Percentiles & *z*-scores

About the Lesson

In this activity, students will be asked to calculate percentiles, *z*-scores, and probabilities using normal distributions. As a result, students will:

- Be able to graphically determine percentiles, *z*-scores, and probabilities using normal distributions.
- Be able to use calculations to determine percentiles, *z*-scores, and probabilities using normal distributions.

Vocabulary

- standardized normal curves
- *z*-scores
- Percentiles

Teacher Preparation and Notes

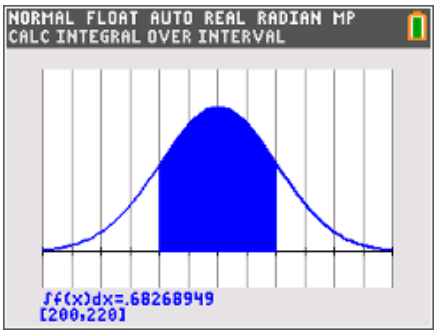
- The graphical display of the normal probability graph greatly helps students with the understanding of this topic. It might be best to have the normal probability graph drawn on the board for easy reference as the class progresses through the activity.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

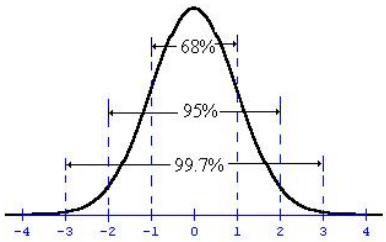
- Percentiles_and_z_scores_Student.pdf
- Percentiles_and_z_scores_Student.doc

Review of Normal Curve Properties

Before beginning the activity, review with students that a density curve has the following properties:

- A smooth curve
- Is always on or above the horizontal axis
- An area of exactly 1 underneath it
- An area under the curve within a range of values is the proportion of all observations that fall in the range

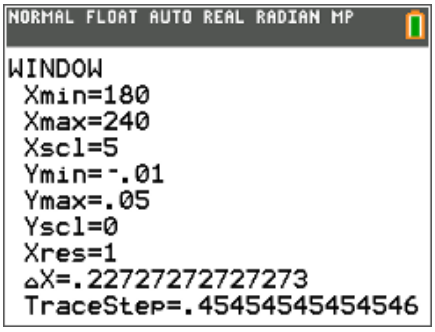
The first page of the worksheet is meant as a review of the properties of normal curves along with a visual representation of the corresponding percentages. Students should know this diagram to allow them to make connections and see relationships. This page reviews *z*-scores and percentiles.



Problem 1 – Given *x*-values, Finding Percentages

Students are given a range of *x*-values and asked to find the probability (or percent) of data that fall within the interval. The worksheet provides step-by-step instructions to find the answer using a graph.

Setting up the window is a challenge for students. The *x*-values should be the mean + or – three standard deviations. The *y*-values need to be small. Encourage students to think about a rectangle with a base of *x*-length and an area of 1. This can become the starting point for the **ymax** value. Use a **ymin** that is negative and with a similar magnitude as the **ymax**.



1. Using the normal distribution graph from earlier, determine the two *x*-values where 99.7% of the data will fall between.

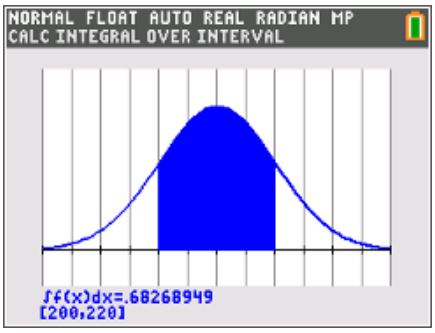
Answer: 180 and 240

2. The area under the curve must be 1. What is a reasonable maximum height for this curve thinking of the *x*-values from Question 1?

Answer: 0.05

Students will then use the **Integral** command ($\int f(x)dx$) from the **Calc** menu (2nd trace) to calculate the area under the curve. It may be important to give a short explanation as to why "integral" is used and not "area." Students do not need to understand the calculus concepts.

A graphical representation (solution) is shown. It is a normal distribution with a mean of 210 and a standard deviation of 10. This graph is helpful because it allows students to visualize what is a difficult topic for many.

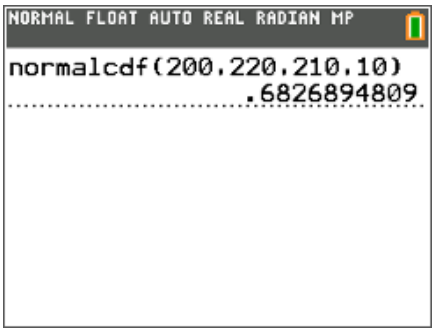


3. What percent of candy bars contain between 200 and 220 calories?

Answer: 68.3%

A numerical solution is also presented. The values students are to enter are shown at the right.

This problem could have been solved without a calculator if one realizes that the values are both 1 standard deviation from the mean.



4. What percent of candy bars contain between 200 and 220 calories?

Answer: 68.3%

5. The length of useful life of a fluorescent tube used for indoor gardening is normally distributed. The useful life has a mean of 600 hours and a standard deviation of 40 hours. Determine the probability that

a. A tube chosen at random will last between 620 and 680 hours.

Answer: 28%

b. Such a tube will last more than 740 hours.

Answer: 0.00023%

Teacher Tip: Ask:

- Is there a difference between the probability of $x > 200$ and $x \geq 200$? Why not? What is the probability $x = 200$?
- What should the endpoint be when only one is given? Can any value be chosen or will any "big" number suffice?
- Why is there not a 100th percentile? Why is there not a 0th percentile?

Problem 2 – Given Percentages, Finding *x*-values

This problem begins with questions that should be answered using the diagram on the worksheet as a reference. The answers are to be written in terms of the mean and standard deviation.

6. Using the mean, μ, and the standard deviation, σ, describe the *x*-value that corresponds to the

a. 50th percentile.

Answer: mean

b. 16th percentile.

Answer: A value one standard deviation below the mean

c. 84th percentile.

Answer: A value one standard deviation above the mean

7. How tall is Mike?

Answer: 75.8 inches

It is important to discuss Question 8. Drawing a diagram helps students visualize what is happening. Students will want to use 4% as the percentage, but they need to recognize that the warranty is more than the average life expectancy, so the percentage is actually 96%. The answer should also be "rounded" up because they can afford to replace up to 4%. So, the answer is 17 months.

8. The lifetimes of zip drives marketed by *Zippers, Inc.* are normally distributed, with a mean lifetime of 11 months and a standard deviation of 3 months.

Zippers, Inc. plans to offer a new warranty guaranteeing the replacement of failed zip drives during the warranty period. It can afford to replace up to 4 percent of its drives.

How many months of warranty should the company offer with these drives? Round your answer to the nearest month.

Answer: 17 months

9. Final grade averages are typically approximately normally distributed with a mean of 72 and a standard deviation of 12.5. Your professor says that the top 8% of the class will receive a grade of A; the next 20%, B; the next 42%, C; the next 18%, D; and the bottom 12%, F.

a. What average must you exceed to obtain an A?

Answer: Because As are the top 8%, this means As are the 72nd to 92nd percentile. Using the **InvNorm** command, the average needed to obtain an A is 89.6

b. What average must you exceed to receive a grade better than a C?

Answer: Because Cs are better than 70%, Cs are between 30th and 72nd percentile. Using the **InvNorm** command, the average needed to obtain a C is 79.3.

c. What average must you obtain to pass the course?

Answer: Because Ds are better than 82%, Ds are between the 12th percentile and 30th percentile. Using the **InvNorm** command, the average needed to obtain a D is 57.3.

Problem 3 – Given *z*-scores, Finding Percentiles and *x*-values

Students are given a *z*-score and asked to find the corresponding percentile. This begins with questions that should be answered using the diagram on page 1.

10. Use the diagram from the first page of the worksheet to help answer the following statements.

a. The *x*-value with a *z*-score = 0 is in the _____ percentile.

Answer: 50th

b. The *x*-value with a *z*-score = –3 is in the _____ percentile.

Answer: 0.15th

c. The *x*-value with a *z*-score = 2 is in the _____ percentile.

Answer: 98.75th

11. Using the mean, μ, and the standard deviation, σ, describe each of the following:

a. The *x*-value with a *z*-score = 0

Answer: the mean

b. The *x*-value with a *z*-score = –3

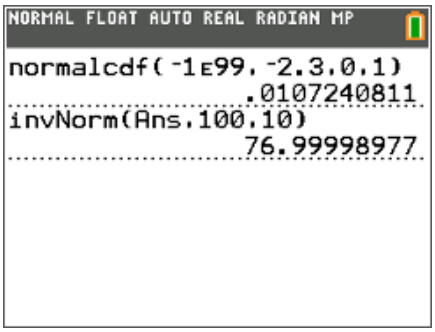
Answer: 3 standard deviations below the mean

c. The *x*-value with a z-score = 2

Answer: 2 standard deviations above the mean

After students graph the normal curve and plot the point (–2.3, 0) explain to students that the area under the curve from the left to –2.3 is the corresponding percentile. This value should be found as before using the **Integral** command. However, the left bound is the **xmin**.

Finding the percentile can be done without graphing. Students will need to type **normcdf(–1E99, –2.3, 0, 1)**.



12. What is the corresponding percentile and *x*-value that has a *z*-score = –2.3 with mean = 100 and standard deviation = 10?

Answer: The percentile is 1.07% and the corresponding x-value is 76.99.

13. In a field, the heights of sunflowers are normally distributed with a mean of 72 inches and standard deviation of 4 inches. Find the corresponding percentile and *x*-value for a sunflower that has a *z*-score of 1.6.

Answer: The percentile is 94.5% and the corresponding *x*-value is 78.4 inches.

14. The shoe sizes of a men's basketball team are normally distributed with a mean of 11.5 and a standard deviation of 1.25. Find the corresponding percentile and *x*-value for a player that has a *z*-score of –3.1.

Answer: The percentile is 0.0967% and the corresponding *x*-value is a size 7.6 shoe.

Question 15 is not straightforward and requires a student to think about what is being asked and make connections. A picture helps students visualize what is being asked.

Have students begin by calculating the *z*-score for the 5th percentile (**invNorm(.05)** = –1.64). Note that for the **invNorm** command the mean is 0 and the standard deviation is 1.

Then have them use the formula $z = \frac{x - \mu}{\sigma}$ to find the mean.

15. A machine is programmed to fill 10-oz containers with a cleanser. However, the variability inherent in any machine causes the actual amounts of fill to vary. The distribution is normal with a standard deviation of 0.02 oz. What must the mean amount be in order for only 5% of the containers receive less than 10-oz? (You will need to use the formula for finding a *z*-score.)

Answer: $-1.64 = \frac{10 - \mu}{.02}$

$10.03 = \mu$

For Question 16, have students begin by calculating the *z*-score for the 3rd percentile (**invNorm(.03) =** –1.88). Then have them find the mean.

16. The weights of ripe watermelons grown at Mr. Smith's farm are normally distributed with a standard deviation of 2.8 lb. Find the mean weight of Mr. Smith's ripe watermelons if only 3% weigh less than 15 lb. (You will need to use the formula for finding a *z*-score.)

Answer: $-1.88 = \frac{15 - \mu}{2.8}$

$20.26 = \mu$