# Exponential Growth

TIMATH.COM: ALGEBRA 2

TEACHER NOTES

ID: 9467

## Activity Overview

In this activity, students will find an approximation for the value of the mathematical constant *e* and to apply it to exponential growth and decay problems. To accomplish this, students are asked to search for the base, *b*, that defines a function $f(x) = b^x$ with the property that at any point on the graph, the slope of the tangent line (instantaneous rate of change) is equal to $f(x)$. The result is approximating the value of *e*—Euler's number and the base of the natural logarithms.

## Topic: Exponential & Logarithmic Functions

- Graph exponential functions of the form $f(x) = b^x$
- Evaluate the exponential function $f(x) = b^x$ for any value of $x$.
- Calculate the doubling time or half-life in a problem involving exponential growth or decay.

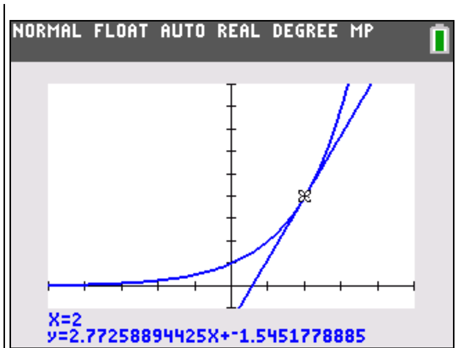
## Teacher Preparation and Notes

- Students encounter the exponential constant e at various levels in their mathematics schooling. It may happen well before they reach calculus, and it is often used without an appreciation for where it originates (or why it is important). A good time to use this activity is when students first encounter e, but it is also appropriate for Precalculus and Calculus students when they are studying derivatives and instantaneous rate of change.
- Prerequisites for the students are: familiarity with graphing and tracing functions on the calculator; an understanding of functions and function notation (both "$y$ =" and "$f(x)$ ="); and an intuitive understanding of rate of change.
- **To download the student worksheet, go to education.ti.com/exchange and enter "9467" in the keyword search box.**

## Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- NUMB3RS – Season 1 – "Identity Crisis" – Exponential Growth (TI-84 Plus and TI-Navigator) — 7727
- Half-Life (TI-84 Plus family) —9287




**This activity utilizes MathPrint™ functionality and includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-83 Plus, TI-84 Plus, and TI-84 Plus Silver Edition but slight variances may be found within the directions.**

**Compatible Devices:**

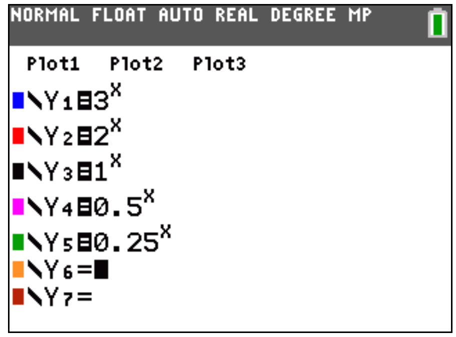
- TI-84 Plus Family
- TI-84 Plus C Silver Edition

**Associated Materials:**

- ExponentialGrowth_Student.pdf
- ExponentialGrowth_Student.doc

Click HERE for Graphing Calculator Tutorials.

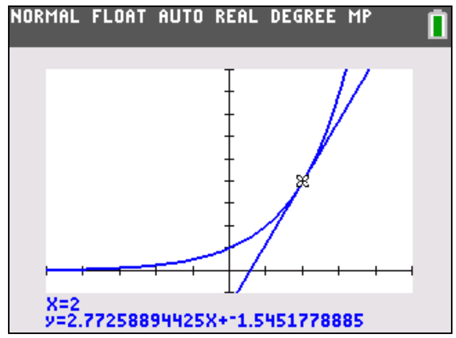
### Problem 1




The activity begins with an investigation of how the value of *b* affects the shape of the graph of $y(x) = b^x$. Students will enter several equations with different values of *b* and examine the graphs of these functions. They are asked to make observations and draw conclusions.

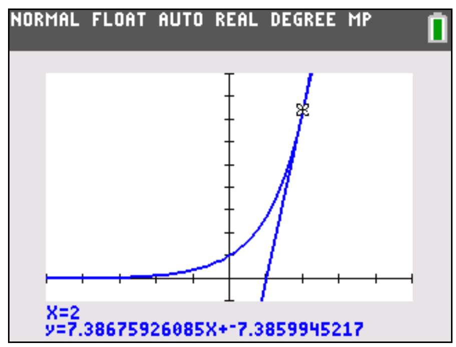
The last question posed in this problem asks students to explain why the value of *b* cannot be negative. It may be worthwhile to discuss this in a whole-class setting.

### Problem 2




In this problem, students work specifically with the graph of $y(x) = 2^x$. Students will draw a tangent line to the curve. Students will explore the relationship between the slope of the tangent line and the value of the function at that point. They should examine several different points and different values of *b*.

### Problem 3




Students are again asked to observe the changing values of the slope of the tangent line and the value of the function—and how they are related.

Students will discover that there is *exactly* one value of *b* for which the slope of the tangent and value of the function are equal—and that this value is a very interesting number!

### Applications

Students are given a series of application problems to apply the knowledge about what they have learned by doing completing this activity.

**Student Solutions**

Problem 1

- Answers may vary. Possible observations: graph gets "steeper" as *b* increases and "flatter" as *b* decreases; always passes through the point (0, 1); increasing when $b > 1$ and decreasing when $0 < b < 1$
- $b = 1$
- Answers may vary. Possible explanation: Even roots of negative numbers are not real numbers. Consider, for example, $(-1)^{0.5} = \sqrt{-1}$, which is not a real number.

Problem 2

- *x*, *f*(*x*), and slope will vary
- the slope is less than *f*(*x*)
- Answers may vary. Possible observations: slope is always positive; as *x* increases, the slope increases; curve never reaches the *x*-axis

*Sample table*

| ***b*** | ***x*** | ***f*(*x*)** | **slope of tangent at *x*** |
|---|---|---|---|
| 2 | 3 | 8 | 5.545 |
| 3 | 1 | 3 | 3.296 |
| 0.5 | 0 | 1 | -0.693 |
| 0.25 | 2 | 0.063 | -0.087 |

Problem 3

- Answers will vary. Possible answer: 3
- Answers will vary. Possible answer: 2
- $b \approx 2.718$
- Answers will vary.

*Sample table*

| ***b*** | $\dfrac{\textbf{slope of tangent at } x}{f(x)}$ |
|---|---|
| 2 | 0.693 |
| 3 | 1.099 |
| 0.5 | –0.693 |
| 0.25 | –1.381 |

*Sample table*

| ***b*** | ***x*** | ***f*(*x*)** | **slope of tangent at *x*** | $\dfrac{\textbf{slope of tangent at } x}{f(x)}$ |
|---|---|---|---|---|
| 2.2 | 0 | 1 | 0.788 | 0.788 |
| 2.4 | 1 | 2.4 | 2.101 | 0.875 |
| 2.6 | 0 | 1 | 0.956 | 0.956 |
| 2.8 | 2 | 7.84 | 8.072 | 1.030 |
| 2.7 | 0 | 2.7 | 0.993 | 0.993 |
| 2.75 | 0 | 2.75 | 1.0116 | 1.0116 |

**Applications**

1. Modeling equation: $P = 1{,}000e^{0.05t}$ (where $P$ is the value and $t$ is the time in years); one year: \$1,051.27; two years: \$1,105.17; five years: \$1,284.03

2. Modeling equation: $P = 500e^{0.5 \cdot 24}$ (where $P$ is population); about 81,000,000

3. Modeling equation: $P = 1{,}000{,}000e^{-0.15 \cdot 10}$ (where $P$ is the volume); about 22.3%

4. Modeling equation for growing snowball: $P = 2e^{0.1t}$ (where $P$ is the weight and $t$ is the time in seconds); 10 seconds: 5.43 pounds; 20 seconds: 14.78 pounds; 45 seconds: 180.03 pounds; 1 minute: 806.86 pounds

   Possible limitations: the modeling equation might not be appropriate after too long a period of time, for example—the snowball may break apart if it gets too big, or it might reach the end of the hill.