Vectors in Component Form

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Student Activity - Answers

7 8 9 10 **11** **12**

TI-Nspire & TI-Nspire CAS | Investigation | Student | 30 min

Introduction

Vectors are used to represent quantities that have both magnitude and direction. There are a number of ways that 2D vectors can be represented. One of these representations involves expressing a vector **r** in terms of unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$. This is known as component form and is expressed as $\underset{\sim}{r} = a\underset{\sim}{i} + b\underset{\sim}{j}$. Use the provided tns file and following information to help answer the questions.

Equipment

For this activity you will need a:

- TI-Nspire or TI-Nspire CAS
- TI-Nspire file: Vectors In Component Form

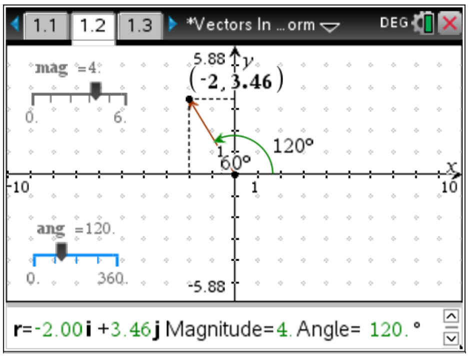
Part of this investigation requires calculations to be performed. Add a Calculator application to perform these calculations.

Visual Representation

Open the TI-Nspire document "Vectors In Component Form".

Navigate to page: 1.2 by pressing ctrl ▶. Use the sliders to rotate the fixed vector around the Cartesian plane.

(**Note:** Only use the sliders to change the magnitude and angle of the vector. **Do not** drag the vector as this will stop the file from working).



Notice that the top slider (**mag**) changes the magnitude (length) of the vector and the bottom slider (**ang**) changes the angle.

Two angles are displayed on the screen. The outside angle represents the angle the vector makes with the positive direction of the horizontal axis and the inside angle is an internal angle of the right-angled triangle thus formed.

At the bottom of the screen, the vector **r** is expressed in component form with magnitude and angle displayed.

(**Note:** It is possible to click on the number given above each slider to change the magnitude and angle size, however do not change the vector by typing over the numbers at the bottom of the page).

Author: S. Julian

Question: 1.

Change the values of the sliders to produce a vector of magnitude 3 that makes an angle of 0° with the positive direction of the horizontal axis; then answer the following questions:

a) Write this vector in component form ($\underset{\sim}{r} = a\underset{\sim}{i} + b\underset{\sim}{j}$).

r = 3**i** + 0**j** (**r** = 3**i**)

b) State where the vector is situated for this angle of 0°?

It is positioned on the horizontal axis pointing in the positive direction

Question: 2.

Change the values of the sliders to produce a vector of magnitude 3 that makes an angle of 90° with the positive direction of the horizontal axis; then answer the following questions:

a) Write this vector in component form ($\underset{\sim}{r} = a\underset{\sim}{i} + b\underset{\sim}{j}$).

r = 0**i** + 3**j** (**r** = 3**j**)

b) State where the vector is situated for this angle of 90°?

It is positioned on the vertical axis pointing in the positive direction

Question: 3.

Change the values of the sliders to produce a vector of magnitude 3 that makes an angle of 60° with the positive direction of the horizontal axis; then answer the following questions:

a) Write this vector in component form ($\underset{\sim}{r} = a\underset{\sim}{i} + b\underset{\sim}{j}$).

r = 1.50**i** + 2.60**j**

b) Which quadrant is the vector situated in for an angle of 60°?

The first quadrant

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Question: 4.

Change the values of the sliders to produce a vector of magnitude 3 that makes an angle of 150° with the positive direction of the horizontal axis; then answer the following questions:

a) Write this vector in component form ($\underset{\sim}{r} = a\underset{\sim}{i} + b\underset{\sim}{j}$).

r = -2.60**i** + 1.50**j**

b) Which quadrant is the vector situated in for an angle of 150°?

The second quadrant

c) Using the answers you have obtained so far, where is the given (outside) angle being measured from?

From the positive side of the horizontal axis in an anti-clockwise direction

d) What is the magnitude of the $\underset{\sim}{i}$ component representing?

The horizontal distance of the end point of the vector from the origin

e) What is the magnitude of the $\underset{\sim}{j}$ component representing?

The vertical distance of the end point of the vector from the origin

Question: 5.

Change the values of the sliders to produce a vector of magnitude 5 that makes an angle of 310° with the positive direction of the horizontal axis; then answer the following questions:

a) Write this vector in component form ($\underset{\sim}{r} = a\underset{\sim}{i} + b\underset{\sim}{j}$).

r = 3.21**i** – 3.83**j**

b) Which quadrant is the vector situated for this angle of 310°?

The fourth quadrant

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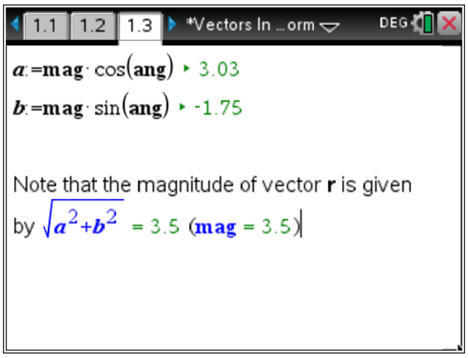
Numerical Representation



Navigate to page: 1.3 by pressing ctrl ▶.

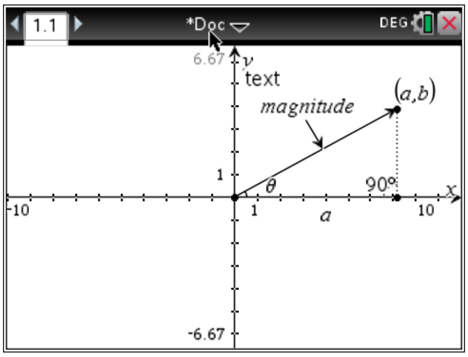
Page: 1.3 displays the magnitude of the **i** and **j** components that are shown on page: 1.2.

Question: 6.

a) Explain why cosine can be used to find the *a*-value (the magnitude of the $\underset{\sim}{i}$ component)?

The coefficient of the **i**-component represents the horizontal distance (base) of a right-angled triangle that has a hypotenuse of length equal to the magnitude of the vector. Using SOH CAH TOA, the horizontal distance that is being found is the adjacent side of this triangle, and therefore cosine is used to find the magnitude.

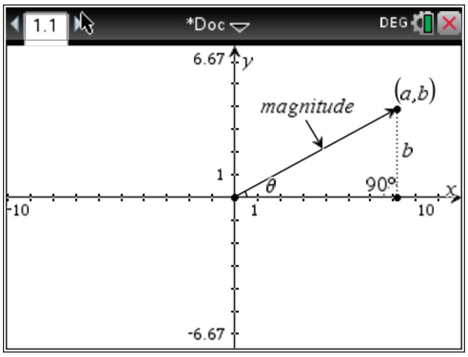
i.e.



b) Explain why sine can be used to find the *b*-value (the magnitude of the $\underset{\sim}{j}$ component)?

The coefficient of the **j**-component represents the vertical distance (height) of a right-angled triangle that has a hypotenuse of length equal to the magnitude of the vector. Using SOH CAH TOA, the vertical distance that is being found is the opposite side of this triangle, and therefore sine is used to find the magnitude.

i.e.



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c) Explain why the magnitude of the vector $\underset{\sim}{r}$ can be calculated using: $\sqrt{a^2+b^2}$.

a represents the horizontal (base) length and *b* represents the vertical (height) length of a right-angled triangle. Since the magnitude is equal to the length of the hypotenuse, Pythagoras' theorem can be used.

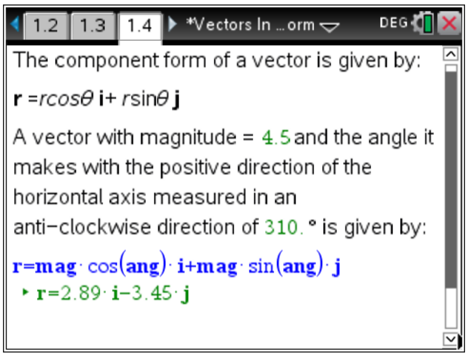
Question: 7.

Using your findings from Question: 6, determine a rule that can be used to find the component form of a vector given the magnitude of the vector $\underset{\sim}{r}$ and the angle it makes with the positive direction of the horizontal axis measured in an anti-clockwise direction, θ.

r = *r*cos θ **i** + *r*sin θ **j**

Navigate to page: 1.4

Confirm your answer for Question: 7.



Extension Questions

Without the use of the TI-Nspire document "Vectors In Component Form", answer the following questions. Then use the "Vectors In Component Form" file to check your answers.

Question: 8.

Express each of the following magnitudes and angles as a vector in component form.

a) Magnitude = 1 and Angle = 20°

r = *r*cos θ **i** + *r*sin θ **j**

= 1 × cos(20°)**i** + 1 × sin(20°)**j**

= 0.94**i** + 0.34**j**

b) Magnitude = 2.5 and Angle = 130°

r = *r*cos θ **i** + *r*sin θ **j**

= 2.5 × cos(130°)**i** + 2.5 × sin(130°)**j**

= -1.61**i** + 1.92**j**

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c) Magnitude = 4 and Angle = 260°

$\mathbf{r} = r\cos\theta\ \mathbf{i} + r\sin\theta\ \mathbf{j}$

$= 4 \times \cos(260°)\mathbf{i} + 4 \times \sin(260°)\mathbf{j}$

$= -0.69\mathbf{i} - 3.94\mathbf{j}$

d) Magnitude = 3.5 and Angle = 330°

$\mathbf{r} = r\cos\theta\ \mathbf{i} + r\sin\theta\ \mathbf{j}$

$= 3.5 \times \cos(330°)\mathbf{i} + 3.5 \times \sin(330°)\mathbf{j}$

$= 3.03\mathbf{i} - 1.75\mathbf{j}$

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