Permutations

ID: 8432

Time required
35 minutes

Activity Overview

This activity is an introduction to permutations. It includes an optional review on factorials and the Fundamental Counting Principle (also called the Basic Counting Rule). Students are then led through the development of the formula for finding n objects taken n at a time and then n objects taken r at a time. They are given several problems to solve. Lastly, an optional extension allows students to use the formula for permutations with repetition.

Topic: Permutations, Combinations, & Probability

- *Use the Factorial Counting Principle to calculate the number of outcomes in a sample space.*
- *Use factorial notation to express the number of permutations and combinations of n elements taken r at a time.*

Teacher Preparation and Notes

- *Problem 1 is an introductory question and Problem 2 is a review of factorials and the Fundamental Counting Principle. Problem 2 can be skipped if this review is not needed in your class.*
- *Many students struggle with permutations and combinations. For this reason, it is important that you do not rush through the activity or assume that any part of the activity is "too easy." A solid foundation on these basic concepts will help them later when the problems become more complex.*
- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *Information for an optional extension is provided at the end of this activity, both on the student worksheet and in the .tns file.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution .tns files and student worksheet, go to education.ti.com/exchange and enter "8432" in the quick search box.***

Associated Materials

- *Permutations_Student.doc*
- *Permutations.tns*
- *Permutations_Soln.tns*

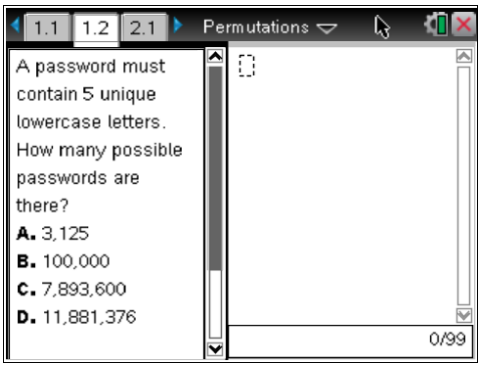
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Permutations and Combinations (TI-Nspire technology) — 12602*

Problem 1 – An introduction

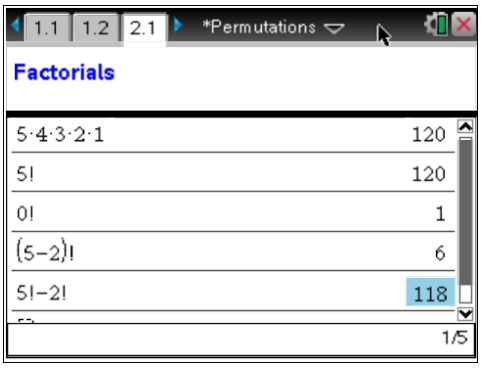
Have students move to page 1.2 and answer the question shown, using whatever reasoning they feel is justifiable. They may use the *Calculator* application on the right side of the page for any necessary calculations. (To move between applications, press ctrl + tab.) Encourage them to share their reasoning with the class.



Assure students that they will be able to answer this question again at the end of the activity, after they have learned how to compute permutations. After working through the activity, this question should be easy!

Problem 2 – Factorials and the Fundamental Counting Principle

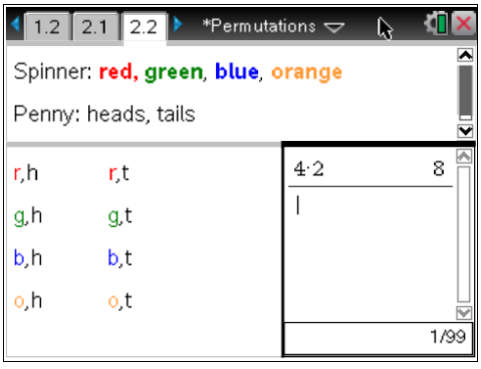
On page 2.1, students will review factorials. The "!" symbol can be found by pressing ctrl + ?!. First they will evaluate the expression $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. Then have students evaluate 5!.



Prompt students for a definition of the factorial of a number. Let them find more factorials if needed.

Next, they should calculate the value of 0!, followed by calculating and comparing (5 – 2)! and 5! – 2! to observe that they are not equivalent expressions. This should be enough review on factorials, although you may have students evaluate more expressions if desired.

On page 2.2, students are to examine all possible outcomes of spinning a spinner and flipping a coin. They can list their answers on their worksheet or in the *Notes* application on the bottom left side of the page. Tell students they can abbreviate using the first letter of each word.

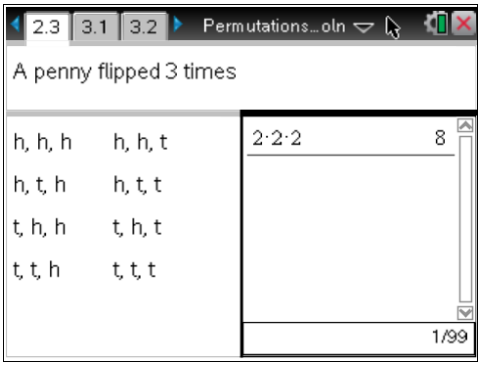


Have students study their list and ask what multiplication expression could have been used to find the answer. If they aren't sure, ask how the list would change if there were 10 colors on the spinner. This should lead to the conclusion that there would be 10 × 2 outcomes—two for each of the 10 colors.

TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 1 at the end of this lesson.

Students can then advance to page 2.3, where they are to find the total number of outcomes when a penny is flipped three times. Students can list the outcomes on the left and/or use the *Calculator* application on the right.



Finish the review by having them state the Fundamental Counting Principle in their own words. If there are *a* ways of performing one action and *b* ways of performing another, then there are $a \cdot b$ ways of performing both actions.

Problem 3 – *n* objects taken *n* at a time

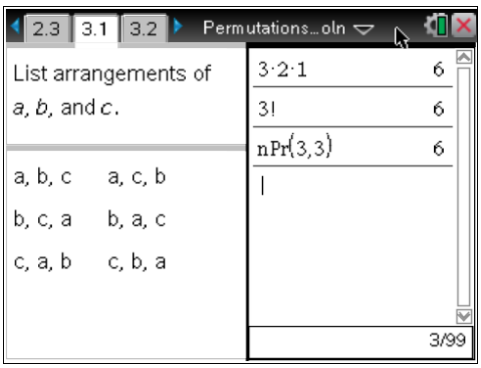
On page 3.1, have students list all the ways the letters *a*, *b*, and *c* can be arranged. Explain that because they are looking for every possible arrangement, they are finding the number of ***permutations***.

Define permutation as an *arrangement* of objects in which order counts. When finding permutations, a different order is a different permutation. Therefore, *a*, *b*, *c* is a different permutation than *a*, *c*, *b* even though they contain the same letters.

Ask students what multiplication expression can be used to find the answer. If needed, prompt them by writing the following on the board:

$$\underline{\text{\# choices for 1st spot}} \times \underline{\text{\# choices for 2nd spot}} \times \underline{\text{\# choices for 3rd spot}}$$

Have them write their expression on the right side of the page. Then have them write an equivalent expression using a factorial.

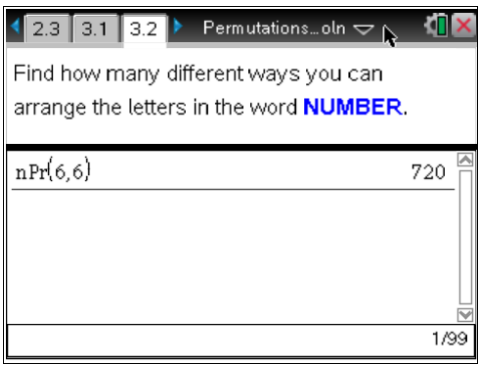


TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 2 at the end of this lesson.

Explain to students that they have found the number of permutations of *n* items taken *n* at a time (meaning all available items were used). Introduce that this can be denoted as ${}_nP_n$. In general, permutations are shown as ${}_nP_r$, where *n* is the number of available items and *r* is how many of those items are actually being arranged. In the example on page 3.1, *r* happens to equal *n*.

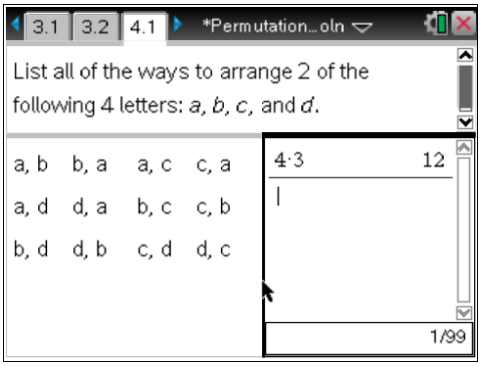
On the handheld, select **MENU > Probability > Permutations** from the *Calculator* application (as shown above, enter values for *n* and *r* (separated by a comma), and press enter. Students should now see that ${}_nP_n = n!$.



Students may use their handhelds to answer the question on page 3.2. The answer should make it clear that it is *not* always easy to write out all arrangements by hand and count them!

Problem 4 – *n* objects taken *r* at a time

On page 4.1, students are asked to list all of the ways to arrange two of four letters.



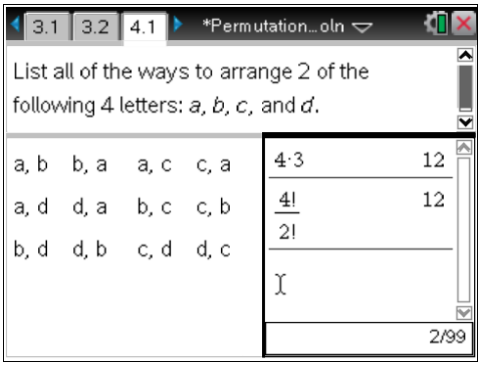
Ask students what multiplication expression can be used to find the answer. If needed, prompt them by writing the following on the board:

$$\frac{\text{\# choices for 1st spot}}{} \times \frac{\text{\# choices for 2nd spot}}{}$$

Ask students if this could be represented as a factorial (no). Ask why not. (It doesn't include $2 \cdot 1$.)

Write the following on the board and ask what multiplication expression should be written in the denominator to make the equation true.

$$\frac{4 \cdot 3 \cdot 2 \cdot 1}{\square} = 12$$



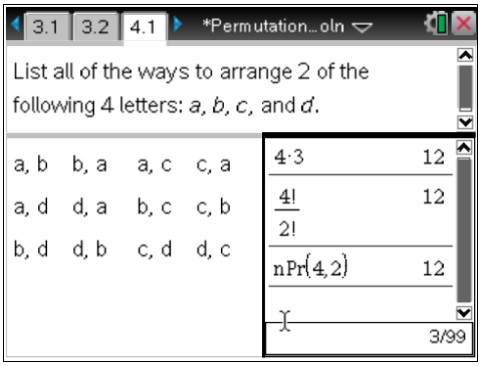
The answer is $2 \cdot 1$, because it would cancel out the $2 \cdot 1$ in the numerator, leaving $4 \cdot 3 = 12$.

Have students write the expression above using factorial notation. They can press ctrl + ÷ to enter the fraction template. Point out that $(4 - 2)! = 2!$

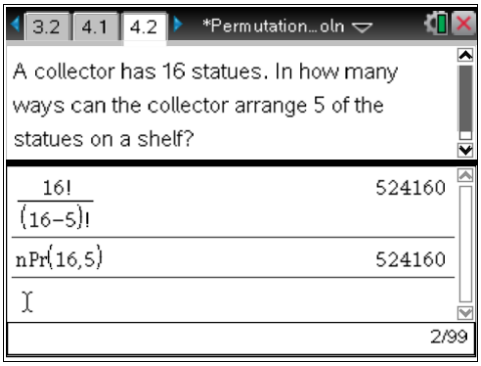
Share the formula for finding the number of permutations of *n* unique objects taken *r* at a time.

$${}_nP_r = \frac{n!}{(n-r)!}$$

Ask that they confirm the answer to the question by using the permutation command.



Encourage students to use both the definition of ${}_nP_r$ and the ${}_nP_r$ function on the handheld to answer the question on page 4.2.



Once again, this answer should show the advantage of using formulas to calculate the number of arrangements over listing and counting by hand.

Problem 5 – Practice

Have students answer the questions posed in Problem 5 on their own. When everyone is finished, review the answers with the students.

Solutions

Page 5.1: (intro question): ${}_nP_r(26,5) = 7,893,600$

Page 5.2: ${}_nP_r(6,6) = 720$

Page 5.3: ${}_nP_r(10,3) = 720$

Page 5.4: ${}_nP_r(26,3) \cdot {}_nP_r(10,5) \cdot 2 = 943,488,000$

Extension

Remind students that the formula ${}_nP_r = \frac{n!}{(n-r)!}$ only applies to arrangements of *n* ***unique*** objects.

The formula for *permutations with repetitions* can be found on page 6.1. Have them use the formula to answer the questions on page 6.2.

The answers are: **PIZZA:** 60; **SUCCESS:** 420: **COOKBOOK:** 840; and **MISSISSIPPI:** 34,650.

TI-Nspire Navigator Opportunities

Note 1
Entire Document, *Screen Capture* and/or *Live Presenter*
Throughout the lesson, you may choose to use screen capture to verify students are following along and able to find the solutions. You may also choose to pick one or more students to lead the class through the lesson. With an overhead projector, the live presenter can show students how to find the factorial and permutation functions on the handheld.

Note 2
Entire Document, *Quick Poll*
You may choose to use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.