Thursday Night PreCalculus, February 8, 2024

Trigonometric Identities: Equations and Inequalities

Problems

1. **(a)** Find all the values of x that satisfy the equation $\sqrt{2}\cos(4x) + 1 = 0$

$$\sqrt{2}\cos(4x) + 1 = 0$$

$$\cos(4x) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$4x = \frac{3\pi}{4} + 2n\pi, \qquad 4x = \frac{5\pi}{4} + 2n\pi$$

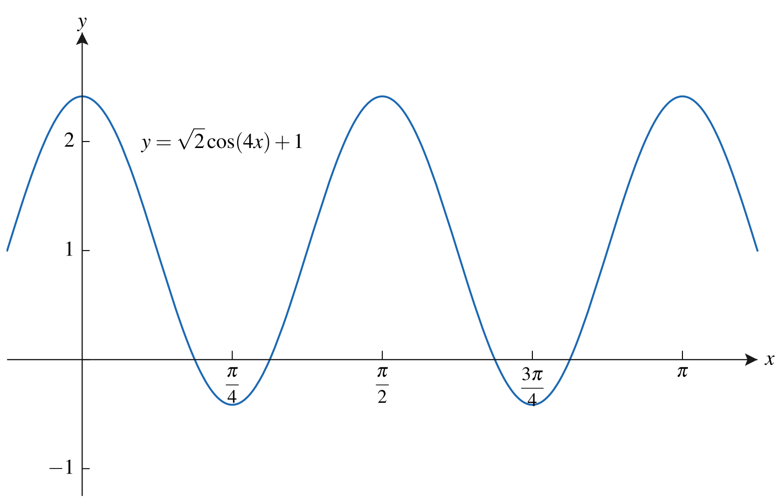
$$x = \frac{3\pi}{16} + \frac{n\pi}{2}, \qquad x = \frac{5\pi}{16} + \frac{n\pi}{2}$$

(b) Find all the values of x in the interval $0 \le x \le \pi$ that satisfy the equation $\sqrt{2}\cos(4x) + 1 < 0$

$$\cos(4x) < -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\frac{3\pi}{4} < 4x < \frac{5\pi}{4} \qquad\qquad \frac{11\pi}{4} < 4x < \frac{13\pi}{4}$$

$$\frac{3\pi}{16} < x < \frac{5\pi}{16} \qquad\qquad \frac{11\pi}{16} < x < \frac{13\pi}{16}$$



2. **(a)** Find all the values of x that satisfy the equation $\frac{1}{\sqrt{3}}\sin(2x) - \frac{1}{2} = 0$.

$$\sin(2x) = \frac{\sqrt{3}}{2}$$

$$2x = \frac{\pi}{3} + 2n\pi, \qquad 2x = \frac{2\pi}{3} + 2n\pi$$

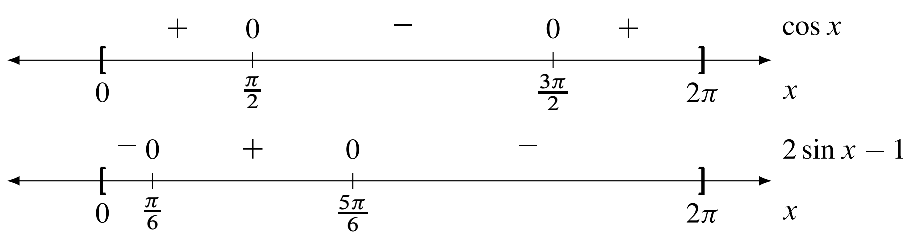
$$x = \frac{\pi}{6} + n\pi, \qquad x = \frac{\pi}{3} + n\pi$$

(b) Find all the values of x in the interval $0 \le x \le 2\pi$ that satisfy the equation $\sin(2x) \le \cos x$.

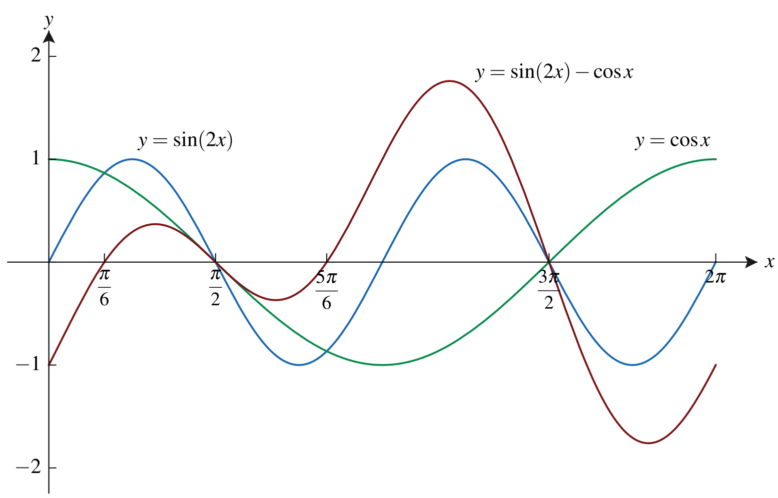
$$2\sin x\cos x \le \cos x \quad \Rightarrow \quad 2\sin x\cos x - \cos x \le 0$$

$$\cos x(2\sin x - 1) \le 0$$

$$\cos x = 0 \quad \Rightarrow \quad x = \frac{\pi}{2}, \frac{3\pi}{2}, \qquad \sin x = \frac{1}{2} \quad \Rightarrow \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$



$$0 \le x \le \frac{\pi}{6}, \qquad \frac{\pi}{2} \le x \le \frac{5\pi}{6}, \qquad \frac{3\pi}{2} \le x \le 2\pi$$



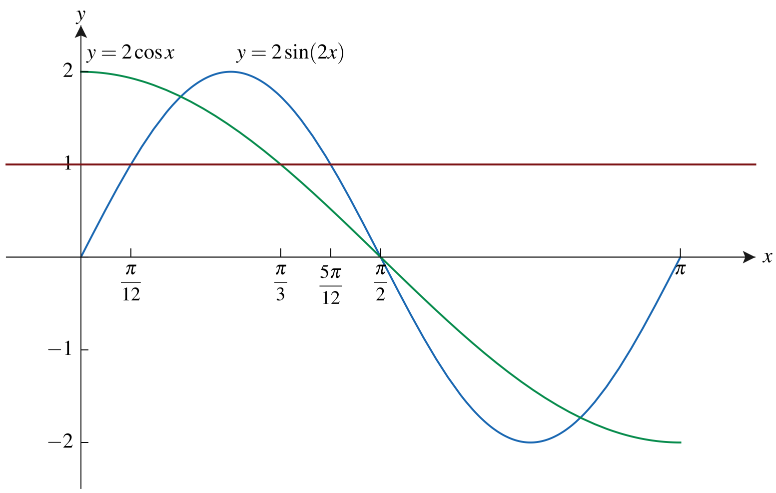
3. What are all the values of θ, $0 \le \theta \le \pi$, for which $2\sin(2\theta) \ge 1$ and $2\cos\theta \ge 1$?

$$2\sin(2\theta) \ge 1 \quad\Rightarrow\quad \sin(2\theta) \ge \frac{1}{2}$$

$$\frac{\pi}{6} \le 2\theta \le \frac{5\pi}{6} \quad\Rightarrow\quad \frac{\pi}{12} \le \theta \le \frac{5\pi}{12}$$

$$2\cos\theta \ge 1 \quad\Rightarrow\quad \cos\theta \ge \frac{1}{2} \quad\Rightarrow\quad 0 \le \theta \le \frac{\pi}{3}$$

Intersection: $\dfrac{\pi}{12} \le \theta \le \dfrac{\pi}{3}$



4. **(a)** Rewrite as an expression in which $\cos x$ appears once and no other trigonometric functions are involved.

$$\frac{1}{1-\sin x}+\frac{1}{1+\sin x}$$

$$\frac{1}{1-\sin x}+\frac{1}{1+\sin x}=\frac{1+\sin x+1-\sin x}{(1-\sin x)(1+\sin x)}$$

$$=\frac{2}{1-\sin^2 x}=\frac{2}{\cos^2 x}$$

(b) Rewrite as an expression in which $\sin x$ appears once and no other trigonometric functions are involved.

$$3\sin x-4\sin^3 x$$

$$3\sin x-4\sin^3 x=(\sin x+2\sin x)-2\sin^3 x-2\sin^3 x$$

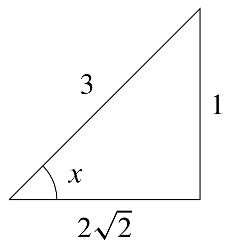
$$=(\sin x-2\sin^3 x)+(2\sin x-2\sin^3 x)$$

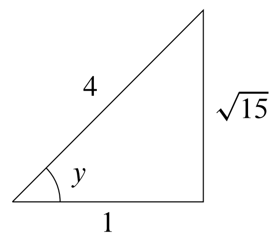
$$=\sin x(1-2\sin^2 x)+2\sin x(1-\sin^2 x)$$

$$=\sin x\cos 2x+2\sin x\cos x\cos x$$

$$=\sin x\cos 2x+\sin 2x\cos x=\sin(x+2x)=\sin 3x$$

5. Suppose $\sin x = \dfrac{1}{3}$ and $\cos y = \dfrac{1}{4}$ where x and y are in the interval $(0, \pi/2)$. Evaluate the expression $\sin(x - y)$.





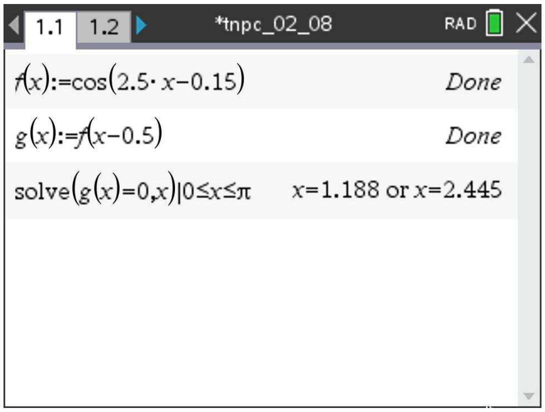
$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

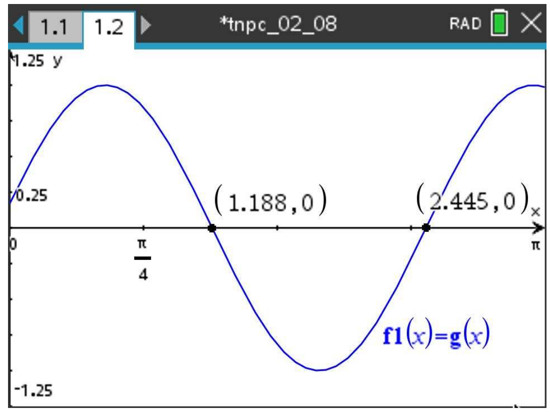
$$= \frac{1}{3} \cdot \frac{1}{4} - \frac{2\sqrt{2}}{3} \cdot \frac{\sqrt{15}}{4}$$

$$= \frac{1}{12} - \frac{\sqrt{30}}{6}$$

$$= \frac{1}{12} - \frac{\sqrt{30}}{6} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{1}{12} - \frac{6 \cdot \sqrt{5}}{6 \cdot \sqrt{6}} = \frac{1}{12} - \sqrt{\frac{5}{6}}$$

6. The function f is given by $f(x) = \cos(2.5x - 0.15)$. The function g is given by $g(x) = f(x - 0.5)$. What are the zeros of g on the interval $0 \leq x \leq \pi$?





$x = 1.188, \;\; 2.445$

Overtime Problems

1. The figures show the graphs of the functions f and g. The function f is defined by $f(x) = \tan^{-1} x$. Find an expression for the function g.