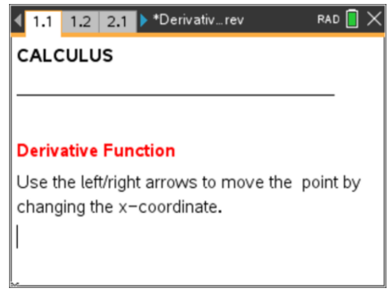
Derivative Function
Student Activity

Name ________________
Class ________________

Open the TI-Nspire document *Derivative_Function.tns.*



If a function **f** is differentiable at $x = a$, then its graph will appear to become linear as you zoom in on the point (a, **f**(a)).

The derivative **f′**(a) is the slope of the tangent line to the graph of $y =$ **f**(x) at the point (a, **f**(a)).

In this activity, you will define a new function, **f′**(x), for the derivative at every value of x.

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1. The graph shown on the left is $y =$ **f**(x) $= x^2$ with one point (a, **f**(a)) boxed in. A magnified “zoomed-in” view of the box is shown on the right with the slope **f′**(a) of the tangent line to the graph at that point. In fact, the graph becomes indistinguishable from the tangent line when you zoom in close. Increase or decrease the value of a by using the up/down arrows.
   a. What is **f′**(2)?

   b. At what value(s) of a is the derivative **f′**(a) = –2?

   c. Fill out the following table of values for a and **f′**(a).

a **=**	–2	–1.3	–0.5	0	0.7	1.5	2.1
f′(*a*) =							

Move to page 2.1.

2. Grab the white point labeled x on the x-axis and move it to see the slope of the tangent line change as you move along the graph of $y =$ **f**(x) $= x^2$.
   a. Describe any pattern you see in the slopes of the tangent lines.

   b. Describe the relationship between each value of x and the slope of the tangent line at (x, **f**(x)).

Move to page 3.1.

3. If you plot the value of the derivative $\mathbf{f'}(x)$ as the y-coordinate for each value x, the ordered pairs $(x, \mathbf{f'}(x))$ trace out the graph of a new function $y = \mathbf{f'}(x)$, the derivative function. Use the up arrow for x in the top window to see the graph of the derivative traced out.
   a. What can you say about the graph of $y = \mathbf{f}(x) = x^2$ when $\mathbf{f'}(x) < 0$?

   b. What can you say about the graph of $y = \mathbf{f}(x) = x^2$ when $\mathbf{f'}(x) > 0$?

   c. What can you say about the graph of $y = \mathbf{f}(x) = x^2$ when $\mathbf{f'}(x) = 0$?

   d. What is the equation of the graph of $\mathbf{f'}(x)$? What is a general rule that gives a relationship between x and $\mathbf{f'}(x)$? Explain.

Move to page 4.1.

4. The graph shown in the left window is of $y = \mathbf{f}(x) = \sin(x)$ with one point $(a, \mathbf{f}(a))$ boxed in. Again, a magnified "zoomed-in" view of the box is shown on the right along with the slope $\mathbf{f'}(a)$ of the tangent line to the graph at that point. Increase/decrease the value of a using the up/down arrows.
   a. What is $\mathbf{f'}(0)$?

   b. At what values of a (in this window) is the derivative $\mathbf{f'}(a) = 0$?

   c. Fill out the following table of values for a and $\mathbf{f'}(a)$.

a	$-\pi$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\mathbf{f'}(\boldsymbol{a})$ **=**								

Move to page 5.1.

5. Grab the white point labeled x on the x-axis and move it to see the slope of the tangent line change as you move along the graph of $y = \mathbf{f}(x) = \sin(x)$.
   a. What can you say about the slope of the tangent line when the graph of $\mathbf{f}(x) = \sin(x)$ is decreasing?

   b. What can you say about the slope of the tangent line when the graph of $\mathbf{f}(x) = \sin(x)$ is increasing?

Move to page 6.1.

6. Use the up arrow for *x* in the top window to plot the graph of the derivative function **f′**(*x*).
   a. What can you say about the graph of y = **f**(x) = sin(x) when **f′**(x) < 0?

   b. What can you say about the graph of y = **f**(x) = sin(x) when **f′**(x) > 0?

   c. What can you say about the graph of y = **f**(x) = sin(x) when **f′**(x) = 0?

   d. Does the graph y = **f′**(x) look familiar? What is the equation of the graph of **f′**(x)? What is a general rule that gives a relationship between x and **f′**(x)? Explain.

Move to page 7.1.

7. Increase the value of *a* using the up/down arrows.
   a. What is **f′**(0)?

   b. For how many values of *a* (in this window) is the derivative **f′**(*a*) = 0?

Move to page 8.1.

8. Grab the white point labeled *x* on the *x*-axis and move it to see the slope of the tangent line change as you move along the graph of y = **f**(x).
   a. For approximately what values of *a* (in this window) is the slope of the graph negative?

   b. For approximately what values of *a* (in this window) is the slope of the graph positive?

Move to page 9.1.

9. Use the up arrow for *x* in the top window to plot the graph of the derivative function **f′**(*x*).
   a. What can you say about the graph of y = **f**(x) when **f′**(x) < 0?

   b. What can you say about the graph of y = **f**(x) when **f′**(x) > 0?

   c. What can you say about the graph of y = **f**(x) when **f′**(x) = 0?