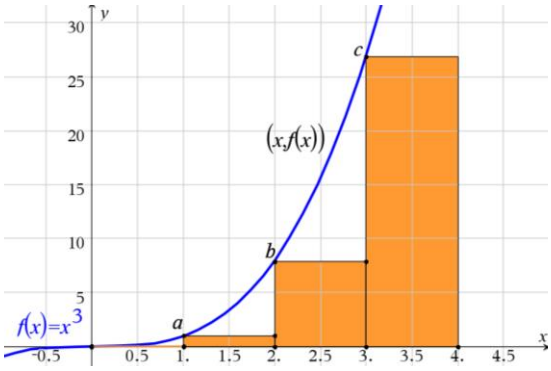
**Formula**:

$$A \approx \sum_{i=1}^{n} f(x_i)\Delta x$$

## Question: 1.

The graph shown opposite is: $f(x) = x^3$.

The rectangles are bound on the left (outer), using a width of 1 unit to approximate the area bound by the curve, the $x$ axis and the lines $x = 0$ to $x = 4$




i) Determine the coordinates of the points a, b and c.

ii) Determine the approximate area according to the graph shown. (Rectangle width = 1 unit)

iii) Determine the approximate area using right bound (outer) rectangles of width 1 unit.

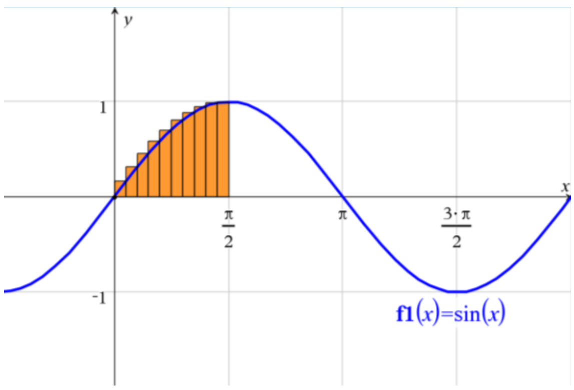
iv) Determine the approximate area using the trapezia.

v) Change the rectangle width to 0.1 units and determine the approximate area using left and right (inner and outer) bound rectangles and hence trapezia.

vi) Using your results, make an 'educated guess' for the exact area.

## Question: 2.

A graph of $y = \sin(x)$ with $x$ measured in radians, is shown opposite. The area is approximated by 10 right bound (outer) rectangles over the domain: $0 \le x \le \frac{\pi}{2}$ .




i) Determine the value of the shaded area.

ii) Use 20 outer bound rectangles to approximate the area over the domain: $0 \le x \le \pi$ .

iii) Use 20 left bound rectangles to approximate the area over the domain: $0 \le x \le \pi$

iv) Which calculation provides the better approximation for this function: 'outer bound' or 'left bound' rectangles?

v) Renee used 20 trapezia over the interval $0 \le x \le \pi$ to approximate the area, how would her result compare to left bound rectangles?

vi) Use the most appropriate method to calculate the approximate area using 40 rectangles over the domain: $0 \le x \le \pi$ and make an educated guess for the exact area.

**Answers on Page 2**



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## Question: 1.

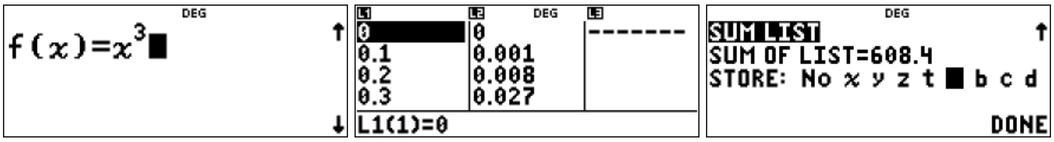
i) a: (1, 1); b: (2, 8); c: (3, 27)

ii) Area = 1 + 8 + 27 = 36 [Left bound or outer bound]

iii) Area = 1 + 8 + 27 + 64 = 100 [Right bound or inner bound]

iv) Area = (36 + 100) ÷ 2 = 68 [Trapezia, average of Left and Right]

v) Define function as f(x), generate x values in List 1 and table in List 2, then sum List 2




Remember to multiple the sum by the column width: 0.1

Left (inner) bound: 60.84

Right (outer) bound: 67.24

Trapezia: 64.04

vi) The area appears to be close to 64. The higher degree polynomial means the 'error' is larger than the quadratic function in the video. The calculations can be repeated using a column width of 0.02, however the lists are limited to 50 elements, so the calculations have to be repeated for intervals: 0 to 1, 1 to 2, 2 to 3 and 3 to 4; this give a result of 64.0016 confirming the 'educated guess' that the limit is approaching 64.

## Question: 2.

i) Shaded area can be computed quickly and efficiently on the calculator: 1.0765.
Make sure the calculator is in radian mode, f(x)=sin(x) and column width is $\pi/20$

ii) Estimating the area with 20 outer bound rectangles from 0 to $\pi$ will simply double the previous answer due to the symmetry of the function: 2.1530

iii) Left bound rectangles require a new calculation as the rectangles beyond $\pi/2$ will be 'inner' bound.
Area ≈ 1.9959

iv) The left bound rectangles from 0 to $\pi$ provide a better approximation as they combine over estimates from 0 to $\pi/2$ and under estimates from $\pi/2$ to $\pi$.

v) The trapezia calculations would provide exactly the same result as left (or right) bound rectangles over the interval from 0 to $\pi$, which would be the same as averaging the inner and outer bound rectangles.
Answer: 1.9959

vi) Using 40 left (or right) bound rectangles will produce the same as 40 trapezia. Area ≈ 1.9990

## Comments:

- Radians are generally used for trigonometric calculations when differential and integral calculus are involved.
- The cosine function is simply a translation of the sine function, so bounded areas are the same.
- Consider the situation where rectangles below the $x$ axis are 'negative' and areas up to key points along the function $f(x) = \sin(x)$ such as $x = \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \ldots\right\}$. The corresponding areas would be:
A = {0, 1, 2, 1, 0, …}. The result is naturally periodic!



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