Variables on Both Sides

MATH NSPIRED

Math Objectives

- Students will recognize that linear equations with variables may have none, one, or an infinite number of solutions.
- Students will identify how many solutions a given linear equation with variables on both sides will have.
- Students will make sense of problems and persevere in solving them (CCSS Mathematical Practice).

Vocabulary

- Linear equations
- Variables
- Expressions
- Equation

About the Lesson

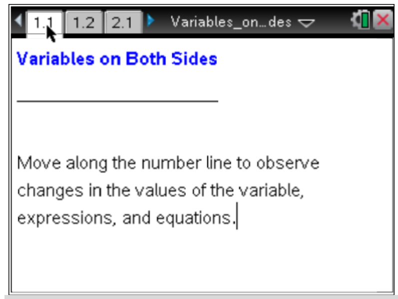
- The Variables on Both Sides lesson is intended to develop student understanding of the number of possible solutions to equations that have variables on both sides. The activity involves observing that linear equations with variables on both sides can have no solution, one unique solution, or an infinite number of solutions.
- Students will move the point on the arrow below the number line and observe the changes that take place in the expressions on each side and make the connection between the two expressions and the equation.

Related Lessons

- Prior to this lesson: Visualizing Equations
- After this lesson: Self Check with Equations

TI-Nspire™ Navigator™ System

- Use Quick Poll to check student understanding.
- Use Live Presenter to engage and monitor students.
- Use Teacher Edition computer software to review student documents.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Variables_on_Both_Sides_Student.pdf

Variables_on_Both_Sides_Student.doc

TI-Nspire document

Variables_on_Both_Sides.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor (arrow) until it becomes a hand (👆) getting ready to grab the point. Also, be sure that the word point appears. Then press ctrl 👆 to grab the point and close the hand (✊). When finished moving the point, press esc to release the point.

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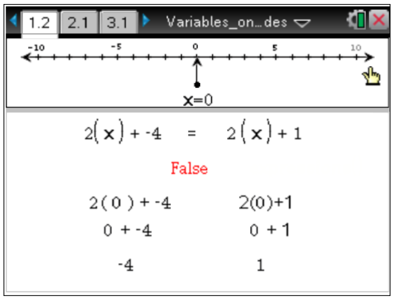

As a result of problems #1–3, students should be able to recognize the conditions necessary for a linear equation with variables on both sides to have no solution.

1. a. As you grab the point to move the arrow beneath the number line, what changes?

   <u>Sample answers:</u> Student responses may include changes in the value of x, the number in parentheses in both expressions, and the value of both of the expressions.

   b. What stays the same?

   Answer: The expressions, the = symbol, and the word “false.”

2. a. Describe the differences in the values of the expressions on the left side and the right side.

   <u>Sample answers:</u> Student responses can include: The value of the expression on the right is always larger than the one on the left. The value of the expression on the left is always smaller than the one on the right. The difference is 5.

b. Move the arrow to try several new values for x. What is true about the difference in the values of the expressions?

Answer: The difference is always 5

3. Gail says that if she were asked to solve the equation $2x + -4 = 2x + 1$, she could find a value of x that would be a solution. Eric says, "That's impossible." Who is correct? Justify your answer.

Answer: Eric is correct. There is no value of x that would make this equation true. The value of the expression on the right is always 5 units more than the value of the expression on the left. They are never equal because the left side adds a negative four to $2x$ and the right side adds 1 to $2x$ so the right side will always be larger.

> **Teacher Tip:** Be sure that this explanation is understood by the students as this is the kind of reasoning that will enable them to recognize when equations will or will not have solutions.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

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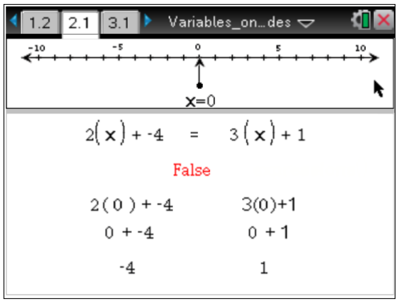

As a result of problems #4–5, students should be able to recognize the conditions necessary for a linear equation with variables on both sides to have one solution.

4. a. Examine the expressions on the left and right sides. Describe the differences between the expressions and their values.

Sample answers: The difference will vary depending on whether students have changed the value of x; for example, if x is at –2, the values of the expressions are –8 and –5 so the difference is either 3 or –3. –5 – (-8) = 3. –8 – (-5) = –3.

> **Teacher Tip:** This might be an opportunity to review operations on negative numbers.

b. Find x so that the difference between the two expressions is 8

Answer: The difference will be 8 when *x* is 3.

c. Find x so that the difference between the two expressions is 4.

Answer: The difference will be 4 when x is –1.

> **Teacher Tip:** For discussion, consider asking: Can you find more than one answer for either of these? There is only one answer because at –5 for *x*, the difference is 0 and as *x* increases or decreases from –5, the differences in the values of the expressions get larger and larger. Some students may note that for each increase or decrease in the value of *x*, the difference grows by one.

5. Gail says that if she were asked to solve the equation $2x + -4 = 3x + 1$, she could find a value of *x* that would be a solution. Eric says “That’s impossible.” Who is correct? Justify your answer.

   Answer: Gail is correct. There is one value of *x* that would make this equation true. The solution of the equation is –5. When –5 is used for *x*, both expressions are equal and have a value of –14. There are no other values for *x* that will make both expressions have the same value because as *x* increases or decreases from –5, the difference between the two expressions gets larger and larger.

6. Predict what would happen if the 2 in the left side of the equation were a 3. Explain your reasoning. Change the 2 to a 3 and see if you are correct.

   Answer: There is no value of *x* that would make this equation true. This is because both sides would have a $3x$. In one case, you would be adding a negative four to the $3x$, and in the other adding a one, so they can never be the same value at the same time.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

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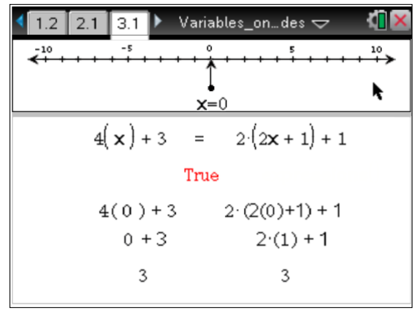
As a result of problems #7–9, students should be able to recognize the conditions necessary for a linear equation with variables on both sides to have an infinite number of solutions.



7. a. As you move the arrow for point *x*, what changes**?**

   <u>Sample answers:</u> Student responses can include: the value of *x*, the number in parentheses in both expressions, the value of the expressions.

   b. What stays the same?

   <u>Answer:</u> The expressions stay the same. Students might note that the value of the expressions on the left and right side are always equal.

How many solutions are there to the equation $4x + 3 = 2(2x + 1) + 1$? Explain your reasoning.
<u>Answer:</u> There are an infinite number of solutions. Every value of *x* gives a true statement because the two expressions are equivalent.

9. Simplify the right side of the equation by distributing and combining like terms. Does this support your response to #8?

   <u>Answer:</u> This expression is identical to the expression on the left side, and so every value of *x* gives a true statement.

10. Describe the characteristics of an equation that would have the solution given below. (Hint: Review the equations that you have explored in this activity.) Also, write an example of an equation for each solution.

    a. no solution (empty set).

    Answer: When like terms are combined, the terms with a variable in the expression on each side have the same coefficients but different constants added to that term.
    Example: $4x + 7 = 4x + 5$

    b. one solution

    Answer: When like terms are combined, the terms with a variable in the expression on each side of the equation have different coefficients.
    Example: $4x + 7 = 3x + 8$.

    > **Teacher Tip:** To ensure that students don't confuse an equation with no solution and one with a solution of zero, ask students to give an example of an equation with solution of zero.

    Example: $4x + 5 = 3x + 5$

    c. infinitely many solutions

    Answer: When like terms are combined, the terms with a variable in the expressions on each side of the equation have the same coefficient and the expressions have the same constant.
    Example: $4x + 7 = 4x + 7$

TI-Nspire Navigator Opportunity: Quick Poll or Screen Capture
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the discussion of this activity, the teacher should ensure that students are able to:

- determine the number of solutions for a linear equation with variables on both sides
- grasp vocabulary associated with the activity: expression, equation, coefficient, constant

To determine whether students have met the goals of the activity, the teacher might give students the expressions $2x + 5$ and $3x - 2$ and ask them if it is possible to change one number in one of the expressions to create an equation from the two expressions that has no solutions, an infinite number of solutions, and one unique solution.

Assessment

The questions posed in this activity form a most suitable basis for student assessment, and so may be offered as the basis for either informal or, if desired, formal assessment of this topic.

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Note 1

Question 3, *Quick Poll*: Use the Open Response feature and using a verbal prompt, ask the students to create an equation with variables on both sides of the equal sign that has no solution. Note: hopefully some students will submit equations where the coefficient of x is not 1. For example: $3x + 5 = 3x - 4$. After the equations are submitted, show the results to the class and decide which are correct and which are not, and why or why not.

Note 2

Question 6, *Quick Poll*: Use the Open Response feature and using a verbal prompt, ask the students to create an equation with variables on both sides of the equal sign that has exactly one solution. For example: $3x + 5 = 5x - 4$. After the equations are submitted, show the results to the class and decide which are correct and which are not, and why or why not.

Note 3

Question 10, *Quick Poll*: or Screen Capture Have students answer submit equations that satisfy the following conditions. If you use Quick Poll, use the Open Response feature. Once you collect the equations, show them to the students. Assign each student one of the answers and be able to say if the equation does satisfy the requirements.

1. Submit an equation that has no solution.
2. Submit an equation that has exactly one solution.
3. Submit an equation that has an infinite number of solutions.
4. Submit an equation that has zero as its only solution. (optional)