Equation of a Circle
Teacher Notes

7 8 9 **10** 11 12

TI-Nspire CAS | Investigation | Student | 120 min

Aim
The aim of this investigation is to develop the equation of a circle of radius, *r*, centred at the origin, and to explore how the equation changes when the centre is translated parallel to the *x* and *y* axes.

National Curriculum Statement: Describe, interpret and sketch parabolas, hyperbolas, **circles** and exponential functions and their transformations (ACMNA267)

Elaboration: Applying transformations, including translations, reflections in the axes and stretches to help graph parabolas, rectangular hyperbolas, **circles** and exponential functions.

Equipment
For this activity you will need:

- TI-Nspire CAS
- TI-Nspire CAS document – Equation of a circle

Introduction – Setting up the calculations
During this activity, students will need to use the TI-Nspire file: “Equation of a circle”. This file can be distributed using TI-Navigator, the TI-Nspire docking station or the teacher/student software. To distribute the file using the Teacher software, use the **Tools** menu and select the **Transfer Tool.** Locate the TI-Nspire file on your computer and then start the transfer. Once the file is transferred to the first handheld, unplug the handheld and continue plugging in each student’s handheld device. Once all the students have the file, stop the transfer. Note that students can also transfer files from one handheld device to another from within the **My Documents** folder. Note also that multi-port USB connectors can be used to transfer files to several computers at the one time.

Part 1 – Equation of a circle centred at the origin
This activity requires access to the “Equation of a circle” TI-Nspire document. This document should be loaded on your device before proceeding.

Once the document is on your handheld, press [**home**] and select **My Documents**. Locate the document and press [**enter**] to open.

The location of the file depends on the selected location during the file transfer.

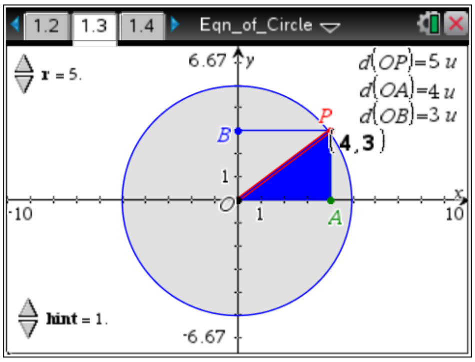
Author: Frank Moya

Step 1 – Adjust the radius of the circle

Navigate to page **1.3** and use the spinner to adjust the radius of the circle to 5 units.

To adjust the spinner, select it by moving the cursor to the spinner and pressing [**enter**]. Then press the up and down arrows to adjust the radius. To deselect the spinner, press [**esc**].

The point ***P*** is a movable point on the circle. Points ***A*** and ***B*** mark the coordinates of *P* on the *x* and *y* axes, respectively.

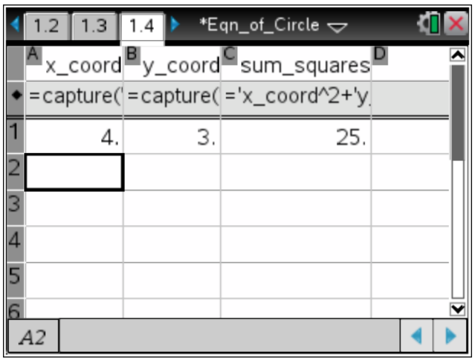


Step 2 – Observe the data capture page

Navigate to page **1.4**. The distances from the origin to *P*, *A* and *B*, d(*OP*), d(*OA*) and d(*OB*), respectively, are shown.

As *P* moves around the circle, the values of the *x* and *y* coordinates of *P* are captured in columns *A* and *B* on page **1.4**.

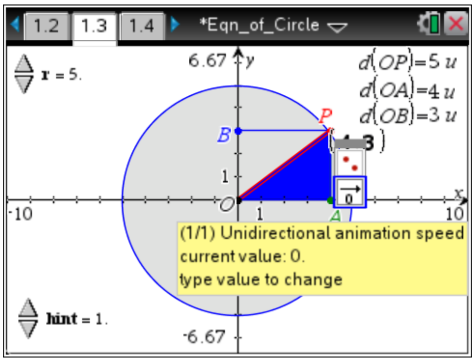
The value of $x^2 + y^2$, the sum of squares of the coordinates of *P*, is automatically calculated in column **C**.



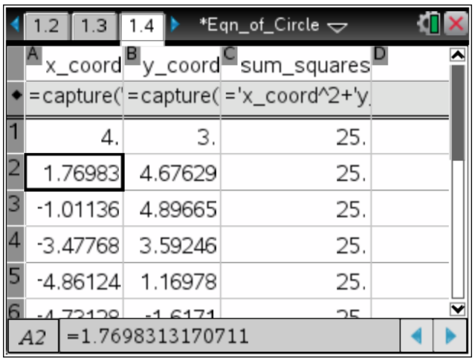
Step 3 – Animate point *P* around circle of radius 5 and capture the coordinates

Navigate back to page **1.3**. To animate the point:

- Move the cursor to point *P* until the 'open hand' symbol appears.
- Press [**ctrl]** > **[menu]**. From the context menu that appears, select **Attributes**.
- Press the down arrow once, input the number [**9**] (fast speed) and press [**enter**].
- When point *P* has moved through a complete revolution around the circle, press [**esc**] to stop the animation.



Navigate back to page **1.4**. The *x* and *y* coordinates will be captured and $x^2 + y^2$ calculated.



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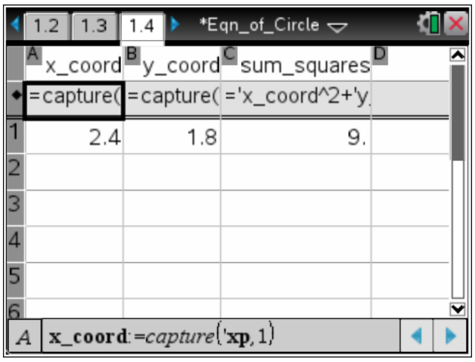
Later in the activity, you will be asked to clear this collected data. This will enable you to collect new data for circles with different radii. To clear the collected data, move the cursor into each of the ‘**capture**’ or ‘**coord**’ cells in the second row and press [**enter**] twice. If you see any messages or warnings, select **OK**.



Questions

1. What do you notice about the value of the sum of squares, $x^2 + y^2$, irrespective of the coordinates of *P*?

When the radius of the circle is 5, the value of the sum of squares of the coordinates of *P*, ($x^2 + y^2$) is equal to 25 in all cases.

2. Explain why the value of the sum of squares ($x^2 + y^2$) is unchanged despite the *x* and *y* coordinates of *P* changing (Hint: look at ΔOPA and ΔOPB).

Triangles *OPA* and *OPB* are right-angled triangles with hypotenuse *OP*. By Pythagoras’ Theorem, the square of the hypotenuse, $(d(OP))^2$, is always equal to $(d(OA))^2 + (d(OP))^2 = x^2 + y^2$.

3. Use your findings from Questions 1 and 2 above to complete the equation of a circle of radius 5:

$x^2 + y^2 = 25$

Step 4 – Change the radius of the circle and capture the *x* and *y* coordinates of *P*

- Use the spinner on page **1.3** to change the radius of the circle to a value of your choosing.
- Navigate to page **1.4** and reset the data in columns **A**, **B** and **C**.

Step 5 – Animate *P* around the circle of your chosen radius and capture the coordinates

- Use the method described in **Step 3** above to animate the point ***P***.
- Navigate to page **1.4** and observe the value of $x^2 + y^2$ in the cells of column **C**.

Repeat Steps 4 and 5 above for three other values of *r*.

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4. For the values of *r* that you investigated, complete the table for the equations of the circles.

Radius	**Sum of squares of the coordinates of *P* (column C of page 1.4)**	**Equation of circle**
$r=5$	$(\text{d}(OA))^2+(\text{d}(OB))^2=25$	$x^2+y^2=25$
The answers below will vary, depending of the values of r chosen by the student. All answers will be of the form $x^2+y^2=r^2$.		
$r=2$	$(\text{d}(OA))^2+(\text{d}(OB))^2=4$	$x^2+y^2=4$
$r=3$	$(\text{d}(OA))^2+(\text{d}(OB))^2=9$	$x^2+y^2=9$
$r=4$	$(\text{d}(OA))^2+(\text{d}(OB))^2=16$	$x^2+y^2=16$
$r=6$	$(\text{d}(OA))^2+(\text{d}(OB))^2=36$	$x^2+y^2=36$

5. Use what you have learnt from completing the table above to write down the radius, *r*, of the circles with the following equations.

(a) Equation: $x^2+y^2=100$, $r=10$

(b) Equation: $x^2+y^2=81$, $r=9$

(c) Equation: $x^2+y^2=1$, $r=1$

(d) Equation: $x^2+y^2=2$, $r=\sqrt{2}$

(e) Equation: $x^2+y^2=10$, $r=\sqrt{10}$

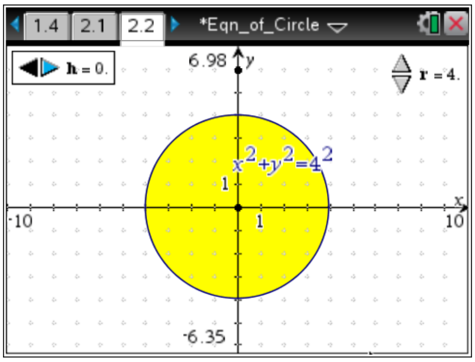
(f) Equation: $x^2+y^2=\frac{1}{4}$, $r=\frac{1}{2}$

Reflection on the learning: discussion of the above results and the sharing of learning with others should provide opportunities for students to reflect on and formalise the **key idea**: the equation of a circle of radius *r*, centred at the origin, is $x^2+y^2=r^2$. Conversely, if a circle has equation $x^2+y^2=a$, this is equivalent to $x^2+y^2=\left(\sqrt{a}\right)^2$, and the circle is therefore centred at the origin and has a radius of $\sqrt{a}$.

Part 2 – Equation of a circle translated parallel to the *x*-axis

Navigate to page **2.2** of the "Equation of a circle" TI-Nspire document. Use the spinner labelled **r** to adjust the radius of the circle to 4 units. The equation of the circle is displayed as $x^2 + y^2 = 4^2$.

Use the spinner labelled **h** to translate an image of the circle to the left and to the right. Note the equation and coordinates of the centre of the translated circle.



6. Use your observations about the effect of changing the value of h to complete the table below.

Radius	Translation parallel to *x*-axis	Coordinates of centre	Equation of circle
$r = 4$	$h = 0$	(0, 0)	$x^2 + y^2 = 16$
$r = 4$	$h = 1$	(1, 0)	$(x - 1)^2 + y^2 = 16$
$r = 4$	$h = 2$	(2, 0)	$(x - 2)^2 + y^2 = 16$
$r = 4$	$h = 5$	(5, 0)	$(x - 5)^2 + y^2 = 16$
$r = 4$	$h = 9$	(9, 0)	$(x - 9)^2 + y^2 = 16$
$r = 4$	$h = -1$	(–1, 0)	$(x + 1)^2 + y^2 = 16$
$r = 4$	$h = -2$	(–2, 0)	$(x + 2)^2 + y^2 = 16$
$r = 4$	$h = -4$	(–4, 0)	$(x + 4)^2 + y^2 = 16$
$r = 4$	$h = -6$	(–6, 0)	$(x + 6)^2 + y^2 = 16$
$r = 4$	$h = -8$	(–8, 0)	$(x + 8)^2 + y^2 = 16$

7. Explore other values of r and h and record the results in the table below.

Radius	Translation	Centre	Equation of circle
Results will depend on the values of r and h chosen. The centre will be of the form (h, 0) and the equation of the circle of the form $(x - h)^2 + y^2 = r^2$			
$r =$ 2	$h =$ 4	(4 , 0)	$(x - 4)^2 + y^2 = 4$
$r =$ 3	$h =$ 2	(2 , 0)	$(x - 2)^2 + y^2 = 9$
$r =$ 5	$h =$ –5	(–5 , 0)	$(x + 5)^2 + y^2 = 25$
$r =$ 6	$h =$ –1	(–1 , 0)	$(x + 1)^2 + y^2 = 36$

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8. Use what you have learnt from completing the tables above to write down the radius, *r*, and the coordinates of the centres of the circles with the following equations.

(a) Equation: $(x-2.5)^2+y^2=64$, $r=8$ centre (2.5, 0)

(b) Equation: $\left(x+\frac{1}{2}\right)^2+y^2=400$, $r=20$ centre $\left(-\frac{1}{2},0\right)$

(c) Equation: $(x+5)^2+y^2=6$, $r=\sqrt{6}$ centre (–5, 0)

(d) Equation: $\left(x-\frac{8}{3}\right)^2+y^2=36$, $r=6$ centre $\left(\frac{8}{3},0\right)$

(e) Equation: $(x-12)^2+y^2=1$, $r=1$ centre (12, 0)

(f) Equation: $(x+8.3)^2+y^2=14$, $r=\sqrt{14}$ centre (–8.3, 0)

9. Use what you have learnt to write the equations of the circles with the following radii and centres.

(a) Radius: $r=9$, centre: $(11,0)$ Equation $(x-11)^2+y^2=81$

(b) Radius: $r=1$, centre: $\left(-\frac{1}{3},0\right)$ Equation $\left(x+\frac{1}{3}\right)^2+y^2=1$

(c) Radius: $r=7$, centre: $(3.4,0)$ Equation $(x-3.4)^2+y^2=49$

(d) Radius: $r=\sqrt{5}$, centre: $(-6,0)$ Equation $(x+6)^2+y^2=5$

(e) Radius: $r=16$, centre: $\left(\frac{12}{5},0\right)$ Equation $\left(x-\frac{12}{5}\right)^2+y^2=256$

(f) Radius: $r=2\sqrt{3}$, centre: $(-10,0)$ Equation $(x+10)^2+y^2=12$

(Note: $\left(2\sqrt{3}\right)^2=2^2\times\left(\sqrt{3}\right)^2=12$)

Reflection on the learning: discussion of the above results and the sharing of learning with others should provide opportunities for students to reflect on and formalise the **key idea**: the equation of a circle of radius *r*, centred at (*h*, 0), can be viewed as a translation of the circle $x^2 + y^2 = r^2$, *h* units parallel to the positive *x*-axis. The equation of the image under the translation is $(x - h)^2 + y^2 = r^2$. When $h < 0$, say, $h = -3$, the translation will be 3 units to the left and the equation of the image will be $(x - (-3))^2 + y^2 = r^2$, or less cumbersome: $(x + 3)^2 + y^2 = r^2$.

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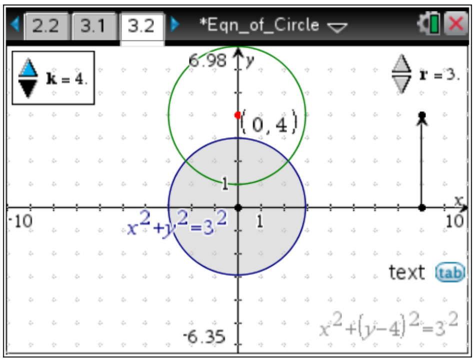
Part 3 – Equation of a circle translated parallel to the *y*-axis

Navigate to page **3.2** of the "Equation of a circle" TI-Nspire document. Use the spinner labelled **r** to adjust the radius of the circle to 3 units. The equation of the circle is displayed as $x^2 + y^2 = 3^2$.

Use the spinner labelled **k** to translate an image of the circle to up and down. Note the equation and coordinates of the centre of the translated circle.



10. Use your observations about the effect of changing the value of *k* to complete the table below.

Radius	Translation parallel to *y*-axis	Coordinates of centre	Equation of circle
$r = 3$	$k = 0$	(0, 0)	$x^2 + y^2 = 9$
$r = 3$	$k = 1$	(0, 1)	$x^2 + (y - 1)^2 = 9$
$r = 3$	$k = 3$	(0, 3)	$x^2 + (y - 3)^2 = 9$
$r = 3$	$k = 5$	(0, 5)	$x^2 + (y - 5)^2 = 9$
$r = 3$	$k = -2$	(0, –2)	$x^2 + (y + 2)^2 = 9$
$r = 3$	$k = -4$	(0, –4)	$x^2 + (y + 4)^2 = 9$
$r = 3$	$k = -6$	(0, –6)	$x^2 + (y + 6)^2 = 9$

11. Explore other values of *r* and *k* and record the results in the table below.

Radius	Translation	Centre	Equation of circle
Results will depend on the values of *r* and *k* chosen. The centre will be of the form (0, *k*) and the equation of the circle of the form $x^2 + (y - k)^2 = r^2$			
$r = 2$	$k = 3$	(0, 3)	$x^2 + (y - 3)^2 = 4$
$r = 4$	$k = 5$	(0, 5)	$x^2 + (y - 5)^2 = 16$
$r = 5$	$k = -2$	(0, –2)	$x^2 + (y + 2)^2 = 25$
$r = 6$	$k = -6$	(0, –6)	$x^2 + (y + 6)^2 = 36$

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12. Use what you have learnt from completing the tables above to write down the radius, r, and the coordinates of the centres of the circles with the following equations.

(a) Equation: $x^2+\left(y+\frac{3}{4}\right)^2=1$, $r=1$ centre $\left(0,-\frac{3}{4}\right)$

(b) Equation: $x^2+(y-5.3)^2=49$, $r=7$ centre (0, 5.3)

(c) Equation: $x^2+(y-9)^2=7$, $r=\sqrt{7}$ centre (0, 9)

(d) Equation: $x^2+(y+5)^2=\frac{16}{9}$, $r=\frac{4}{3}$ centre (0, –5)

13. Use what you have learnt to write the equations of the circles with the following radii and centres.

(a) Radius: $r=4$, centre: $(0,-3)$ Equation $x^2+(y+3)^2=16$

(b) Radius: $r=1$, centre: $\left(0,\frac{2}{3}\right)$ Equation $x^2+\left(y-\frac{2}{3}\right)^2=1$

(c) Radius: $r=\sqrt{3}$, centre: $(0,1.5)$ Equation $x^2+(y-1.5)^2=3$

(d) Radius: $r=\frac{5}{2}$, centre: $\left(0,-\frac{1}{2}\right)$ Equation $x^2+\left(y+\frac{1}{2}\right)^2=\frac{25}{4}$

Reflection on the learning: discussion of the above results and the sharing of learning with others should provide opportunities for students to reflect on and formalise the **key idea**: the equation of a circle of radius r, centred at $(0, k)$, can be viewed as a translation of the circle $x^2 + y^2 = r^2$, k units parallel to the positive y-axis. The equation of the image under the translation is $x^2 + (y - k)^2 = r^2$. When $k < 0$, say, $k = -2$, the translation will be 2 units down and the equation of the image will be $x^2 + (y - (-2))^2 = r^2$, or less cumbersome: $x^2 + (y + 2)^2 = r^2$.

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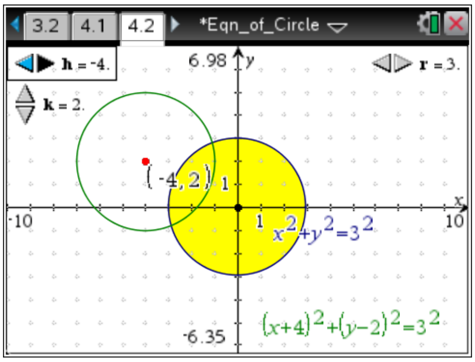
Part 4 – Equation of a circle with centre at any point on the plane

Navigate to page **4.2** of the "Equation of a circle" TI-Nspire document. Use the spinner labelled **r** to adjust the radius of the circle to the value specified in the table below.

Use the spinners labelled **h** and **k** to translate the centre of an image of the circle to the coordinates specified in the table below.



14. Note the equation of the translated circle and of the value of **h** and **k** that translate the centre of the circle from the origin to the desired point on the plane. Complete the table below.

Radius	Coordinates of the centre	Value of *h*	Value of *k*	Equation of circle
$r=3$	(2, 1)	2	1	$(x-2)^2+(y-1)^2=9$
$r=4$	(1, 2)	1	2	$(x-1)^2+(y-2)^2=16$
$r=5$	(−3, 2)	−3	2	$(x+3)^2+(y-2)^2=25$
$r=3$	(3, −2)	3	−2	$(x-3)^2+(y+2)^2=9$
$r=2$	(−6, −2)	−6	−2	$(x+6)^2+(y+2)^2=4$
$r=4$	(6, 2)	6	2	$(x-6)^2+(y-2)^2=16$
$r=3$	(−5, −4)	−5	−4	$(x+5)^2+(y+4)^2=9$
$r=2$	(−5, 3)	−5	3	$(x+5)^2+(y-3)^2=4$
$r=1$	(5, −3)	5	−3	$(x-5)^2+(y+3)^2=1$
$r=5$	(−4, −1)	−4	−1	$(x+4)^2+(y+1)^2=25$

15. Use what you have learnt from completing the table above to write down the radius, **r**, and the coordinates of the centres of the circles with the following equations.

(a) Equation: $(x-3.5)^2+(y+1.2)^2=1$ $r=1$ centre (3.5, −1.2)

(b) Equation: $(x+2.7)^2+(y+3.1)^2=64$ $r=8$ centre (−2.7, −3.1)

(c) Equation: $\left(x-\frac{7}{8}\right)^2+\left(y-\frac{9}{10}\right)^2=144$ $r=12$ centre $\left(\frac{7}{8},\frac{9}{10}\right)$

(d) Equation: $(x+6.4)^2+(y-5.1)^2=\frac{25}{4}$ $r=\frac{5}{2}$ centre (−6.4, 5.1)

(e) Equation: $\left(x-\frac{5}{6}\right)^2+(y+3)^2=6$ $r=\sqrt{6}$ centre $\left(\frac{5}{6},-3\right)$

(f) Equation: $(x+0.7)^2+(y+1.3)^2=2.25$ $r=1.5$ centre (−0.7, −1.3)

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Reflection on the learning: discussion of the above results and the sharing of learning with others should provide opportunities for students to reflect on and formalise the **key idea**: the equation of a circle of radius r, centred at (h, k), can be viewed as a translation of the circle $x^2 + y^2 = r^2$, h units parallel to the positive x-axis and k units parallel to the positive y-axis. The equation of the image under the translation is $(x - h)^2 + (y - k)^2 = r^2$. When $h < 0$ and/or $k < 0$, say, $h = -1$ and $k = -4$, the translation will be 1 unit left and 4 units down. The equation of the image will be $(x - (-1))^2 + (y - (-4))^2 = r^2$, or less cumbersome: $(x + 1)^2 + (y + 4)^2 = r^2$.

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