Solving Percent Problems

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will solve one-step and multi-step percent problems.
- Students will investigate percents greater than 100%.
- Students will use proportional relationships to solve multi-step percent problems.

Vocabulary

- percentage
- base
- rate

About the Lesson

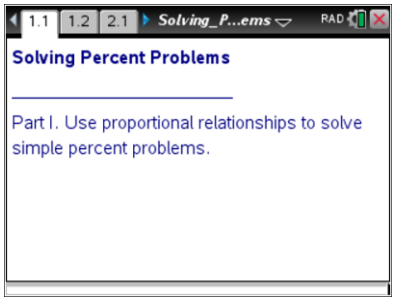
- This lesson involves solving word problems dealing with percents by using visual and numerical representations of percents.
- As a result, students will:
  - Grab and drag points to change the sizes of rectangles that represent the base and the percentage in each problem to create a visual representation for each problem.
  - Analyze the percent equation for each problem, and make connections between verbal, visual, and numerical representations in order to establish patterns in solving percent problems.

TI-Nspire™ Navigator™

- Send and transfer a document.
- Use Class Capture to monitor student progress and understanding.
- Use Live Presenter to let students demonstrate and explain their answers.
- Use Quick Poll to assess students' understanding of percent problems.
- Use Class Analysis and Slide Presentation to assess students' understanding of percent problems.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions given may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Solving_Percent_Problems_Student.pdf
- Solving_Percent_Problems_Student.doc

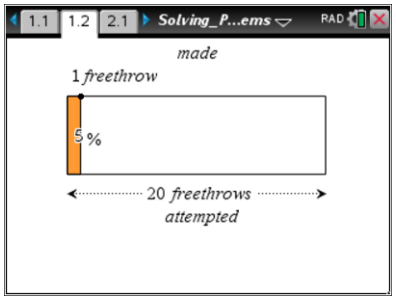
TI-Nspire documents

- Solving_Percent_Problems.tns
- Solving_Percent_Problems_Assessment.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand ready to grab the point. Then press ctrl to grab the point and close the hand.

Move to page 1.2.



The girls' basketball team attempted 20 free throws in a game. The larger rectangle represents the total number of attempted free throws. The shaded rectangle represents the number of free throws made.

1. How does the area of the shaded rectangle compare to the area of the large rectangle?

   Answer: The area of the shaded rectangle represents a percentage of the area of the whole rectangle. This corresponds to the percentage of free throws made.

   Tip: Students can grab the point on the top of the rectangle and drag the point to the left and right in order to understand the visual model for the problem. Consider demonstrating how to grab and drag a point, and emphasize that the area of the larger rectangle represents the total number of attempted free throws and the area of the shaded rectangle represents the number of free throws that were made.

2. Grab and drag the point on the top of the rectangle to answer the following questions:
   a. If the team scored on 12 free throws, what percent of their shots did they make?

      Answer: The team made 60% of the shots, $\frac{12}{20} \cdot \frac{5}{5} = \frac{60}{100} = 60\%$ (Students will drag the point on the rectangle until the label above the rectangle indicates 12 free throws made. The shaded rectangle will then show 60%.)

   b. What fraction(s) can be used to represent 60%?

      Answer: 60% = $\frac{60}{100} = \frac{12}{20} = \frac{6}{10} = \frac{3}{5}$.

Teacher Tip: Fractions are equivalent when they differ by a factor of 1, for example, $\frac{12}{20} = \frac{6}{10}\cdot\frac{2}{2} = \frac{6}{10}\cdot 1 = \frac{6}{10}$. Thus, equivalent fractions are simplified by factoring out values of 1 using The Identity Property of 1. Fractions are simplified to expedite computation. However, students need to realize that sometimes in real-life problems this process can lead to values that are harder to interpret. For example $\frac{3}{5}$ is harder to interpret as percent than $\frac{60}{100}$.

Tech Tip: It might be beneficial for students to use the Scratchpad to simplify the fraction. Press ctrl ÷ to get the fraction template to use to type in the fraction ($\frac{\square}{\square}$), then press enter to see the simplified form.

Tech Tip: It might be beneficial for students to use the Calculator application to simplify the fraction. Select + to add a Calculator page to the document. Tap to bring up the keyboard to access the fraction template.

Note that if students do this, the addition of the new page will cause a discrepancy between instructions and page numbers for the remainder of this document.

This Calculator page can be used for all future scratchpad references.

c. What is the decimal representation of 60%?

Answer: $60\% = \frac{60}{100} = \frac{6}{10} = .60 = 0.6$.

Tech Tip: If the Scratchpad is used in part b to simplify the fraction, now press ctrl enter to see the result in decimal form. Press esc to exit the Scratchpad.

d. How many free throw shots did the team make if they made 70% of their shots?

Teacher Tip: You might want to suggest a rewording of this question in the form '70% of 20 is what number'?

Answer: The team made 14 shots. 70% $\cdot$ 20 = $\frac{70}{100} \cdot 20$ = 14. (Students drag the point on the rectangle until the shaded rectangle indicates 70% of the shots were made.)

e. How many shots would they make if the team had made 90% of their shots? How many free throws would they miss?

Teacher Tip: You might want to suggest a rewording of this question in the form '90% of 20 is what number'?

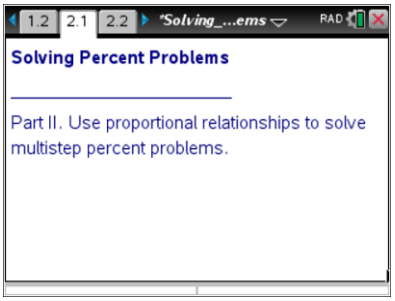
Answer: The team would have made 18 shots and missed 2 shots. (Students can drag the point until the shaded rectangles shows 90% of the shots were made. The model indicates this means 18 free throws were made; 90% of 20 = .90 x 20 = 18; 20-18 = 2).

Teacher Tip: Encourage students to explain the answers to the questions provided by the visual model in the TI-Nspire. Students can set up numerical representations of the visual model to support their answers. Some students might use the equation form as suggested above, while some students might use a proportion such as $\frac{70}{100} = \frac{x}{20}$ or $\frac{90}{100} = \frac{x}{20}$.

TI-Nspire Navigator Opportunity: *Quick Poll, Class Capture, and Live Presenter*
See Note 1 at the end of this lesson.

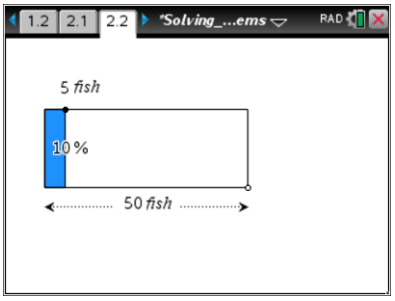
Move to page 2.1.

Now you will apply your knowledge of percents to solve multistep percent problems.



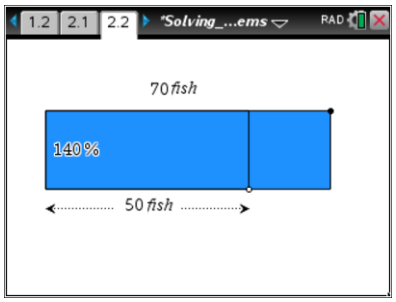
Move to page 2.2.

There are 50 fish in a small stocked pond. The Game and Fish Department adds fish to the pond. As a result, the total number of fish is now 140% of the original number.



3. Grab and drag the point on the top of the rectangle to answer the following questions.
   a. How many fish are now in the pond?

Answer: There are now 70 fish in the pond, $140\% \cdot 50 = \frac{140}{100} \cdot 50 = 70$. (Grab and drag the point to the right until shaded rectangle is 140% of the original rectangle)



   b. How many fish were added to the pond?

Answer: 70 – 50 = 20 fish were added to the pond.

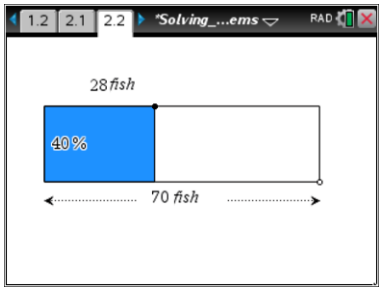
   c. What is the visual representation of 140%? Draw the picture you see on your calculator. Why does the blue rectangle get longer than the white rectangle?

Answer: The shaded rectangle is larger than the original rectangle because 140% represents more than the original amount.

   d. Use the number of fish to write a ratio that represents140%?

Answer: $\frac{70}{50}$; $140\% = \frac{140}{100} = \frac{70}{50}$ (other representations: 70 to 50 and 70:50)

Grab and drag the point on the bottom of the rectangle to indicate that the total number of fish in the pond is now 70 fish.



4. Some of the fish in the pond are caught the next week, and only 28 fish remain in the pond. Grab and drag the point on the top of the rectangle to find the new percentage of the total.
   a. What percentage of fish have been caught?

Answer: 28 fish remain in the pond which corresponds to 40% of the fish that are in the pond now; $\frac{28}{70}$ x 100% = 40%.

Thus 60% of fish have been caught (100% – 40% = 60%).

   b. What is the base (the total number you started with) in your calculations?

Answer: The base is the total number of fish in the pond before any fish were caught, so the base is 70.

c. What is the rate (the decimal representation of the percentage) in your calculations?

Answer: The rate is a decimal representation of the percentage, so the rate is 40% = 0.4.

Tech Tip: In this problem, students have to change the base for their calculations from 50 to 70 in order to find the percentage of fish that remain in the pond in relation to the total number of fish. To change the base, students should drag the open point on the bottom of the rectangle to change the number of fish in the pond to 70. Then they can use the closed point on the top of the rectangle to find the percentage which corresponds to 28 fish remaining. You might want to encourage students to show numerical calculations that support their answers. Students can use Scratchpad by pressing [doc] to show their calculations.

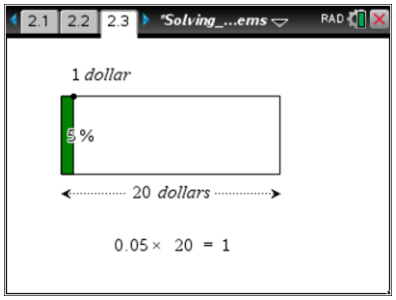
Teacher Tip: You might want to encourage students to rephrase the question in the form "what percent of 70 is 28"? Then when students show numerical calculations that support the answer, they may use:

- the algebraic equation $x \cdot 70 = 28$ and produce the rate, or
- the proportion $\frac{x}{100} = \frac{28}{70}$ and produce the percentage.

It might be helpful to compare the two approaches and discuss the similarities and differences. The terms base, rate, and percentage will provide a good foundation for understanding algebraic representations for exponential growth and decay.

TI-Nspire Navigator Opportunity: *Quick Poll, Class Capture, and Live Presenter*
See Note 1 at the end of this lesson.

Move to page 2.3.



5. It is your best friend's birthday and you treat him/her to lunch. The bill is $20. Grab and drag the point on the top of the rectangle to answer the following questions:
   a. The waitress provided excellent service, and you want to leave a 15% tip. How much tip should you leave?

Answer: The tip is $3; 15% = .15; .15 x 20 = 3

Teacher Tip: You might want to remind students 15% is 10% + 5% and discuss how students could use mental math to find the answer.

b. What does the equation shown on Page 2.3 represent?

Answer: The equation shows how to calculate the percentage given the rate and the total.

c. What is the base in this equation? Why?

Answer: The base is $20; the base is the total amount of the bill.

d. What is the rate in this equation? Why?

Answer: The rate is 0.15 which is the decimal equivalent of 15%.

Teacher Tip: Help students to understand that the % symbol means "per cent" or "per hundred." The percentage is a way to express the number as a fraction of 100 so 15% = $\frac{15}{100}$ or its decimal equivalent, 0.15. Calculations cannot be performed with symbols. Thus, 15% x 20 = $\frac{15}{100}$ x 20 = .15 x 20.

e. What is the percentage in this problem?

Answer: The percentage is 15%.

f. With the tip included, what is the total bill? What percent of the original bill is the total bill that includes the tip?

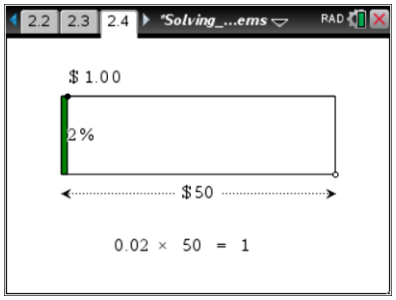
Answer: The total bill is $23 = $20 + $3; $\frac{23}{20} = 1 + \frac{3}{20}$ = 100% + 15% = 115%. A second approach is then $23 = 1.15(20).

Teacher Tip: You might want to ask students whether percentage that is larger than 100% is always meaningful in a real-life problem. For example, ask students to compare the basketball and fish problems and whether it makes sense to discuss percentages that are larger than 100% in the contexts of these two problems.

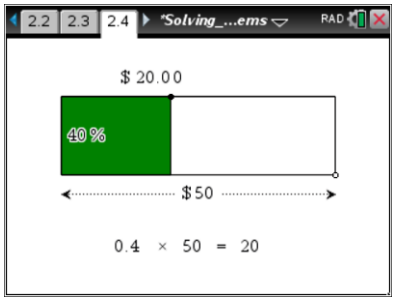
Also encourage students to show two methods for solving this problem. One method involves multiplying first, then adding. The second method involves adding the percentage to 100% first, then multiplying the associated rate x base. Consider the second approach to provide a good foundation for understanding how to represent exponential growth and decay.

TI-Nspire Navigator Opportunity: *Quick Poll, Class Capture, and Live Presenter*
See Note 1 at the end of this lesson.

Move to page 2.4.

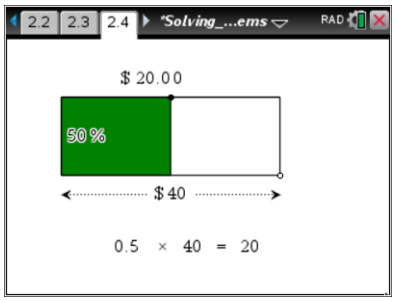


6. While shopping, Courtney finds a dress she likes at Store A for $50 at a discount of 60%. She finds the same dress at Store B for $40 at a discount of 50%. Grab and drag the point on the bottom of the rectangle to change the cost of the dress before the discount. Grab and drag the point on the top of the rectangle to find percentage of the cost.
   a. Where should she buy the dress? Why?



Answer: The dress at Store A cost 40% (100% - 60%) of $50 which is $20. The dress at Store B cost 50% of $40 which is $20. It doesn't matter where she buys the dress. The price is the same.

   b. What equation is used to calculate 40% of $50? What are the base, the rate, and the percentage in this equation?



Answer: .4 x 50 = 20; The base is 50 because it is the total cost of the dress. The rate is .4 because it is the decimal equivalent of 40%. The percentage is 40%.

   c. What equation is used to calculate 50% of $40? What are the base, the rate, and the percentage in this equation?

Answer:.5 x 40 = 20; The base is 40 because it is the total cost of the dress. The rate is .5 because it is the decimal equivalent of 50%. The percentage is 50%.

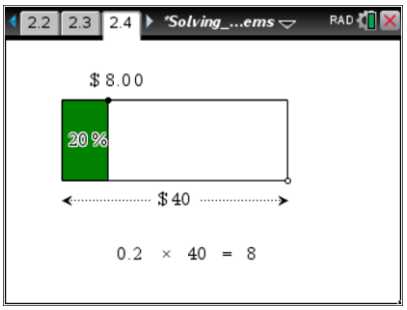
Teacher Tip: Students can approach this question differently. Some students first find the discount on each dress, so for the dress from Store A they would use the representation 60% of $50 to find the discount of $30 and then subtract that from the cost of the dress to find the final discounted price of $50 - $30 - $20. Other students will follow the method described above. Encourage students to share their approaches, so they can see that both methods lead to correct answers; however, subtraction from 100% first is preferred as a foundation for understanding exponential decay. Help students to recognize that in order to find a solution, they should determine the base and the rate from the word problem and then multiply these quantities.

TI-Nspire Navigator Opportunity: ***Quick Poll, Class Capture, and Live Presenter***
See Note 1 at the end of this lesson.

7. Describe how you can find the percentage given the base and the rate.

   Answer: In order to find the percentage that will be used to find a solution, use the rate given in the problem and multiply by 100%. Then add this to 100% if the quantity is increasing (e.g., a restaurant bill tip), but subtract this from 100% if the quantity is decreasing (e.g., a discount).

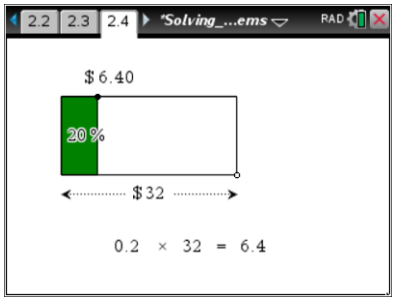
8. Courtney continues shopping for shoes in another store. She finds dress shoes and sandals that she likes at the same original price of $40. She has a 20% off coupon that she can use on the dress shoes that are already marked down 20%. The sandals are marked 40% off, but the store will not accept the coupon on the purchase of the sandals. She can only buy one pair of shoes, and she wants to spend the least amount of money.



   a. What is the discounted price of the dress shoes?

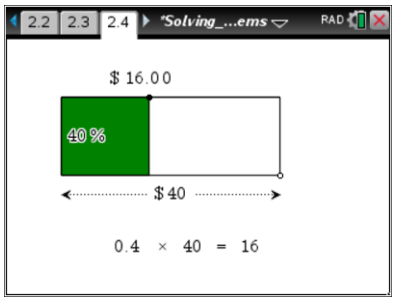
   Answer: If the price of the dress shoes is $40, then 20% off of $40 would be $8 (0.2 x 40 = 8). These dress shoes would cost $32 before the coupon is used.

b. What is the price of the dress shoes with the coupon?



Answer: When the 20% off coupon is used, 20% of $32 will be $6.40 (0.2 x 32 = 6.40). Courtney's final cost for the dress shoes using the coupon would be $32 – 6.40 = $25.60.

c. What is the discounted price of the sandals?



Answer: The discount of 40% of $40 on the sandals would give a discount of $16 (0.4 x 40 = 16). The final cost for the sandals is $40 - $16 = $24.

Teacher Tip: Students might find the discount and then subtract from the original cost or they might subtract the discount from 100% and use that answer to calculate the reduced cost. Example: Again, two approaches are possible: students might find 40% of $40 and then subtract $16 from $40 or they might take 60% of $40 to get $24. Encourage students to consider different approaches to the problem, as the second approach will aid them in their understanding of exponential growth and decay.

d. Is there a difference in the price of the shoes? Why?

Answer: Yes. A one-time 40% discount of an original price is higher than a 20% discount followed by another 20% discount of the same price. When we use a 20% discount on a discounted price, we apply the 20% discount to only 80% of the original price, thus the additional discount is only 0.2 x 80% or 16%. Thus the total discount is 36%. 36% < 40%.

Teacher Tip: It is important to help students understand that the percent of any amount is proportional to the base amount. Thus, in the first case, the total discount cannot be found by adding 20% + 20%. Instead, what is paid for the shoes is 80% of the original, followed immediately by 80% of that answer, or 0.8(0.8 x original) = 0.64 x original. Therefore, the total discount is found by 1 – 0.64 = 36%.

TI-Nspire Navigator Opportunity: ***Quick Poll, Screen Capture, and Live Presenter***
See Note 1 at the end of this lesson.

9. You are now going to shop for a hoodie. In the store you find two hoodies that you like at the same original price. The first hoodie is marked down 15%, and the second hoodie is marked down 25%. You have a coupon for 10% that can only be applied to the first hoodie.
   a. Record the price of these hoodies below. Choose the price between $20 and $60.

      The original price of each hoodie is $ _____.

      Sample Answers: Answers will vary; however, students can only choose a whole number for the original price when using the visual model.

      Tech Tip: Students can use the visual model on Page 2.4 to help them solve this problem or to verify their calculations.

   b. What is the discounted price of the first hoodie? What is the base, and what is the rate in the equation?

      Sample Answers: Answers will vary. Students should be working with the equation, discounted price = 0.85 x original price. The base is the original price and the rate is 0.85. Alternatively, students can find the discount = 0.15 x original price, and then the base is the original price and the rate is 0.15. Then they will need to subtract the discount from the original price.

   c. What is the price of the hoodie with the coupon? What is the base, and what is the rate in the equation?

      Sample Answers: Answers will vary. Students should be working with the equation final price = 0.9 x discounted price. The base is the discounted price, and the rate is 0.9. Alternatively, students can find the coupon discount = 0.10 x discounted price, then the base is the discounted price and the rate is 0.10. Then they will need to subtract the coupon discount from the discounted price.

   d. What is the discounted price of the second hoodie? What is the base, and what is the rate in the equation?

      Sample Answers: Answers will vary. Students should be working with the equation discounted price = 0.75 x original cost. The base is the original cost, and the rate is 0.75. Alternatively, students can find the discount = 0.25 x original price, then the base is the original price and the rate is 0.25. Then they will need to subtract the discount from the original price.

e. Is there a difference in the price of the hoodies? Why?

Answer: The hoodie with the 25% single discount will cost less than the hoodie that has a 15% discount and a 10% coupon. This is because 0.9 x (0.85 x original cost) = 0.765 x original cost > 0.75 x original cost.

Teacher Tip: Generate discussion that compares two consecutive discounts of an item, A% and B%, and one time discount (A+B)% on the same item. Help students generalize this situation for any price of the item to see that consecutive discounts will produce a higher priced item than a single time discount equal to the sum of all consecutive discounts. Generate discussion of why this is happening.

TI-Nspire Navigator Opportunity: *Class Capture and Live Presenter*
See Note 2 at the end of this lesson.

Wrap Up

As a result of this lesson, teachers should ensure that students are able to:

- Connect verbal, visual, and numerical representations of problems involving percents, including percents larger than 100%.
- Identify base, rate, and percentage using visual models and the percent equation.
- Understand the meaning of percents greater than 100%, and apply this understanding to problem solving.
- Use proportional relationships to solve multi-step percent problems.

Assessment

Send the file *Solving_Percent_Problems_Assessments.tns* to students. After students complete the assessment, collect the files.

Answers: B, C, C, C, False, B&C

TI-Nspire Navigator Opportunity: Portfolio and Slide Presentation
See Note 3 at the end of this lesson.

TI-Nspire Navigator

Note 1

Quick Poll, Class Capture, and Live Presenter

Use Open Response in Quick Poll to collect students' answers to the questions with numerical answers. Use Class Capture to monitor students' progress throughout the lesson. Live Presenter can be used at any point to let students demonstrate the use of the visual model and explain their answers. Encourage students to demonstrate different approaches to solving percent problems. For example, in problem 4a some students could find percentage of remaining fish first and then subtract that from 100% to find percentage of caught fish. Other students could find the number of caught fish first, and then determine percentage of the caught fish out of total.

Note 2

Class Capture and Live Presenter

Use Class Capture to monitor students' work on their own problems and select several different situations for sharing. Use Live Presenter to allow students to explain their solutions and answer questions 9a – 9e.

Note 3

After collecting files from the students, use the Portfolio to open *Solving_Percent_Problems_Assessments.tns* in order to analyze students' responses. Use the Slide Presentation to discuss student answers with the class.