

Rational Thinking

Teacher Answers

7 8 9 10 11 12



TI-Nspire CAS



Investigation



Student



45min

Aim

Is it possible to find a number that when multiplied by itself will equal 20?

Equipment

For this activity you will need:

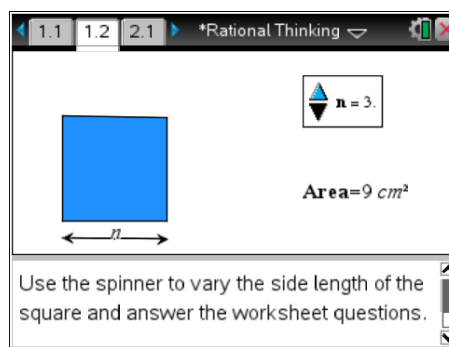
- TI-Nspire CAS
- TI-Nspire CAS file (tns): Rational Thinking

Rational Investigation:

Open the TI-nspire document “Rational Thinking”.

Navigate to page 1.2

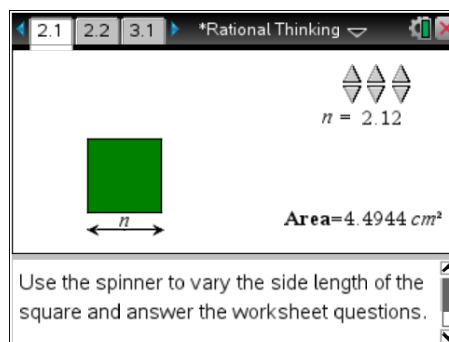
Move the mouse over the spinner and use it to change the side length and subsequent area of the square.



1. Adjust the side length of the square (using the spinner) to obtain the following areas:
 - a. Area = 25cm^2 Side length = **5cm**
 - b. Area = 16cm^2 Side length = **4cm**
 - c. Area = 9cm^2 Side length = **3cm**
2. If a square has an area of 20cm^2 the side length would need to be more than **4** cm and less than **5** cm.

Navigate to problem 2.1

On this page three spinners are used to adjust the side length of the square. The first spinner changes the units, the second changes tenths and the third changes hundredths.



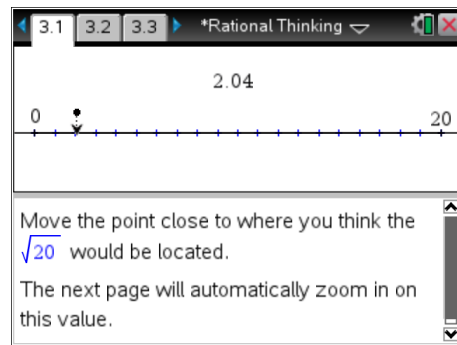
3. Adjust the spinners to make the area of the square as close to 20cm^2 as possible.
 - a. What is the side length that produces an area as close as possible to 20cm^2 ? **4.47cm**
 - b. A square with area 20cm^2 has a side length between **4.47cm** and **4.48cm**.
4. Adjust the spinners to make the area of the square as close to 10cm^2 as possible.
 - a. What is the side length that produces an area as close as possible to 10cm^2 ? **3.16cm**
 - b. A square with area 10cm^2 has a side length between **3.16cm** and **3.17cm**.
5. Adjust the spinners to make the area of the square as close to 8cm^2 as possible.
 - a. What is the side length that produces an area as close as possible to 8cm^2 ? **2.83cm**
 - b. A square with area 8cm^2 has a side length between **2.82cm** and **2.83cm**.
 - c. How many spinners do you think would be required to create the exact side length for a square with area 8cm^2 ? **Infinitely many... or its not possible. (Discussion point)**

Navigate to problem 3.1

On this page there is a single point that can be moved along the number line. The aim is to move the point as close to $\sqrt{20}$ as possible.

Due to the scale, it may not possible to get the point really close; however the next page zooms in on your estimate allowing you to make a better approximation.

Adjust the position of the point then answer the next question.

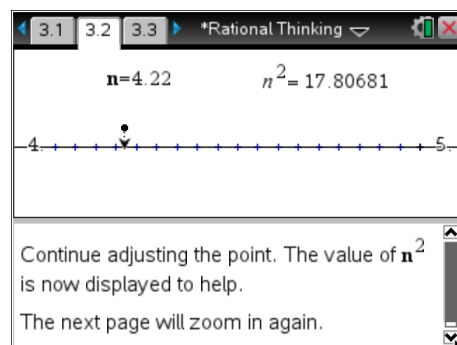


6. My estimate that I made for $\sqrt{20}$ using this moveable point was **answers will vary $\approx 4.47\text{cm}$** .

Navigate to problem 3.2

This page zooms in on your approximation for $\sqrt{20}$.

Move the point closer to your estimate of $\sqrt{20}$, this time the computed value of your estimate is displayed to provide some assistance.

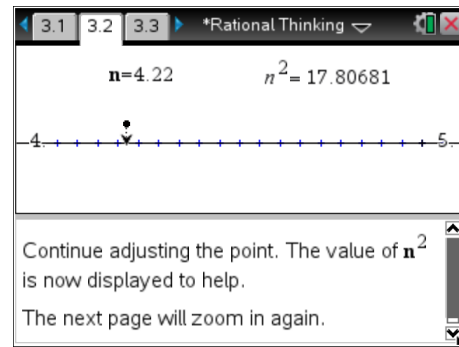


7. The $\sqrt{20}$ lies between: **4.0cm** and **5.0cm**. (Answers will vary – these values should be the first set of zoomed points).

Navigate to problem 3.3

This page zooms in even closer to your approximation for $\sqrt{20}$.

Move the point closer to your estimate, notice that n^2 is displayed with more decimal places as your approximation improves the accuracy of the answer must also improve.



8. My estimate for $\sqrt{20}$ is: **4.47214cm**.
9. My estimate squared is: **20.000036cm²**.

Babylonian Investigation:

In the previous investigation feedback for the estimate was provided by seeing the estimate squared. The estimate could be increased or decreased accordingly. The Babylonian method uses a different method for feedback. To understand how this feedback works, consider the following:

$$\sqrt{81} = 9$$

This means: $9 \times 9 = 81$

Therefore $\frac{81}{9} = 9$

This concept can be used if the square root of the number is not known:

Estimate $\sqrt{20} \approx 4$ Guess (4)

But $\frac{20}{4} = 5$ Feedback (5)

Therefore $\sqrt{20} \neq 4$ But the answer must lie between the guess: 4 and the feedback: 5.

A reasonable step forward would be to average the guess and the feedback:

New Estimate: $\frac{4+5}{2} = 4.5$ New Guess (4.5)

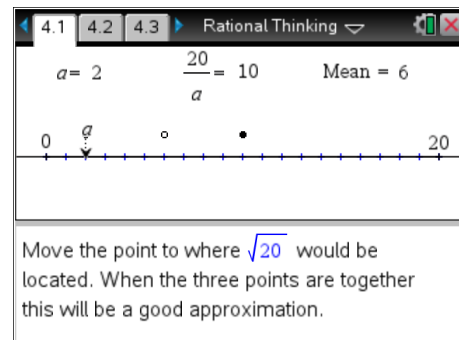
But $\frac{20}{4.5} = 4.4$ Feedback (4.444...)

As this process is repeated the approximation improves.

Navigate to page 4.1

Move point a and observe how the other two points move. The feedback is the bold point. The average is the hollow point.

When all three points are together this will be a good approximation.

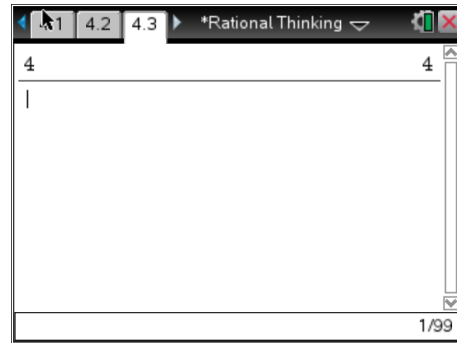


10. My first estimate using the Babylonian method for $\sqrt{20}$ is: **4.47cm**.

Navigate to page 4.3

For illustrative purposes suppose $\sqrt{20}$ is estimated as 4.

Enter the number 4 and press [Enter]

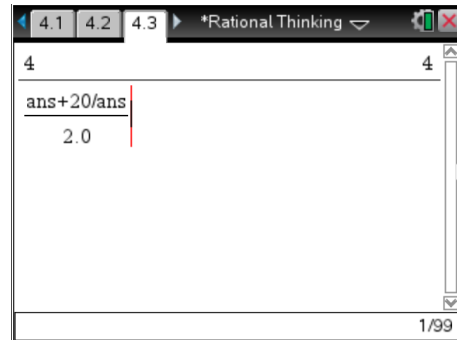


Feedback can be generated using the following:

$$\frac{ans+20/ans}{2.0}$$

Note: For *ans* Press CTRL + (-)

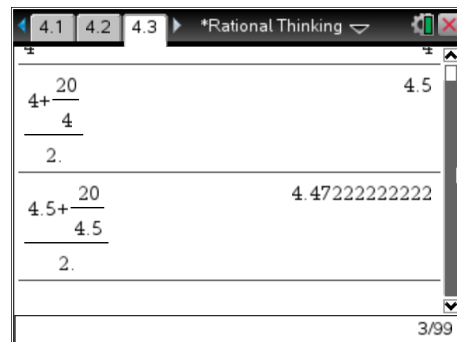
Use 2.0 for the denominator as this will produce a decimal answer when the calculator is in the 'automatic' calculation mode.



Press [Enter] twice (as shown opposite) and notice how the reference to **ans** is replaced with the actual value.

The original estimate for $\sqrt{20}$ is still just visible at the top of the screen. The first feedback answer is 4.5 followed by 4.4722...

This second feedback response is a relatively close approximation as $4.7222^2 \approx 20.0008$



11. Determine the approximate value for the $\sqrt{20}$ after three (3) iterations of this process.

4.472135956

12. When the feedback answer ceases to change, the value has been computed to the capacity of the calculator. How much iteration was required to compute the approximate value for $\sqrt{20}$?

Four iterations (4.4721359549996)

13. Use the Babylonian method for computing approximate values for the square root of the following numbers. Include your original estimate and a justification for your estimate, the approximate answer (8 decimal places) and the number of iterations required to obtain the answer.

- $\sqrt{60}$ Estimate: 7cm (Between 7 & 8). Answer ≈ 7.74596692 (4 iterations)
- $\sqrt{46}$ Estimate: 6cm (Between 6 & 7). Answer ≈ 6.78232998 (5 iterations)
- $\sqrt{29}$ Estimate: 5cm (Between 5 & 6). Answer ≈ 5.38516481 (5 iterations)
- $\sqrt{80}$ Estimate: 8cm (Between 8 & 9). Answer ≈ 8.94427191 (4 iterations)

14. Do you think it is possible to calculate the exact value for $\sqrt{20}$? **No – discussion point!**