Metallic Numbers

Student Activity

7 8 9 **10** **11** 12

TI-Nspire™ Investigation Student 30 min

Introduction

The famous Fibonacci sequence 1, 1, 2, 3, 5, 8 ... involves the recursive sequence definition: $t_{n+2} = t_n + t_{n+1}$.
The ratio between consecutive terms for the Fibonacci sequence as $n \to \infty$ is known as the Golden Ratio.

$$\text{Golden Ratio: } \lim_{n\to\infty} \frac{t_{n+1}}{t_n} = \phi$$

In this investigation you will explore a small variation on the Fibonacci sequence: $t_{n+2} = t_n + at_{n+1}$ where a is a natural number. In this investigation these variants on the Fibonacci sequence will be referred to as "Levels", for example Fibonacci Level 2 means that $a = 2$ in the recursive definition above. The original Fibonacci sequence is therefore Fibonacci Level 1 with $a = 1$.

Fibonacci Level 2: In search of the Silver Ratio

This sequence starts as: 1, 1, 3, 7, 17, 41, 99 …

Each successive term is equal to "the previous two terms plus another helping of the previous term." This can be expressed more succinctly using mathematical notation as:

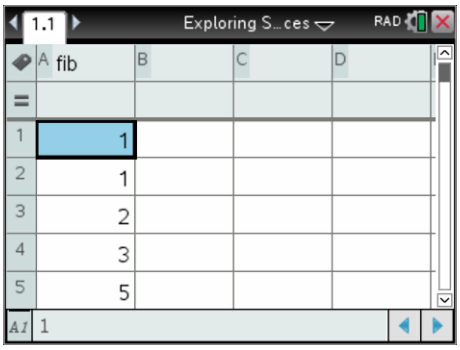
$$t_{n+2} = t_n + 2t_{n+1}$$

The first two numbers can still be set as 1 and 1.

Use either a recursive formula or an appropriate sequence command to generate the first 50 terms of the Fibonacci Level 2 sequence.

Call the list: FIB2

Insert a Calculator Application in preparation for your exploration.



Question: 1.

Explore the ratio between consecutive Fibonacci Level 2 terms, this is called the 'Silver' ratio.
Note: Any term in the sequence can be recalled by typing: FIB2[#] where # represents the term number.

Calculator Tip!

Insert a Notes application and a slider called 'n'. Set the minimum value of the slider to 1, the maximum to 50 with increments of '1'. In a maths box type:

$$\frac{\text{Fib2[n+1]}}{\text{Fib2[n]}}$$

Adjust the slider to see how the ratio between consecutive terms changes.

Author: P. Fox

Question: 2.

Change the first two terms in the Fibonacci Level 2 sequence and check to see if this changes the long term value of the ratio between consecutive terms.

Question: 3.

Let x represent any term in the sequence and y the next term.

a) Explain the two formulas below:

$$r_n = \frac{y}{x} \quad \text{and} \quad r_{n+1} = \frac{2y+x}{y}$$

b) Assuming the ratio between consecutive terms is approximately equal as $n \to \infty$ determine the value of the ratio.

Fibonacci Level 3: In search of the Bronze Ratio

The bronze ratio refers to the ratio between consecutive terms of the level 3 Fibonacci sequence. The general formula for the sequence $t_{n+2} = t_n + at_{n+1}$ therefore becomes: $t_{n+2} = t_n + 3t_{n+1}$

Question: 4.

Create a new list in the spreadsheet application called Fib3, generate the first 50 terms of the level 3 sequence and explore the ratio between consecutive terms as n increases.

Question: 5.

Set up two formulas similar to those from Question 3 and hence determine the exact value for the bronze ratio.

Fibonacci Level *n*: The Metallic Ratios

The general term for the ratio between consecutive terms for $t_{n+2} = t_n + at_{n+1}$ is referred to as a Metallic ratio.

Question: 6.

Determine an expression for the general form of the Metallic ratios. Check your answer using $a=1$, $a=2$ and $a=3$.

Question: 7.

For the golden ratio (ϕ) the following relationships hold:

$$\phi = \frac{1}{\phi} + 1 \qquad \phi^2 = \phi + 1 \qquad \phi^1 + \phi^2 = \phi^3$$

Do any of the above relationships hold for the silver or bronze ratio?

Author: P. Fox

Question: 8.

Calculate the approximate value for each of the following and comment on your finding as the quantity of 'embedded' fractions increases.

a) $1+\dfrac{1}{1+1}$

b) $1+\dfrac{1}{1+\dfrac{1}{1+1}}$

c) $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+1}}}$

d) $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+1}}}}$

e) $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\ldots}}}}}$ (In this case add as many fractions as possible)

Question: 9.

Calculate the approximate value for the following 'embedded' fraction and comment on the result.

$$2+\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{2+\ldots}}}}}$$

Author: P. Fox