Number Sums

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Student Activity

7 8 9 **10** **11** 12

TI-Nspire Investigation Student 90 min

Introduction

Count the number of cans in the stack shown opposite.

Think about the way you counted the cans. Did you count them from the top down, one at a time? Did you look at the number of cans in each row and add these numbers together? Now imagine a stack 30 rows high; would you count them the same way? Our brains are wired to look for patterns. This investigation uses combinations of numeric, visual and algebraic strategies to explore this and related problems by observing and building patterns.

Numeric – Visual Approach

The can stack can be tackled by working with sums of:

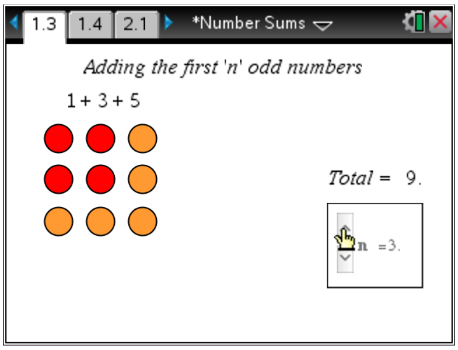
- Odd numbers
- Even numbers
- Whole numbers

Odd Numbers

Open the TI-Nspire document "Number Sums".

Read the instructions and then navigate to page: 1.3.

Adjust the slider, observe the diagram, expression and total (sum) displayed on the screen.



Question: 1.

Explain how the visual representation relates to the sum of the first *n* odd numbers.

Question: 2.

Write an equation for the sum ***s*** of the first ***n*** odd numbers. Start your equation "s =".
Explain how the visual representation helps produce the equation.
[Include your rule on page 1.4 of the TI-nspire document]

Question: 3.

Use your formula to determine and check the sum of the first 20 odd numbers.

Author: P. Fox

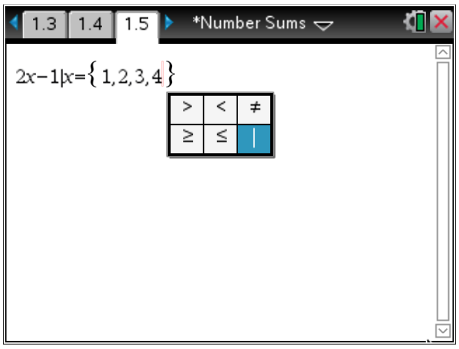
A sequence of numbers can be generated by the rule:

$$2x-1 \quad \text{where} \quad x \in \{1,2,3\ldots\}$$

To see how this 'rule' works, enter the rule shown on the calculator page (1.5) and substitute the set of numbers:

{1, 2, 3, 4}

The '|' symbol can be found by pressing [**Ctrl**] + [**=**]



Question: 4.

What numbers are produced for the calculation: $2x-1|x=\{1,2,3,4\}$?

Show a sample calculation to illustrate and explain why the rule produces these types of numbers.

Question: 5.

The word 'sum' is a built in command. Type: **sum(** then copy and paste the previous into the sum command and press [**enter**]. Use this approach to find the sum of the first 10 odd numbers and use it to check your rule from question 2.

The use of the 'sum' command is good for small sets of numbers or when the numbers already exist in a list. For larger sets it is more convenient to use a rule to generate the numbers. The Greek letter Σ is used in mathematics to represent the 'sum of'. The calculator has this functionality built in so it can be used to find the 'sum of' a set of numbers generated by a rule:

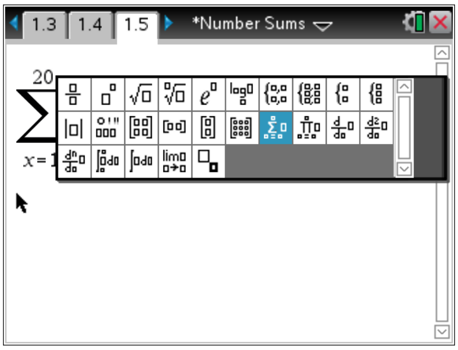
$$\sum_{start}^{finish} rule$$

The sum template can be found in the template fly-out or by selecting 'sum' from the calculus menu. The rule being used in this example has the variable x. So the sum of the first 20 odd numbers would include:

Start: $x = 1$ -- *initial value used by the rule*

Finish: 20 -- *final value used by the rule*

Rule: $2x-1$ -- *rule used to generate the numbers*



Question: 6.

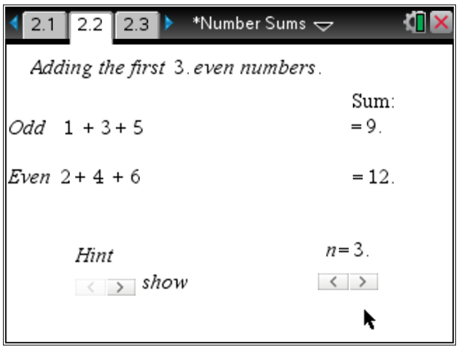
Use the calculator to determine the answer to: $\sum_{x=1}^{20} 2x-1$.

How does the answer compare to that obtained in question 3.

Author: P. Fox

Even Numbers

Navigate to page 2.2, adjust the value of *n* and observe the sequence of even and odd numbers with the corresponding sums.



Use the 'hint' option to reveal more clues for developing a rule for the sum of the first *n* even numbers.

Question: 7.

Describe (in words) the relationship between the sum of the first *n* even numbers and the sum of the first *n* odd numbers.

Question: 8.

Use your rule for the sum of the first *n* odd numbers to help write a rule for the sum of the first *n* even numbers. [Include your rule on page 2.3 of the TI-nspire document.]

Question: 9.

What numbers are produced for the calculation: $2x \mid x=\{1,2,3,4\}$? Show a sample calculation to illustrate. [Use page 2.4 of the TI-nspire document to check your answers.]

Question: 10.

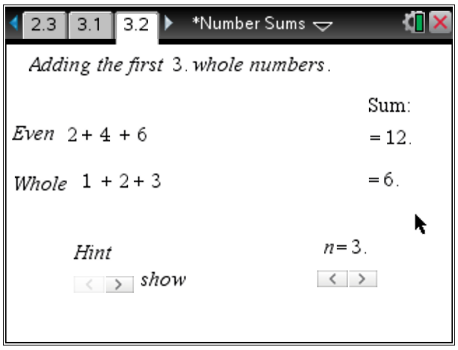
Use your calculator to determine $\sum_{x=1}^{20} 2x$ and explain each component of this expression.

Question: 11.

Use your formula to calculate the sum of the first 20 even numbers and compare it to: $\sum_{x=1}^{20} 2x$.

Whole Numbers

Navigate to page 3.2, adjust the value of *n* and observe the sequence of even and whole numbers with the corresponding sums.



Use the 'hint' option to reveal more clues for developing a rule for the sum of the first *n* whole numbers.

Question: 12.

Describe (in words) the relationship between the sum of the first *n* even numbers and the sum of the first *n* whole numbers.

Question: 13.

Write a rule for the sum of the first *n* whole numbers. [Include your rule on page 3.3]

Question: 14.

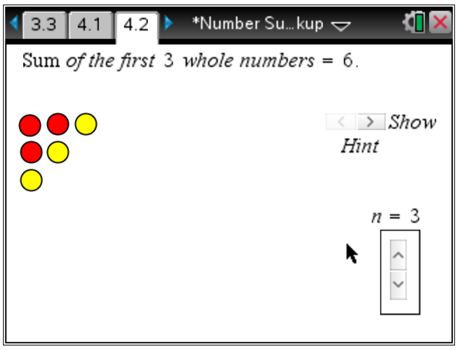
Explain how the sum of the first *n* whole numbers relates to the can stacking problem and hence determine the number of cans in a stack 30 rows high.

Author: P. Fox

Visual Approach

Looking at things from a different angle or imagining extra pieces can sometimes help to see patterns that you may not have previously noticed.

Navigate to page 4.2, adjust the value of *n* and observe the way the pattern develops.



Use the 'hint' option to reveal items that add to the visualisation of this pattern.

Question: 15.

How does this pattern relate to the original can stacking problem?

Use the 'Hint' option to reveal additional visual clues for this problem.

Question: 16.

Describe the overall shape produced as *n* is changed. (Hint: on)

Question: 17.

When $n = 4$, calculate the 'area' of the shape produced. (Hint: on)

Question: 18.

Adjust the value of *n* and describe the length and width of the shape produced. (Hint: on)

Question: 19.

Remove the 'hint' and write a rule for the pattern. [Include your rule on page 4.3]

Numerical Approach

The focus here is on numerical strategies only. Consider calculating the sum of the first 10 whole numbers. This can be done quickly and easily in your head; 1 + 2 + 3....9 + 10, however this technique will be very time consuming when trying to calculate the sum of the first 100 whole numbers.
Consider a more alternative, more efficient strategy which incorporates multiplication via grouping:

$(1 + 10) + (2 + 9) + (3 + 8)$ ....

This strategy is displayed on the page 5.2 of the TI-Nspire document. The numbers are purposefully grouped so that multiplication can be used in conjunction with addition.

Question: 20.

Use the grouping strategy to calculate the sum of the first 10 whole numbers. (Show working)

Question: 21.

Use the grouping strategy to calculate the sum of the first 100 whole numbers. (Show summary)

Question: 22.

Generalise the grouping strategy to write a rule for the sum of the first *n* whole numbers.

Question: 23.

Compare the formulas created in each key question: 13, 19 and 22.

Author: P. Fox