Determining Area
ID: 8747

<div align="right">

Time required
45 minutes

</div>

Activity Overview
This activity begins by presenting a formula for the area of a triangle drawn in the Cartesian plane, given the coordinates of its vertices. Given a set of vertices, students construct and calculate the area of several triangles and check their answers using the Area tool in Cabri Jr. Then they use the common topological technique of dividing a shape into triangles to find the area of a polygon with more sides. They are challenged to develop a similar formula to find the area of a quadrilateral.

Topic: Matrices
- *Calculate the determinant of a matrix.*
- *Applying a formula for the area of a triangle given the coordinates of the vertices.*
- *Dividing polygons with more than three sides into triangles to find their area.*
- *Developing a similar formula for the area of a convex quadrilateral.*

Teacher Preparation and Notes
- *This activity is designed to be used in an Algebra 2 classroom.*
- *Prior to beginning this activity, students should have an introduction to the determinant of a matrix and some practice calculating the determinant.*
- *Information for an optional extension is provided at the end of this activity. Should you not wish students to complete the extension, you may have students disregard that portion of the student worksheet.*
- ***To download the student worksheet and Cabri Jr. files, go to education.ti.com/exchange and enter "8747" in the quick search box.***

Associated Materials
- *Alg2Week03_DeterminingArea_worksheet_TI84.doc*
- *TRIANGLE (Cabri Jr. file)*
- *HEPTAGON (Cabri Jr. file)*

Suggested Related Activities
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.
- *Triangle in the Matrix (TI-84 Plus family) — 14401*
- *Operating on Matrices (TI-84 Plus family) — 11358*

Problem 1

Students are to start the Cabri Jr. application and open the file **TRIANGLE**. A triangle is shown on the screen.

Using the **Coord. & Eq.** tool they will find and record the coordinates of the vertices of the triangle.
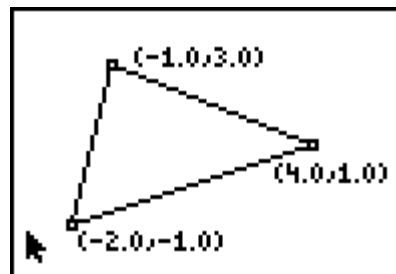
Before exiting the program to return to the Home screen, students need to save their changes by pressing ⃞Y= to open the file menu and choosing **Save**. Then press ⃞2nd ⃞[QUIT] to exit the *Cabri Jr.* app.
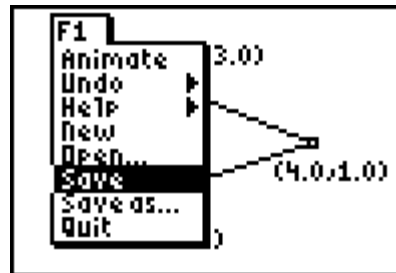
The next step is to create the matrix that is used in the formula. Students should make sure that all of the *x*-values are in the *first row* and all of the *y*-values are in the *second row*.
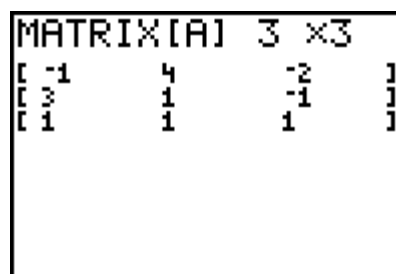
Now students are to calculate the formula: **0.5* abs (det [A])**. They are told to find the **abs(** and **det(** commands in the **Catalog**.
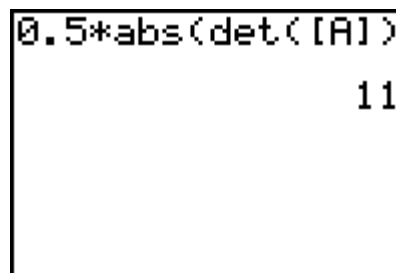
Students can also find the **abs(** command by pressing ⃞MATH and moving to the NUM men. The **det(** command can be entered by pressing ⃞2nd ⃞[MATRIX] and moving to the MATH menu.

Explain to students that to type the matrix **[A]**, go to the matrix menu and choose **[A]** from list of Names.

When students return to Cabri Jr., they are to use the **Area** tool to confirm their answer.

Students will then use the formula and Cabri Jr. to find the area of two different triangles. Students can either start with a new Cabri Jr. file or use the existing TRIANGLE file. Directions for using the existing file are given on the worksheet. Directions for using a new file are given below.
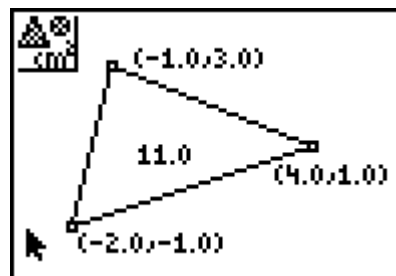
Step 1: Open a new *Cabri Jr.* file and turn on the **Axes** from the Hide/Show menu.

Step 2: Use the **Triangle** tool to draw a triangle. Use the **Coord. & Eq.** tool to display the coordinates. Move the vertices to the correct locations. (Press ALPHA to grab the point.)

Step 3: Use the **Area** tool to find the area of the triangle.
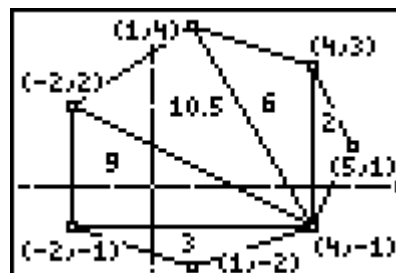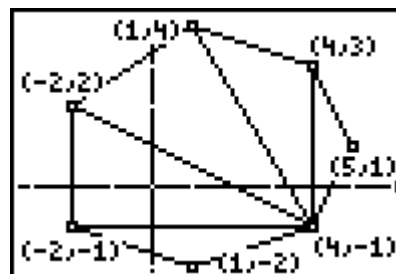
Problem 2

In the second problem, students are to find a formula for the area of a convex polygon. When students open the Cabri Jr. file HEPTAGON, they will see a heptagon.

Students are to use the same method from Problem 1 to find and record the coordinates of the vertices.

After dividing the polygon into triangles using the **Triangle** tool, students should find the area of each triangle. They should make sure to draw all of the triangles with the tool even though when the second to last triangle is drawn, all of the triangles as visible. They can use the Home screen to add up the area of the five triangles.

To use the formula, students need to enter the coordinates of each vertex into its own separate matrix.

Note: If the extension is to be used, students should create and save the new file for the quadrilateral.

Problem 3

Through trial and error, students will find a formula for the area of a quadrilateral given its vertices. Using the coordinates of the vertices, shown below, for the quadrilateral in the Exercise for Problem 2, students will already know the area.

(–3, –1), (0, 3), (5, 0) and (1, –1)

For the exercise, students are to use new vertices of a quadrilateral to confirm the formula they found.

Extension – Signed area

As an extension, students may return to the quadrilateral in drawn in the Exercise of Problem 2 and drag one vertex across the figure so that the quadrilateral is no longer convex. This figure is called a *crossed polygon*. Find the area of the crossed quadrilateral using both the determinant formula and the **Area** tool. Discuss why the results differ. The determinant formula finds the **signed area**, meaning that if part of the polygon crosses over itself, that portion has a negative area. Drag the vertices to create a polygon with a negative signed area.

Solutions

Problem 1

- x_1: −1 y_1: 3
- x_2: −2 y_2: −1
- x_3: 4 y_3: 1
- area = 11 square units

Exercises

1. 13 square units
2. 15.5 square units

Problem 2

x_1: 5 y_1: 1

x_2: 4 y_2: 3

x_3: 1 y_3: 4

x_4: −2 y_4: 2

x_5: −2 $y5$: −1

x_6: 1 y_6: −2

x_7: 4 y_7: −1

- area of heptagon = 30.5 square units

Exercise

- area of quadrilateral = 16.5 square units

Problem 3

$$\text{Area of a quadrilateral} = \frac{1}{4} \left| \det \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix} \right| \text{ OR } \frac{1}{4} \left| \det \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \right|$$

Answer will vary.