Arithmetic Sequences

T³
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Teacher Notes & Answers

7   8   9   10   **11**   12

TI-30XPlus MathPrint™   Worksheet   Student   45 min

Dining Table

Teacher Notes:

There are several ways students can generate sequences on the TI-30XPlus MathPrint calculcator.

- Recursively using the ANS option
- Storing and recalling values stored in memory
- Sequence command (Lists)
- Function definition

This activity uses each method to ensure students can explore problems using a variety of techniques. A Powerpoint file has also been included for teacher use to help students see the pattern for each question.

The world's longest dining table measured 3,189.83m. The table was used to serve the evening meal to 7000 Muslims at the end of Ramadan. The images below show the configuration of seats around a collection of rectangular dining tables in a typical restaurant.

Question: 1.

i) Assuming the pattern continues, how many people could be seated at the fourth configuration?
    Answer: Each configuration attracts another 2 seats, therefore 10 people.

ii) Explain why the number of seats at successive configurations forms an arithmetic sequence.
    Answer: Each new configuration adds the same quantity of seats (2).

iii) How many people could be seated at the 10th configuration?
    Answer: Students may use formal calculations or create a table to arrive at the answer: 22 people.

iv) The world record breaking dining table was set out similar to the pattern shown above, determine the approximate space (width) available to each person. (Show your calculations)
    Answer: If the table included place settings at either end: $7000 = 4 + 2(x - 1)$ ∴ $x = 3499$. The space for each person was therefore: $3189/3499 \approx 91$cm

There are four ways you can generate these types of sequences on your calculator:

- Recursion (home screen)
- Sequence command (Lists)
- Formulas (memory allocations)
- Functions (efficient use of formulas and memory allocations)

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TEXAS INSTRUMENTS

Using your calculator - Recursion

The first term in the dining table sequence is 4, entering this value may seem trivial but is required to initiate the recursion process.

D

4 enter

The value '4' is now stored as the answer (ANS); this is the value that will change with each calculation.
Now type the following:

⊙► **B**

+ **2** enter

Notice that the abbreviation 'ans' automatically appears. This represents the configuration for the second table. The value stored in 'ans' is now updated to 6.

Continue pressing enter and counting, the screen shown opposite shows this calculation in progress; the most recent calculation is for the number of seats at the 5th table.

Keep counting and pressing enter until you reach the configuration for the 10th table.
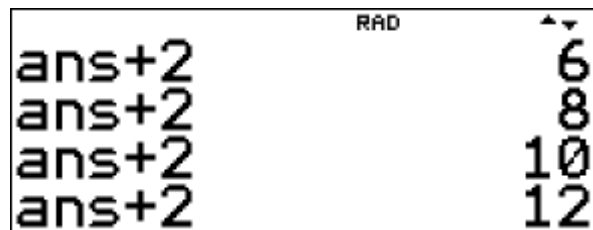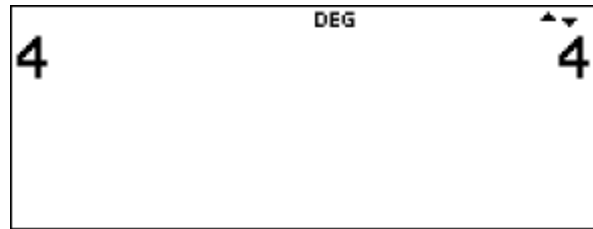
Using your calculator - Sequence

The recursion approach is quick and easy; however, you have to count carefully!

The sequence command can generate the entire sequence and store it in a list. To navigate to the sequence command press:

stat-reg/distr stat-reg/distr   **C**

data **data** **◄** **3**

From here you can select where you want to store the sequence. Navigate to your preferred list, then press:
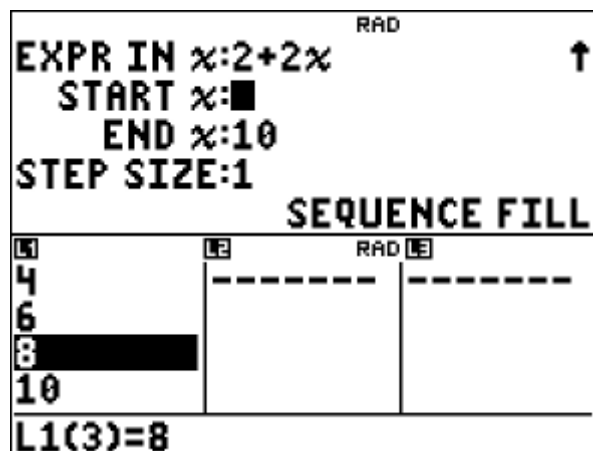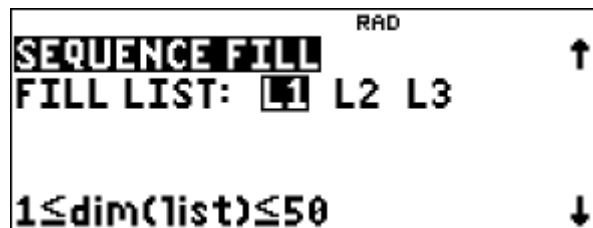
enter

Enter the formula shown opposite: $2x + 2$.

Start the sequence at $x = 1$ and end with $x = 10$ with a step size of 1, then select: Sequence Fill.

The screen shown opposite has L1(3) = 8 highlighted.

The 3 represents the term number in the sequence (table configuration) and the 8 represents the term (quantity of seats), scroll down to see the remaining values.

TEXAS INSTRUMENTS

Using your calculator - Formulas

The nth term of an arithmetic sequence can be computed using the formula:

$$a_n = a_1 + (n-1)d$$

$$a_n = n^{th} \ term \qquad a_1 = first \ term$$
$$n = \ term \ number \qquad d = common \ difference$$

In the dining table problem, the first term is 4 (number of seats at the first table), this value can be stored in 'a' on the calculator.

Note: The variable key is a multi-tap key.

Type in the following:

4 [sto→] [x^{yzt}_{abcd}] ...... [enter]

The next value to store is the common difference d = 2.

Type in the following:

2 [sto→] [x^{yzt}_{abcd}] ...... [enter]

Suppose you want to calculate the 10th term of the sequence, just enter the formula and use '10' for 'n'.

The advantage of this method is that you can simply recall the previous calculation and change the 10 for another term number. Alternatively, if the first term changes, define a new value for 'a' and then recall the expression and the new term will be recalculated.
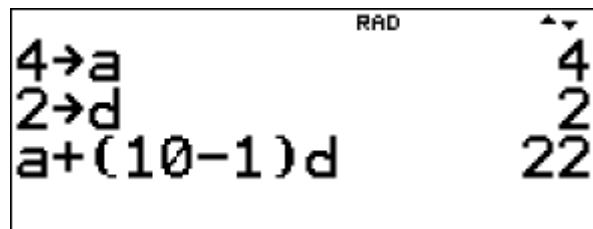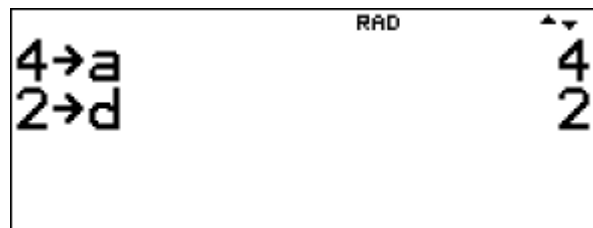
Using your calculator - Functions

This approach uses the formula but has an additional advantage, it is even more convenient! Press:

[table] [enter]

The function editor allows us to define a rule, f(x) has been selected.

Enter the formula as before, but this time use 'x' for the term number, then press enter to lock in the function for f(x), and again to lock in the function for g(x).*

*g(x) could be used for the 'sum' of an arithmetic sequence!

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TEXAS INSTRUMENTS
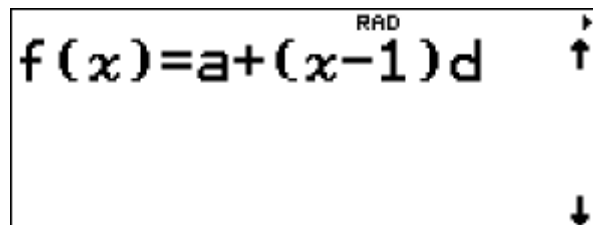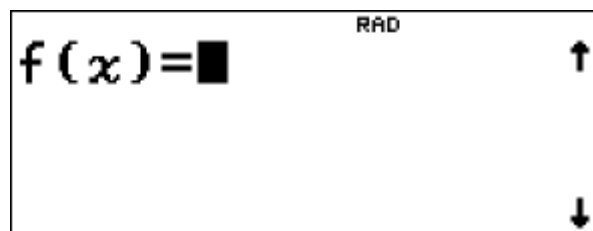
The Table set up can automatically generate a table of values or prompt for any value of x. Match the selections shown opposite and then navigate and accept calculate (calc).
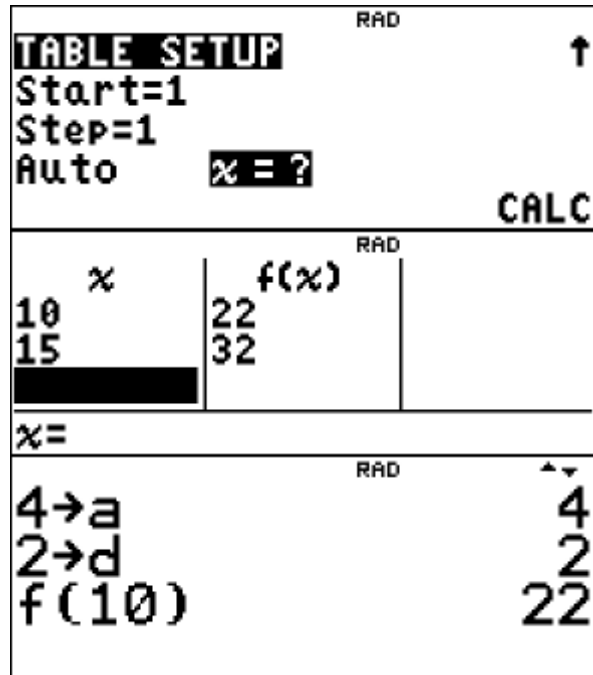
Enter the term number(s) you want to compute and the term will automatically be completed using the stored values for 'a' and 'd'. The table automatically updates when the stored values are changed.

Functions can also be recalled from the home screen.

To access the function, press:

expr-eval   **B**

| table | 2 |

Enter the term number to be calculated and press enter.

Question: 2.

A restaurant has multiple table bookings and configurations. For each of the following configurations in the restaurant, determine the total number of people that can be seated.

i) Craig's restaurant has 6 tables set up as the following configurations: 2, 2, 3, 4, 5 and 5. How many people can be seated in total?
Hint: Use the sequence command and generate a list starting at 1 and ending at 6.
Answer: $t_2 = 6$, $t_3 = 8$, $t_4 = 10$ and $t_5 = 12$ ∴ Total seating = 6 + 6 + 8 + 10 + 12 + 12 = 54

Teacher Notes: Repeated calculations is where the function definition provides an expedient approach.
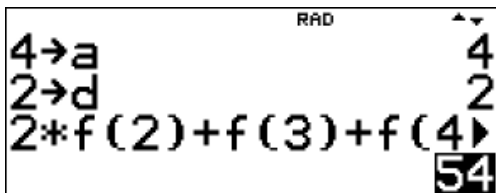
ii) Hayley runs a wedding venue and has 8 tables set up with the following configurations: 6, 6, 7, 7, 8, 8, 9 and 9. How many guests can be seated in total?

Answer: $t_6 = 14$, $t_7 = 16$, $t_8 = 18$ and $t_9 = 20$, students can generate these using the lists and either manually or automatically compute the sum of the list, and double their answer: 136.

Teacher Notes: If the function has already been defined, students could use the same approach as the previous question, uses lists to generate only the values required (see below) or they could use the summation formula!
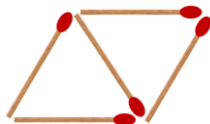
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TEXAS INSTRUMENTS
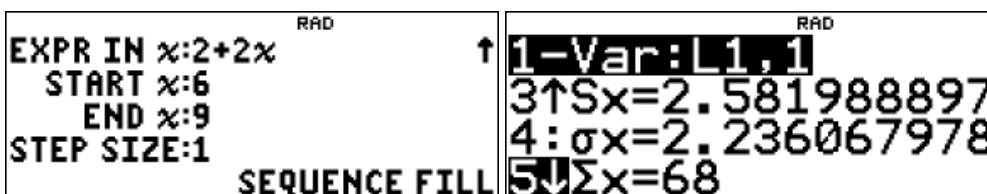
An alternative approach (covered in other activities and videos) would be to use the sum tool located in the maths menu.

Match Stick Patterns

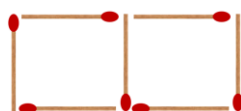| Pattern 1 | Pattern 2 | Pattern 3 | Pattern 4 |

Question: 3.

Matchsticks can be used to form all sorts of patterns. A single box of matches contains precisely 50 matches. The following questions relate to the (above) pattern.

i) How many matches in the first pattern?
   Answer: 3

ii) How many matches are required for each additional pattern?
   Answer: 2

iii) Generate the sequence using a list to help determine quantity of triangles formed using the above pattern, from a single box of matches.
   Answer: 24 (This requires 49 matches.)

iv) Cameron has created 99 triangles. How many boxes of matches did he use?
   Answer: t_{73} = 3 + 98 x 2 = 199 ∴ 4 boxes of matches, with one match left over!

| Pattern 1 | Pattern 2 | Pattern 3 | Pattern 4 |

Question: 4.

A series of squares is formed using matchsticks. Using the pattern above:

i) How many matches in the first pattern?
   Answer: 4

ii) How many matches are required for each additional pattern?
   Answer: 3

iii) Generate the table feature to help determine quantity of squares formed using the above pattern, from a single box of matches.
   Answer: 16 (49 matches required)

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TEXAS INSTRUMENTS