

Graphical designs

Teacher Notes and Answers

7 8 **9** 10 11 12



TI-Nspire



Investigation



Student



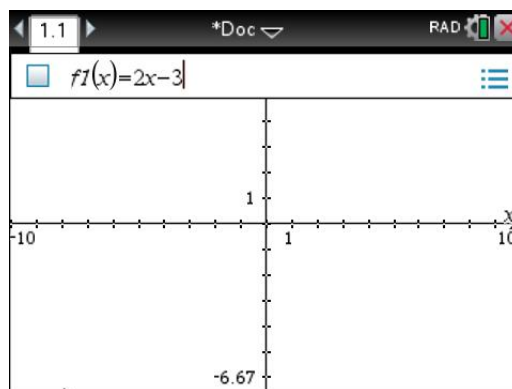
90 min

Introduction

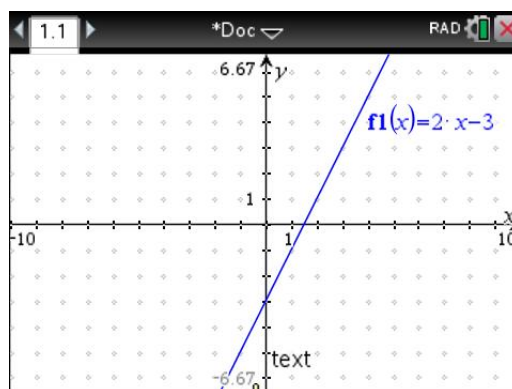
Function graphs can be used to create various designs. In this task you will be challenged to reproduce designs using your knowledge of linear and quadratic functions and their graphs. We will start by reviewing the plotting of linear function graphs on the TI-Nspire CAS, using the linear function with rule $y = 2x - 3$.

On the TI-Nspire CAS

- Press **HOME-1** to create a new document, and then press **2** to add a **Graphs** page.
- Type the rule **f1(x) = 2x - 3** and then press **ENTER**.
- Press **MENU-2-6** to select **View** and then press **2** (Dot Grid).



The graph should appear as shown at right. (Note the function label has been retained in this case.) The end values of the axes show that the function is graphed in the window with dimensions $[-10, 10]$ by $[-6.67, 6.67]$, which is the window we will use for this exploration. This window has the advantage that it uses the same scale on both the x - and the y -axes.



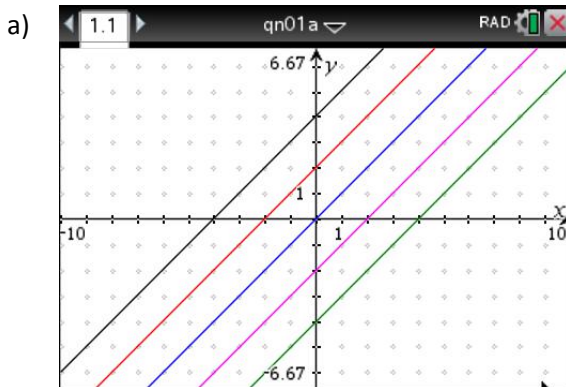
Part 1: Designs with lines

A linear function has rule of the form $y = ax + b$. In the example above, we chose $a = 2$ and $b = -3$. Try various values of a and b to generate the following designs.

Question 1.

Use your knowledge of linear function graphs to construct the following designs in the standard window shown. (Press **MENU-4** to select **Window** and then press **5** to select **Zoom – Standard**.) When you have successfully completed each design, record the function rules that you have used.

[Note: To edit an existing rule or to enter additional rules, press either **TAB** or **CTRL-G**. This will display the function rule editing line at the top of the page. After each design has been completed, you can delete all the graphs in one step by pressing **MENU-1-6**.]



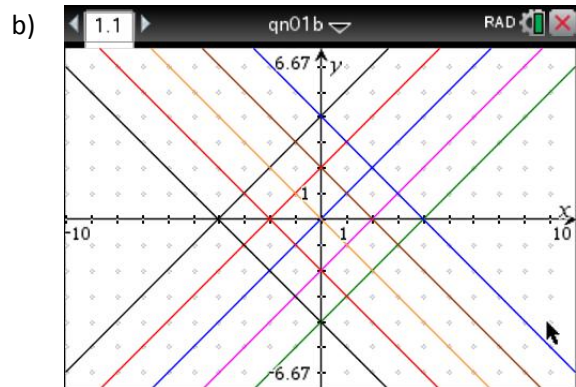
$$y = x$$

$$y = x+2$$

$$y = x+4$$

$$y = x-2$$

$$y = x-4$$



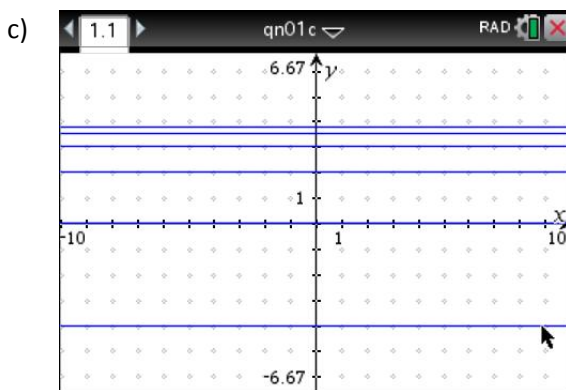
$$y = -x+4$$

$$y = -x+2$$

$$y = -x$$

$$y = -x-2$$

$$y = -x-4$$



$$y = -4$$

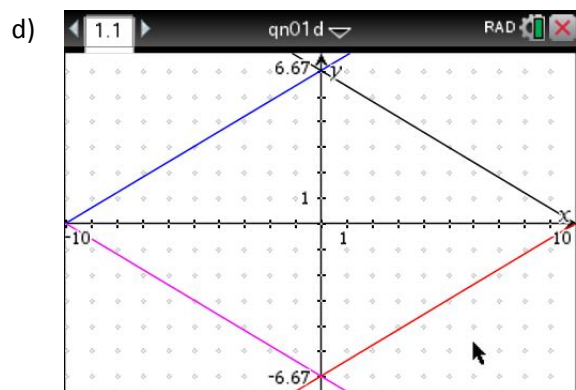
$$y = 0$$

$$y = 2$$

$$y = 3$$

$$y = 3.5$$

$$y = 3.75$$

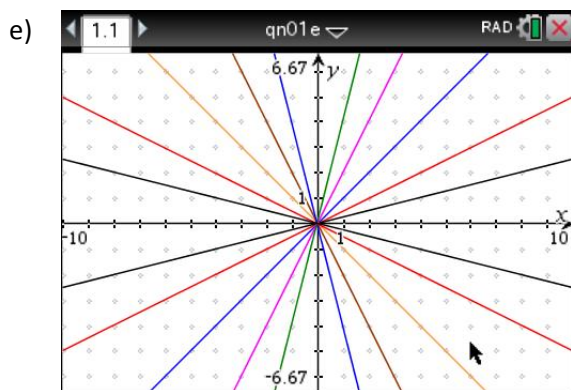


$$y = 6/10x+6$$

$$y = 6/10x-6$$

$$y = -6/10x+6$$

$$y = -6/10x-6$$



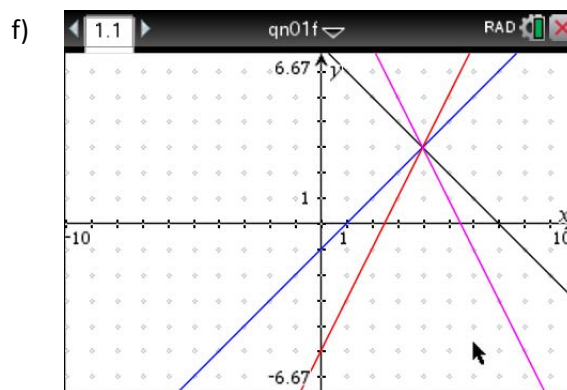
$$y = x \text{ and } y = -x$$

$$y = 2x \text{ and } y = -2x$$

$$y = 4x \text{ and } y = -4x$$

$$y = x/2 \text{ and } y = -x/2$$

$$y = x/4 \text{ and } y = -x/4$$



$$y = (x-4)+3$$

$$y = 2(x-4)+3$$

$$y = -(x-4)+3$$

$$y = -2(x-4)+3$$

Question 2.

Summarise what you have discovered about the effect of the values of a and b on the graph of linear function with rule $y = ax + b$.

a) Effect of a

The value of a determines the gradient of the linear function graph.

b) Effect of b

The value of b determines the y -intercept of the linear function graph.

Question 3.

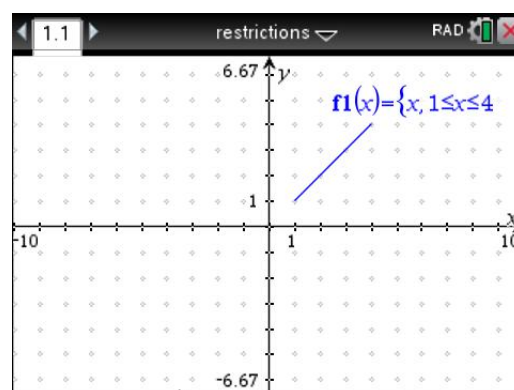
Use the graphs of linear functions to create your own designs.

Part 2: Line designs with restrictions

It is possible to graph only a part of a linear function by restricting the x -values to which the function applies. This is called 'restricting the domain' of a function. For example, using the TI-Nspire CAS, the graph of $y = x$ can be drawn for $1 \leq x \leq 4$, by using the following syntax (*ignore spaces*):

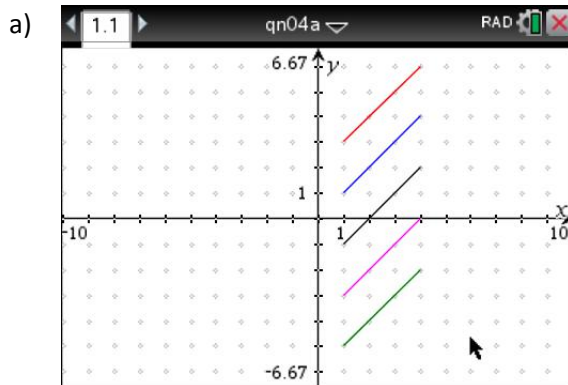
$$\mathbf{f1(x) = x | 1 \leq x \leq 4}$$

Note that the key for the symbol " $|$ " can be found by pressing CTRL and then " $=$ ". Observe that only a segment of the graph has been drawn (see right).



Question 4.

Use this syntax and your knowledge of linear function graphs to construct the following designs in the standard window. (Press **MENU-4** to select **Window** and then press **5** to select **Zoom – Standard**). Show the rules used (including any domain restrictions).



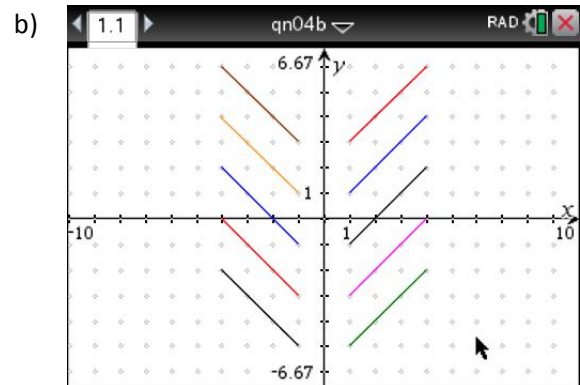
$$y = x \mid 1 \leq x \leq 4$$

$$y = x + 2 \mid 1 \leq x \leq 4$$

$$y = x - 2 \mid 1 \leq x \leq 4$$

$$y = x - 4 \mid 1 \leq x \leq 4$$

$$y = x - 6 \mid 1 \leq x \leq 4$$



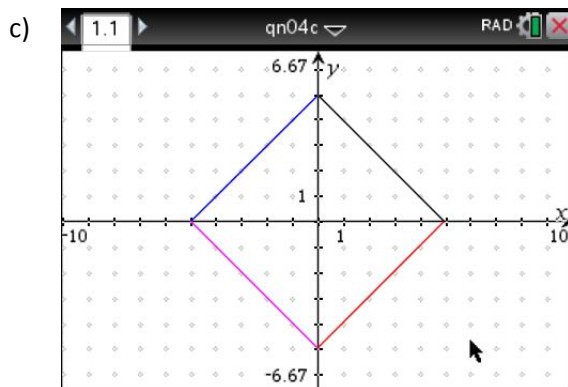
$$y = -x \mid -4 \leq x \leq -1$$

$$y = -x + 2 \mid -4 \leq x \leq -1$$

$$y = -x - 2 \mid -4 \leq x \leq -1$$

$$y = -x - 4 \mid -4 \leq x \leq -1$$

$$y = -x - 6 \mid -4 \leq x \leq -1$$

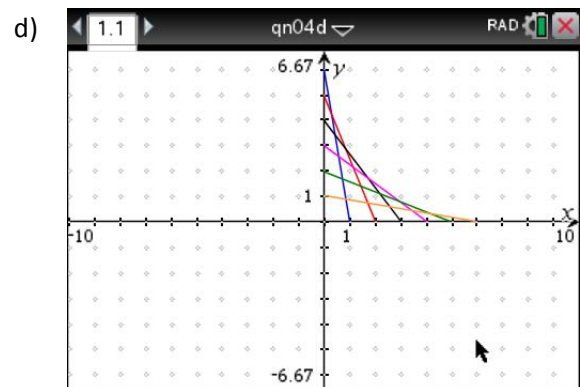


$$y = x + 5 \mid -5 \leq x \leq 0$$

$$y = -x - 5 \mid -5 \leq x \leq 0$$

$$y = x - 5 \mid 0 \leq x \leq 5$$

$$y = -x + 5 \mid 0 \leq x \leq 5$$



$$y = -6x/1 + 6 \mid 0 \leq x \leq 1$$

$$y = -5x/2 + 5 \mid 0 \leq x \leq 2$$

$$y = -4x/3 + 4 \mid 0 \leq x \leq 3$$

$$y = -3x/4 + 3 \mid 0 \leq x \leq 4$$

$$y = -2x/5 + 2 \mid 0 \leq x \leq 5$$

$$y = -1x/6 + 1 \mid 0 \leq x \leq 6$$

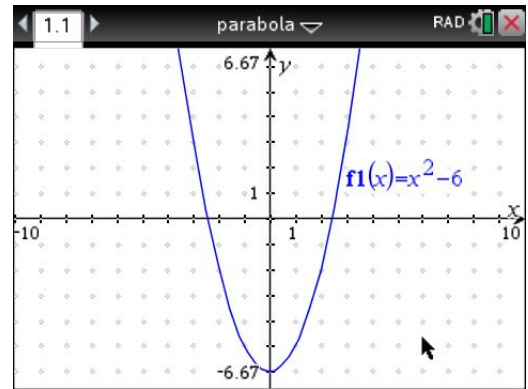
Extension: Use the graphs of linear functions with restrictions to come up with your own design. Use domain restrictions if you wish.

Part 3: Parabola designs

It is of course possible to create designs using other function types, such as the graphs of quadratic functions (called parabolas). Quadratic functions have the forms listed below. Each is useful for determining some aspect of its graph, such as its axes intercepts and the turning point of the parabola.

- Expanded form: $y = ax^2 + bx + c$
- Factorised form: $y = a(x - d)(x - e)$
- Turning point form: $y = a(x - h)^2 + k$

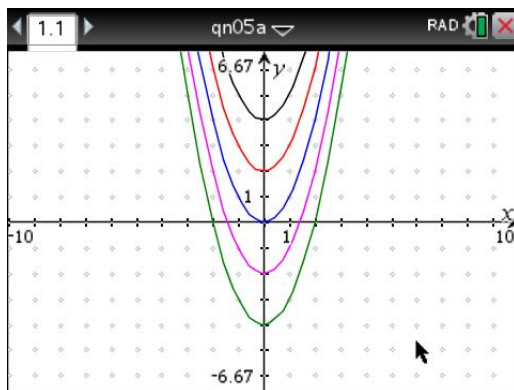
For instance, in the example shown, the expanded form has been used with $a = 1$, $b = 0$ and $c = -6$.



Question 5.

Use your knowledge of quadratic function graphs to construct the following designs in the windows shown. In this question, use the expanded form of the rule ($y = ax^2 + bx + c$). When you have successfully completed each design, record the function rules you have used.

a)



$$y = x^2$$

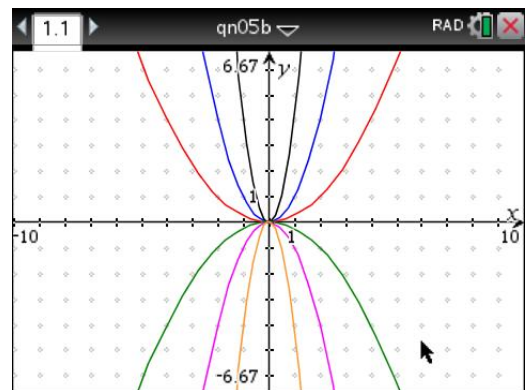
$$y = x^2 + 2$$

$$y = x^2 + 4$$

$$y = x^2 - 2$$

$$y = x^2 - 4$$

b)



$$y = x^2$$

$$y = 4x^2$$

$$y = x^2/4$$

$$y = -x^2$$

$$y = -4x^2$$

$$y = -x^2/4$$

Question 6.

Summarise what you have discovered about the effect of the value of a and c on the graph of quadratic function with rule $y = ax^2 + c$.

a) Effect of a

The value of a determines the dilation factor in the y -direction.

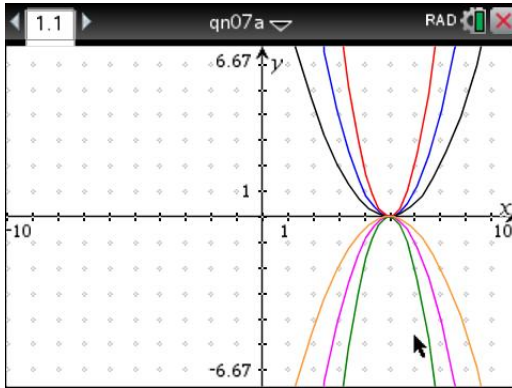
b) Effect of c

The value of c determines the y -intercept of the linear function graph.

Question 7.

Use your knowledge of quadratic function graphs to construct the following designs in the windows shown. In this question, use either the factorised form of the rule ($y = a(x - d)(x - e)$) or the turning point form ($y = a(x - h)^2 + k$). When you have successfully completed each design, record the function rules you have used.

a)



$$y = (x-5)^2$$

$$y = 2(x-5)^2$$

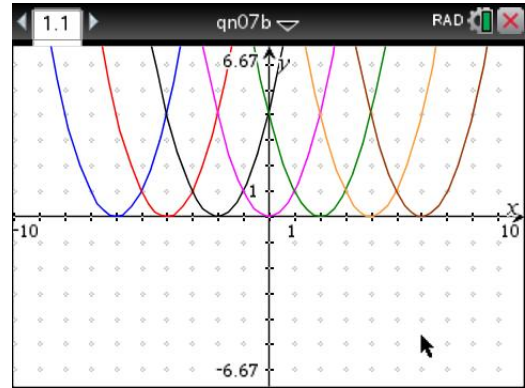
$$y = 0.5(x-5)^2$$

$$y = (x-5)^2$$

$$y = -2(x-5)^2$$

$$y = -0.5(x-5)^2$$

b)



$$y = (x+6)^2$$

$$y = (x+4)^2$$

$$y = (x+2)^2$$

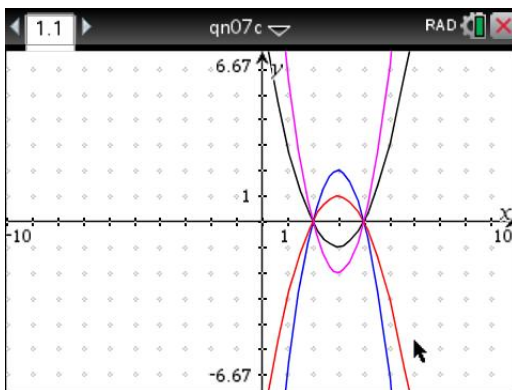
$$y = x^2$$

$$y = (x-2)^2$$

$$y = (x-4)^2$$

$$y = (x-6)^2$$

c)



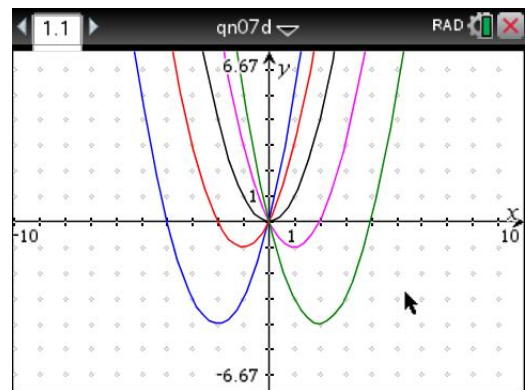
$$y = (x-2)(x-4)$$

$$y = 2(x-2)(x-4)$$

$$y = -(x-2)(x-4)$$

$$y = -2(x-2)(x-4)$$

d)



$$y = x(x+4)$$

$$y = x(x+2)$$

$$y = x^2$$

$$y = x(x-2)$$

$$y = x(x-4)$$

Question 8.

Summarise what you have discovered about the effect of the value of a , b and c on the graph of linear function with rule $y = a(x - d)(x - e)$.

a) Effect of a

The value of a determines the dilation factor in the y -direction

b) Effect of d and e

The values d and e determine the location of the x -intercepts.

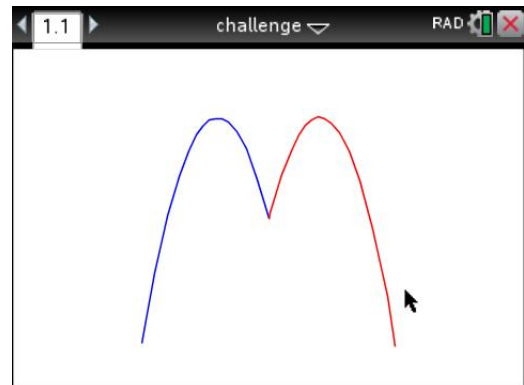
Challenge: Make the Big M sign!

In order to do this, you will need to use what you learned in the previous explorations, including how to incorporate domain restrictions. (Note, to hide the coordinate axes, press **MENU-2-5**.) When you have succeeded in making this shape, record the two function rules you used, including the domain restriction.

Sample answer given, many others possible

$$y = -x^*(x+4) \mid -5 \leq x \leq 0$$

$$y = -x^*(x-4) \mid -5 \leq x \leq 0$$



Teacher notes

- This task provides a creative way for helping Years 9–10 students learn about transformations of simple linear and quadratic functions. The focus is on the students discovering what effect each parameter in the function (e.g. a , b) might have on the shape and location of its graph.
- The exploration works well if you tell the students very little, and let them learn by the ‘do, reflect, modify’ cycle. It may work better at the start of a related unit on linear or quadratic functions, or on the section of that unit related to graphing.
- There are a couple of calculator procedures students will need to master at the start of the task, but thereafter students will be using the standard viewing window, and so only the function rules need be changed. It is useful for students to see the following demonstrated.
 - Entering a function rule
 - Showing the grid
 - Hiding the function rule label
- The graph labels are a bit difficult to view on the calculator window, and are best hidden from the design.
- Emphasise that the scale is the same on each axis. Ask students how each design might be altered if we chose a different scale on each axis. (Perhaps demonstrate an alternative window with different scales.)
- Function notation is used by the TI-Nspire CAS. Depending on the students’ exposure to this notation, you may need to explain the relationship between $f(x)$ -values and y -values. You might also need to explain the concept of ‘domain’ in this context as a defined set of x -values.
- Entering a number of parameters in one function definition can be accomplished with lists. For example, the graphs in question a part i can be described by $f1(x) = x + \{-4, -2, 0, 2, 4\}$. Some students are more comfortable entering each rule in a separate line.
- Part 3 focuses on parabola designs, and could be omitted if you would prefer to only cover linear function graphs (Parts 1 and 2).