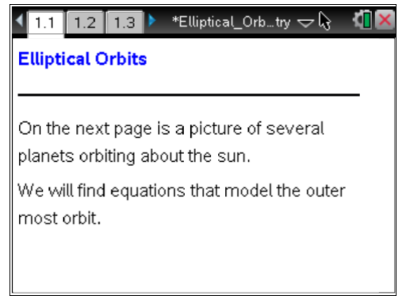


Math Objectives

- Students will model a curve from a picture on the graph screen.
- Students will apply the general equation of an ellipse whose center is not at the origin.
- Students will solve an equation of an ellipse in x and y for the variable y.

Vocabulary

- ellipse
- vertices
- general equation of an ellipse

About the Lesson

- This lesson involves generating equations of best fit for an ellipse.
- As a result students will:
  - Be able to calculate values for parameters for the general equation of an ellipse from the graph of an ellipse whose center is not the origin.
  - Create two functions to model the equation of an ellipse, one is the top half and the other is the bottom half of the ellipse.
  - Verify that the equations do in fact closely model the actual path of a planet in orbit.

TI-Nspire™ Navigator™ System

- Transfer a File
- Use Screen Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

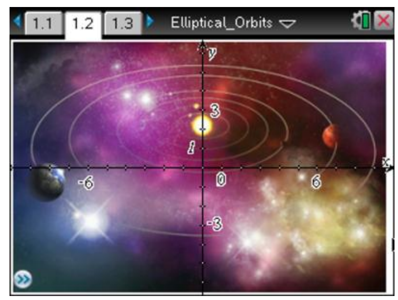
Lesson Files:
Student Activity
Elliptical_Orbits_Student.pdf
Elliptical_Orbits_Student.doc

TI-Nspire document
Elliptical_Orbits.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☝) getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text*. Then press [ctrl] [☝] to grab the point and close the hand (✊).



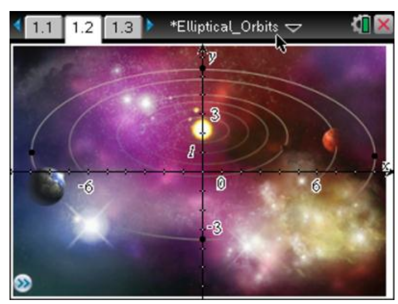
Move to page 1.2.

We are going to model the path of the largest orbit by using the equation of an ellipse.

1. Grab & move the four points near the origin and place the points at each of the four vertices of the ellipse. Notice that the horizontal vertices are **not** on the x-axis, while the vertical vertices *are* on the y-axis. (To grab a point, press [ctrl] [☝] and use the Touchpad arrows to move. Press d to release the point when the ordered pairs show.)

Note: If you make a mistake, or are not happy with where the point is placed, immediately press [ctrl] [esc] to undo the last action.

For reference purposes, we are going to call the four points at the vertices P, Q, R, and S. P is the top vertical point; R is the bottom vertical point. S is the horizontal point on the left side, and Q is the horizontal point on the right side.



2. Write the coordinates in the spaces provided. Include all digits shown on your TI-Nspire handheld screen.

ANSWER:

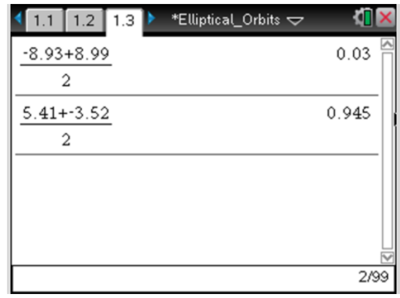
P: (0 , 5.41)
Px Py

S: (-8.93 , 0.881) Q: (8.99 , 0.818)
Sx Sy Qx Qy

R: (0 , -3.52)
Rx Ry

Note: Px stands for the x-coordinate of point P, Py stands for the y-coordinate of point P, and so on.

TI-Nspire Navigator Opportunity: ***Screen Capture and/or Live Presenter***
See Note 1 at the end of this lesson.



Move to page 1.3.

Note: This is a calculator page. Use this page for any calculations that need to be performed. Record the results on this worksheet in the appropriate spaces provided.

The center of the ellipse should be close to the midpoints of segments SQ and PR. To approximate the center, we will calculate the *x*-coordinate of the midpoint of segment SQ, and the *y*-coordinate of the midpoint of segment PR.

3. To find the *x*-coordinate of the midpoint of segment SQ, find the average of the x-coordinates:

Answer:

$$\frac{Sx+Qx}{2} = 0.03$$

TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 2 at the end of this lesson.

4. To find the *y*-coordinate of the midpoint of segment PR, find the average of the *y*-coordinates:

Answer:

$$\frac{Py + Ry}{2} = 0.945$$

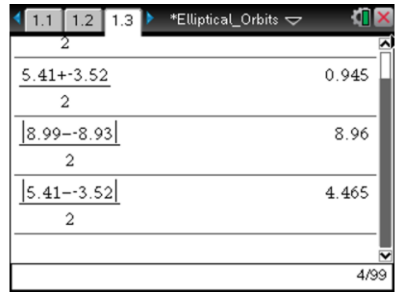
5. Round your final answers to the nearest tenth. The coordinates of the center of the ellipse are approximately:

Answer:

(0 , 0.9)
h k

The general equation of an ellipse is: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The value for **a** is half the length of segment SQ. The value for **b** is half the length of segment PR.

6. Calculate those two values on the worksheet below. Use page 1.3 of the TI-Nspire document to do any computations. Do not round any final answers until told to do so.



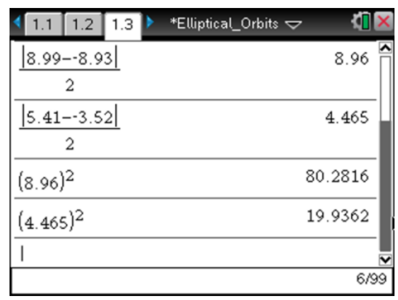
a = half the length of segment SQ = $\dfrac{|Q\,x - S\,x|}{2}$

<u>Answer</u>: 8.96

b = half the length of segment PR = $\dfrac{|P\,y - R\,y|}{2}$

<u>Answer</u>: 4.465

7. Since the equation of the ellipse squares the values for both **a** and **b**, square each of the values, and record your answers below:



<u>Answer</u>:

a^2 = 80.2816 b^2 = 19.9362

8. For our purposes, we will round those values to the nearest tenth:

<u>Answer</u>:
a^2 = 80.3 (to the nearest tenth) b^2 = 19.9 (to the nearest tenth)

> **Teacher Tip:** You might want to have your students round to the nearest whole number to make calculations a bit easier. Explain how rounding can affect the results in the graph.

9. Substitute the values of h, k, a^2, and b^2 into $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$. This is the general equation of an ellipse whose center is at (h, k).

Write your equation here:

<u>Answer</u>:

$$\frac{(x-0)^2}{80.3} + \frac{(y-0.9)^2}{19.9} = 1 \quad \textbf{or} \quad \frac{x^2}{80.3} + \frac{(y-0.9)^2}{19.9} = 1$$

10. Solve for y, and record your work next to each following step:

$$\frac{x^2}{80.3} + \frac{(y-0.9)^2}{19.9} = 1$$

Original

a. First, clear out the denominators by multiplying each term by $a^2 \cdot b^2$. The result should be:

$$b^2 \cdot (x-h)^2 + a^2 \cdot (y-k)^2 = a^2 \cdot b^2$$

b. Subtract the first term (the one involving x) from each side:

$$a^2 \cdot (y-k)^2 = a^2 \cdot b^2 - b^2 \cdot (x-h)^2$$

c. Divide each term by a^2:

$$(y-k)^2 = b^2 - \frac{b^2}{a^2} \cdot (x-h)^2$$

Answers:

a. $19.9x^2 + 80.3(y-0.9)^2 = (80.3)(19.9)$

b. $80.3(y-0.9)^2 = (80.3)(19.9) - 19.9x^2$

c. $(y-0.9)^2 = 19.9 - \dfrac{19.9}{80.3}x^2$

d. Take the square root of each side.

$$y - k = \pm\sqrt{b^2 - \frac{b^2}{a^2}(x-h)^2}$$

(make sure that you have both roots: $\pm$)

e. Add k to each side to finish:

$$y = k \pm \sqrt{b^2 - \frac{b^2}{a^2}(x-h)^2}$$

Answers:

d.

$$y - .9 = \pm\sqrt{19.9 - \frac{19.9}{80.3}x^2}$$

e.

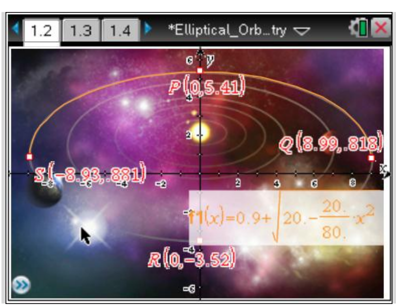
$$y = .9 \pm \sqrt{19.9 - \frac{19.9}{80.3}x^2}$$

Move to page 1.2.

11. Press ctrl G to make the function input line appear, type one of the equations you just obtained into $f1(x)$, and press enter.

For example:

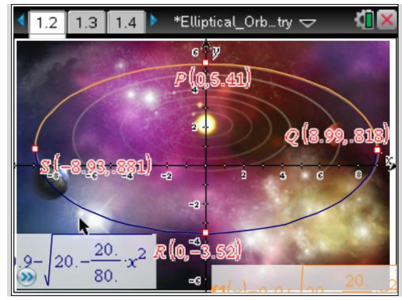
$$f1(x) = k + \sqrt{b^2 - \frac{b^2}{a^2}(x-h)^2}$$



The top half of the ellipse should closely approximate the top half of the path of the planet.

If so, you did all the algebra correctly. If the ellipse does not follow the path of the planet, go back, and recheck your algebra.

12. Press ctrl G to make the function input line appear again, and type the other equation you obtained into $f2(x)$, and press enter.

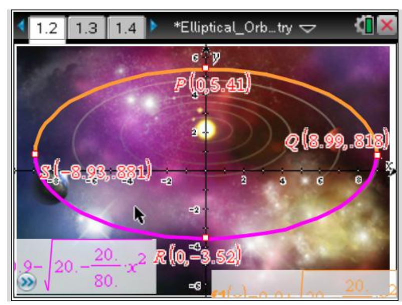


> **Tech Tip:** To copy & paste the equation, put the cursor in the entry line, press & hold the shift key and use the Touchpad arrows to highlight the expression. Press ctrl C to copy and move the cursor to f2(x) and press ctrl C to paste.

The bottom half of the ellipse should closely approximate the bottom half of the path of the planet.
If so, you did all the algebra correctly. If the ellipse does not follow the path of the planet, go back, and recheck your algebra.

> **Teacher Tip:** Students may want to move the equation label out of the way and place it at the bottom of their screen.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 3 at the end of this lesson.



> **Tech Tip:** To make the screen look better, instruct students to make the line thicker by hovering over the graph and pressing ctrl menu > Attributes > Medium. Or, to change the color of the graphs, hover over the graph and press ctrl menu > Color > Line Color.

Extension

Have students model an ellipse whose center is not on either axis.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- What the general equation of an ellipse is, what the different parameters mean, and how they are computed.
- How to solve an equation for *y* in terms of other variables.

Assessment

Ask the students to model the second largest ellipse.

TI-Nspire™ Navigator™

Note 1

Take Screen Captures of each of the student's handhelds to ensure that they are placing the points correctly. You might also elect to make one student the Live Presenter to show how he/she is placing the points.

Note 2

Ask students to go to the calculator part of the Scratchpad and type in the coordinates that they are getting for (h, k). Take a Screen Capture of all of these and discuss which ones are acceptable and which ones might be incorrect. Talk about what is considered acceptable and what isn't.

Note 3

Have the students graph both of the functions on the picture. Then take a Screen Capture of each screen, and show the class how each person did.