Math Objectives

- Students will explore how positive and negative leading coefficients in a function can dilate or reflect the graph of the function.
- Students will graph and analyze absolute value functions.
- Students will analyze the structure of a function and its graph to determine the value of a coefficient in the function.

Vocabulary

- vertical dilation
- reflection
- vertical stretch
- vertical shrink

About the Lesson

- This lesson involves dilating and reflecting different types of function graphs by grabbing points.
- As a result, students will understand:
  - How multiplying a function by a leading coefficient vertically stretches or shrinks a graph.
  - How multiplying by a negative leading coefficient reflects the graph through the *x*-axis.

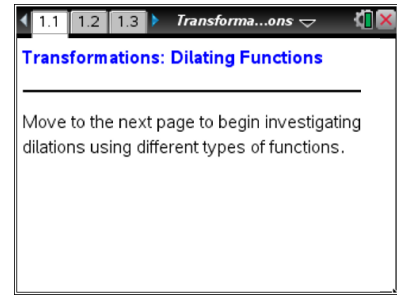## TI-Nspire™ Navigator™

- Send out the *Transformations_Dilating_Functions.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/ calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
- Transformations_Dilating_ Functions_Student.pdf
- Transformations_Dilating_ Functions_Student.doc

TI-Nspire document
- Transformations_Dilating_ Functions.tns
- Transformations_Dilating_ Functions_QP.tns

Discussion Points and Possible Answers

> **TI-Nspire Navigator Opportunity:** *File Transfer*
> **See Note 1 at the end of this lesson.**

> **Tech Tip:** If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (☜) getting ready to grab the point. Press `ctrl` `✎` to grab the point and close the hand (☜). Press `esc` to release the point or deselect other objects. Use the Touchpad to move the point up and down.

> **Tech Tip:** If students experience difficulty grabbing and dragging a point, have them tap and hold the point for a few seconds and then drag the desired point to another location.

Move to page 1.2.

1. Grab and drag the open point on the parabola. Notice that as the point is moved vertically, the value of *a* changes in the equation and hints appear on the left side of the screen.

    a. Using the hints on the left side of the screen, move the open point until there is a reflection, but no stretch or shrink. What is the value of *a*? Describe how the shape of the graph changes.
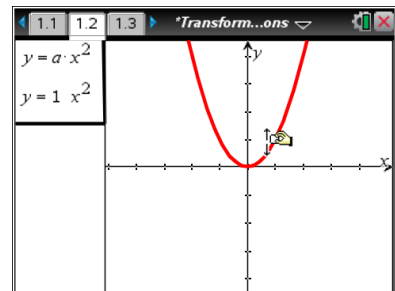
        Answer: $a = -1$. The graph has reflected over the *x*-axis. Students may say things like "the parabola opens down," or "the graph flipped over." Provide students an opportunity to attend to precision with language they use to describe the changes in the graph.
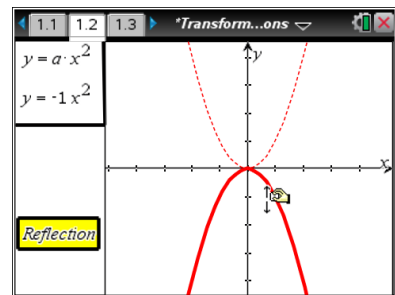

    b. Move the open point so that the hint shows a vertical shrink. What must be true about any value of *a* that makes the graph shrink vertically? Describe how the shape of the graph changes.

        Answer: $|a| < 1$. The graph gets closer to the *x*-axis. The *y*-values have been shrunk vertically. A formal way to say this is that the graph has been compressed vertically.

c. Move the open point so that the hint shows a vertical stretch. What must be true about any value of *a* that makes the graph stretch vertically? Describe how the shape of the graph changes.

Answer: $|a| > 1$. The graph of $y = a \cdot x^2$ gets further away from the *x*-axis. The *y*-values have been stretched vertically. This is called a vertical dilation.
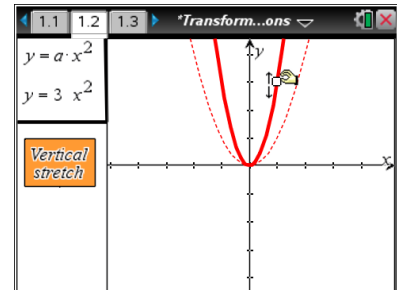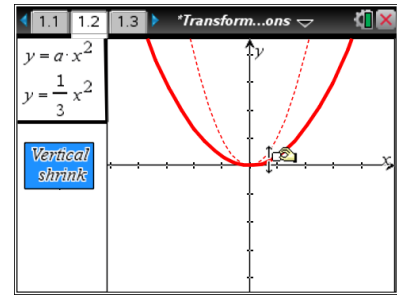
Teacher Tip: Many students and teachers get confused by the "look" of the graph and say that the function is getting wider or narrower. This implies that the x-values are changing. Be sure to clarify that changing the value of *a* in a function like $y = a \cdot f(x)$ causes a vertical dilation, which affects the *y*-values of the function. Limiting the domain can help make this clear. The graph does not get closer to the *y*-axis, but it gets further away from the *x*-axis.

Students may not know what a vertical dilation is. Multiplying a function by a constant causes a vertical dilation. In geometry, a figure can be either enlarged or reduced using a scale factor while the center stays in the same position. Dilations of function graphs can be horizontal or vertical in nature. This document deals only with vertical dilations. Vertical dilations are commonly referred to as vertical stretch or vertical shrink. Horizontal dilations are of the form $y = f(b \cdot x)$. You may want to explore this more depending on the level of your students and the curriculum.

d. What must be true of the value of *a* for there to be both a vertical stretch and a vertical reflection?

Answer: $a < -1$. The value of *a* must be negative to have a vertical reflection. The $|a|$ must be greater than 1 to have a vertical stretch.
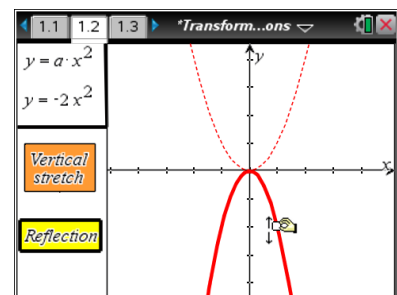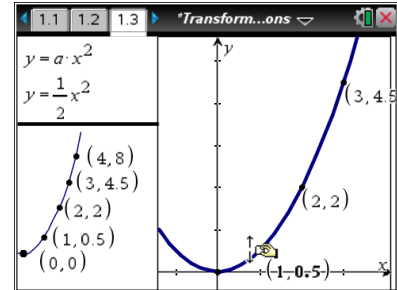
> **TI-Nspire Navigator Opportunity:** *Class Capture* or *Live Presenter*
> **See Note 2 at the end of this lesson.**
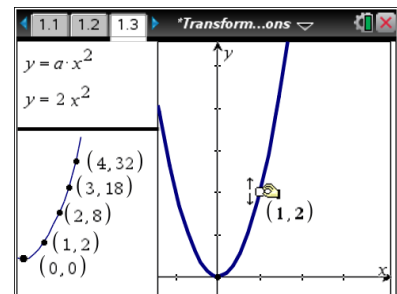
Move to page 1.3.

2. On the bottom left portion of the screen, there is a
   "thumbprint" of the parabola. The thumbprint shows five
   ordered pairs on the parabola and the difference between
   each *y*-coordinate.

   a. When you vertically stretch or shrink (compress) the
      graph by grabbing the open point, what changes in the
      ordered pairs? What remains the same?

   Answer: The *x*-values don't change. The *y*-values get larger when they are vertically
   stretched. And the *y*-values get smaller when they are vertically shrunk (compressed).

   b. Use the thumbprint on page 1.3 to fill in the table below.
      When the value of *a* = 2, the function can be described
      as being vertically stretched by a factor of 2. Looking at
      the table, explain why that description makes sense.

X	$y = 1 \times x^2$	$y = 2 \times x^2$
0	0	0
1	1	2
2	4	8
3	9	18
4	16	32

Answer: The *y*-values in the table have all been multiplied by two. Graphically, the shape of
the graph also looks like it has been grabbed and stretched vertically.

Move to page 2.1.

3. Observe the ordered pairs of the thumbprint view on the left as you move the open point to change the value of *a* in the absolute value graph. Given the equation $y = a \cdot |x|$, what would the value of *a* be if the graph contains the point $\left(-2, -\frac{2}{3}\right)$?
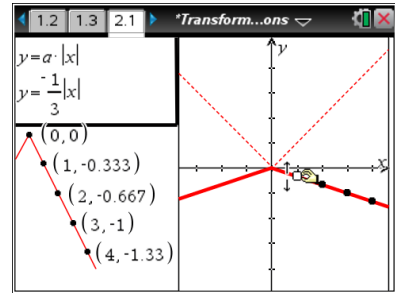
Answer: $a = -\frac{1}{3}$. Substitute the *x* and *y* value of the ordered pair into the equation $y = a \cdot |x|$, then solve for *a*. Or, students could change the value of *a* to $\frac{1}{3}$ and recognize the pattern using the thumbprint.

Move to page 3.1.

4. Observe the ordered pairs as you move the open point of the square root graph.
   a. What is the *y*-value of the function $y = 1 \cdot \sqrt{x}$, when $x = 2$? Why do you think that the ordered pair when $x = 4$ is labeled instead of the ordered pair when $x = 2$?
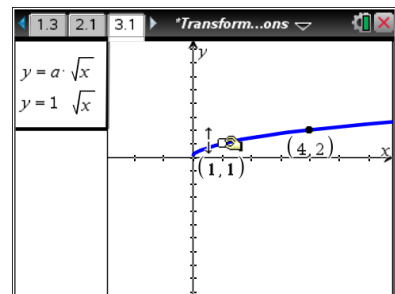
   Answer: $\left(2, \sqrt{2}\right)$. The *y*-value is a nice integer value when $x = 4$. Students should look for these values when they are searching for *x*-values to substitute into the function.

5. Given that the point (7, 12) is a point on the graph of *y = f(x)*, what ordered pair would be on the graph of $y = \frac{1}{3} \cdot f(x)$?

   Answer: The new point would be (7, 4), Students need to know that the *x*-value doesn't change with a vertical stretch or shrink.

6. Describe the transformation(s) that occur to the function $y = g(x)$ if the new function is $y = -4 \cdot g(x)$.

   **Answer:** The graph is reflected through the *x*-axis and vertically stretched by a factor of 4.

7. Given the graph to the right, determine the value of *k* for the square root function $f(x) = k\sqrt{x}$.

    Answer: $f(x) = 4\sqrt{x}$. Students can solve this by substituting the coordinates of the point (4,8) into $f(x) = k\sqrt{x}$ and solving for *k*.
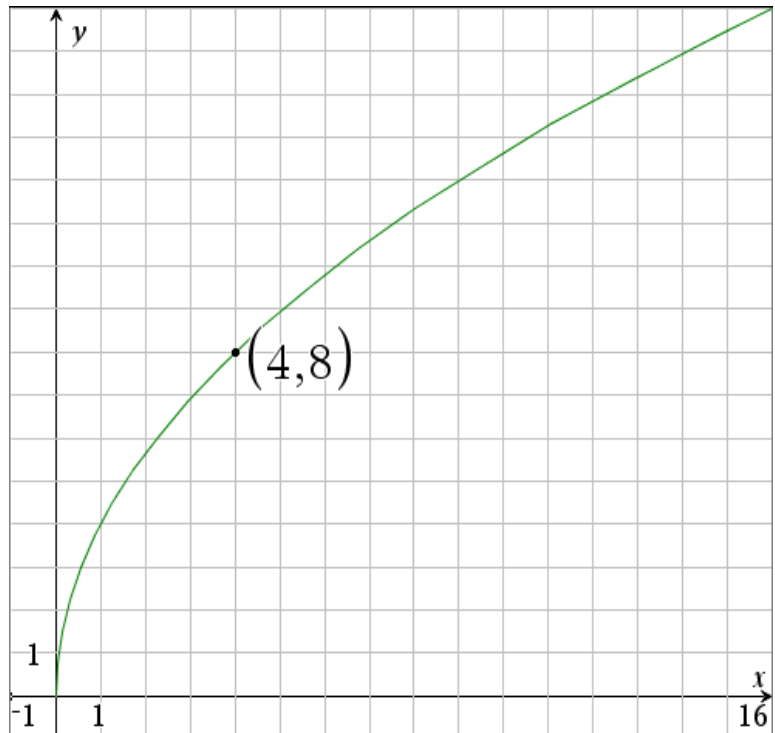
8. What is the equation of the graph to the right?

    Answer: $f(x) = 3|x|$. Students can solve by recognizing that the graph is an absolute value function and calculating the slope.
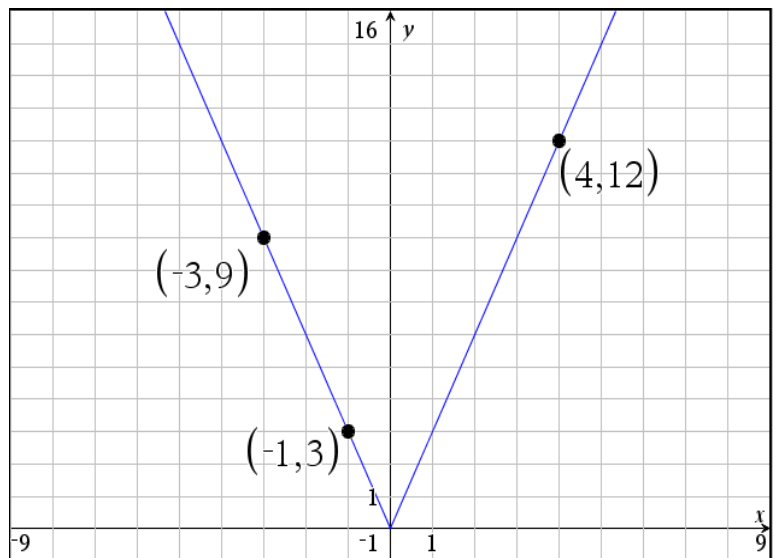
![icons] **TI-Nspire Navigator Opportunity:** *Quick Poll*

See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to identify the leading coefficient of a vertically dilated function.

- How to write the equation of a reflected graph as a multiplication by a negative leading coefficient.

There is an additional TI-Nspire file that can be used as a wrap up.

> **Tech Tip: Transformations_Dilating_Functions_QP.tns**
> contains questions that can be used as Self-Check on the TI-Nspire™
> App on the iPad® or TI-Nspire™ CX handheld. Teachers using the
> TI-Nspire Navigator can send the questions individually as Quick Polls.

TI-Nspire Navigator

Note 1

Entire Document, *File Transfer:* Use File Transfer to efficiently send the TI-Nspire document to the students. Using TI-Navigator will allow students to receive the file without having to leave their seats or use extra cables.

Note 2

Entire Document, *Class Capture* or *Live Presenter:* If students experience difficulty with operation of a file or a question, use the computer software or *Live Presente*r with TI Navigator. You can also use this as a way to facilitate student discussion.

Note 3

Question 6, *Quick Poll:* A *Quick Poll* can be given at the conclusion of the lesson. You can save the results and show them at the start of the next class to discuss possible misunderstandings students may have. The following are some sample questions you can use. Some of these questions might be an extension of the lesson. You can use the file **Transformations_Dilating_Functions_QP.tns** for already written Quick Poll questions.

1. Given that the point (–3, 5) is a point on the graph of $y = f(x)$, what would be the new value of the ordered pair on the graph of $y = 3f(x)$? **Answer:** (–3, 15)

2. Given the function $y = –1(x + 2)^2 + 9$, what are the x-intercepts? **Answer:** (–5, 0) and (1, 0)

3. True/False: The domain of a function is left unchanged by a vertical stretch. **Answer:** True.