ACTIVITY 11

Inverse of Two Temps

Math Objectives:

- Graph scatter plots
- Analyze and graph linear equations
- Compute and model slope
- Derive and apply a conversion equation
- Analyze inverse relations

Materials:

- TI-83/TI-84 Plus Family
- Data from Activity 10

OVERVIEW

Based on the data collected in Activity 10, you will develop and test a mathematical relationship between these two temperature scales and explore the relationship between them. In Activity 10, you found a conversion equation that would calculate the Fahrenheit temperature for any given Celsius temperature. In this activity, you will find a conversion equation that will calculate the corresponding Celsius temperature for any given Fahrenheit temperature. You will also learn about inverse relations.

SETUP

You may have used your calculator for other calculations since the time you performed **Activity 10: Two Hot, Two Cold**. You need to find the data from Activity 10 to use for this activity.

1. Press [STAT] and select **1:Edit** to see the lists displayed. If you do not see the two lists you need, position your cursor **on** the name of any list displayed and press [2nd] [DEL] to access the [INS] (insert) command which will create a blank column to the left of that list. **See Figure 1.**

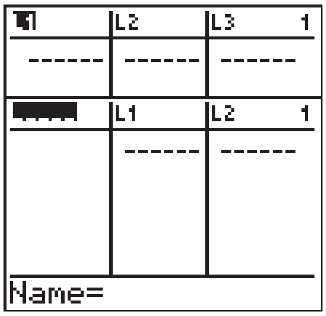


Figure 1

2. Press [2nd] [STAT] to access [LIST] and view the names of all the lists stored in the calculator. Scroll down until you see the names you used from Activity 10. In this example, the data was stored in lists called **CELSI** and **FAHRN**. When **CELSI** is highlighted, press [ENTER]. **See Figure 2**. The list name will be shown across the bottom of the List Editor window. **See Figure 3**. Press [ENTER] to fill in the blank column with this list. **See Figure 4**. Repeat this procedure to have the **FAHRN** list displayed next to the **CELSI** list. **See Figure 5.**

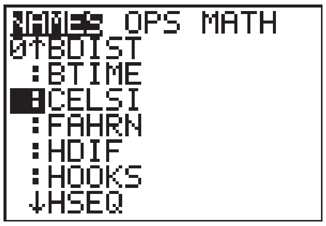


Figure 2

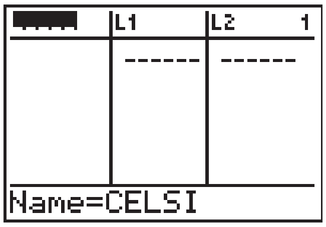


Figure 3

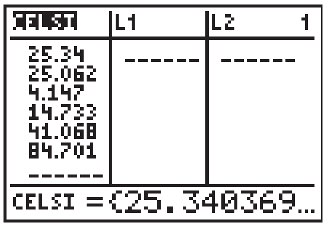


Figure 4

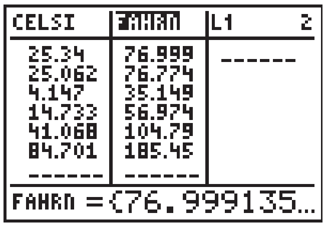


Figure 5

DATA ANALYSIS

3. Set up a scatter plot in **PLOT1** using the temperatures in degrees Celsius as the independent variable (**Xlist**) and the corresponding temperatures in degrees Fahrenheit as the dependent variable (**Ylist**). To do this, position your cursor to the right of the **Xlist:. See Figure 6.**

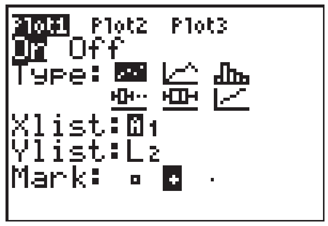


Figure 6

4. Press [2nd] [STAT] to access [LIST] and view the names of all the lists stored in the calculator. Scroll down until you see **CELSI**. When **CELSI** is highlighted, press [ENTER]. **See Figure 7.**

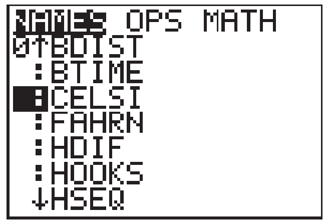


Figure 7

5. Position the cursor behind the **Ylist:** and repeat the procedure above to insert the list called **FAHRN**. Choose the cross for the mark. **See Figure 8.**

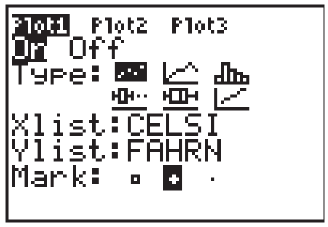


Figure 8

6. To see the graph, press [ZOOM], and then **9:ZoomStat**. **See Figure 9.**

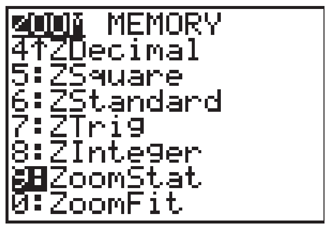


Figure 9

7. When the graph is displayed, press [TRACE]. The lists were sorted before they were stored, so the cursor should highlight the first point and allow you to scroll, left to right, as you press the right arrow key. **See Figure 10.**

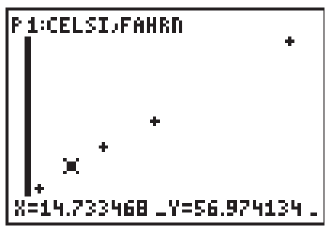


Figure 10

8. Set up a second scatter plot with the lists reversed. **See Figure 11.**

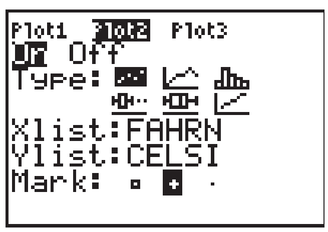


Figure 11

9. Press [ZOOM] and select **9:ZoomStat** to see both scatter plots. **See Figure 12.**

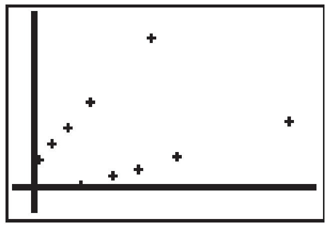


Figure 12

10. Press [TRACE] and notice the **P1** in the upper left corner of the screen. Arrow over several points. Take notice of a particular ordered pair and then press the down arrow key. **See Figure 13.**

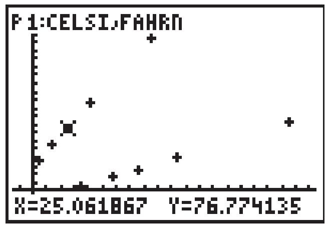


Figure 13

11. The curser has jumped from **P1** to **P2** and the ordered pair is the same number as before only it is now in reversed order. Before leaving this section, explain to students what has happened with the ordered pairs. This is the target concept. The first function takes **A** and maps it into **B** and the second function takes **B** and maps it back into **A**. Let them trace every point and find each corresponding inverse point. **See Figure 14.**

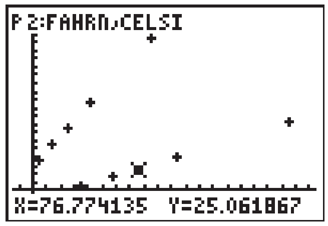


Figure 14

12. From Activity 10, we know the formula for the data in **Plot1**. Enter **9/5X+32** in **Y1** as the conversion equation for Celsius to Fahrenheit. **See Figure 15.**

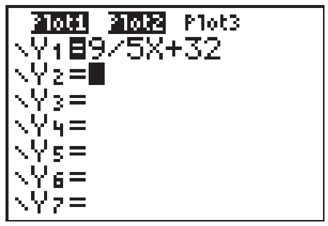


Figure 15

13. Press [GRAPH] to see both scatter plots and this line. **See Figure 16.**

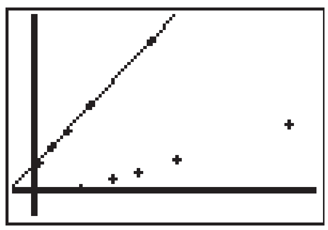


Figure 16

14. Find the other regression equation. There are several methods to find the regression equation. Your choice depends on your goal for the lesson. Because the goal of this lesson is to examine the relationship between a function and its inverse, you can choose a quick method for finding the regression equation. The example here uses the built-in linear regression feature and pastes the equation into **Y2**. To accomplish this, press [STAT] [▸] [CALC] and choose **4:LinReg(ax+b)**. **See Figure 17.**

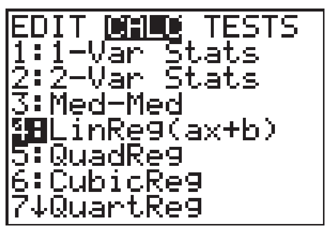


Figure 17

15. You will be taken to the home screen to enter the list names and where you want the equation pasted. Press [2nd] [STAT] to access [LIST] and view the names of all the lists. Scroll down until you see **FAHRN**. When **FAHRN** is highlighted, press [ENTER]. Press the comma key and then repeat the procedure above to enter the list **CELSI**. Press the comma key again and then press [VARS] [▸] to access **Y-VARS** and select **1:Function**. From the list displayed, select **2:Y2**. Press [ENTER] to execute the command. **See Figure 18.**

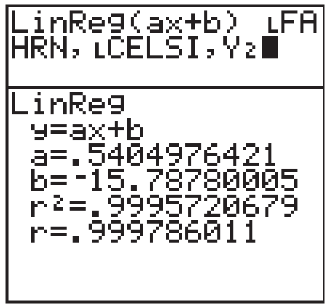


Figure 18

16. Press [Y=] to confirm the equations that will be graphed. **See Figure 19.**

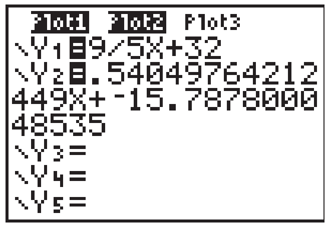


Figure 19

17. Press [GRAPH] to see both scatter plots with both lines. **See Figure 20.**

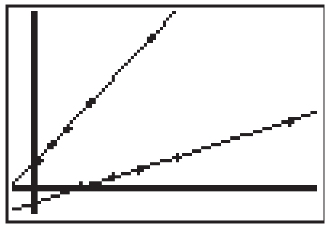


Figure 20

18. When the **points** are traced on a scatter plot, the cursor moves along the points in the order they were entered into the lists. When the points were traced earlier, it was very obvious that two points contained the same values for their coordinates but in reverse order. When **lines** are traced from the [Y=] screen the **X**-value remains the same as you move from one line to the next. Therefore, when tracing along the two lines, it is much harder to find corresponding points whose coordinates are reversed. **See Figures 21–22**. Stress this concept when tracing the individual data points from the scatter plots in the previous sections.

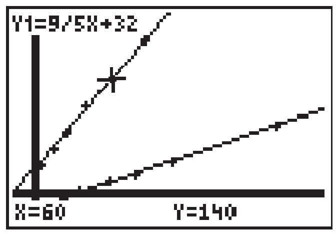


Figure 21

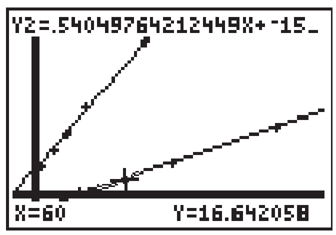


Figure 22

19. An interesting point about a function and its inverse is that if you graph them both on the same coordinate plane, they will have symmetry with respect to the line Y = X. To demonstrate symmetry, type **X** into **Y3**. **See Figure 23.**

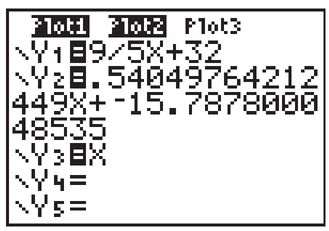


Figure 23

20. Press GRAPH. This graph may not demonstrate the concept of symmetry well. Try explaining symmetry after adjusting the window of the graphing screen. **See Figure 24.**

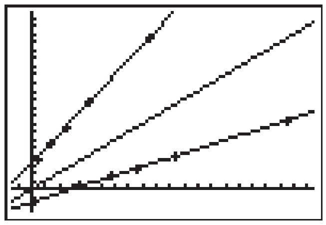


Figure 24

21. Press the WINDOW screen. The settings for the range of **X**-values is approximately 217 and approximately 243 for the **Y**-values. The rectangular shape of the window on the calculator is approximately 1.5 times as long as it is tall. With these settings, there are more **Y** units (243) in a space smaller than the space allowed for the **X** units (217). **See Figure 25**. The resulting appearance of the graph is like taking a piece of silly putty and stretching it in only one direction. To clearly see the symmetry, it is important that the scale be accurate.

```
WINDOW
 Xmin=⁻13.98326…
 Xmax=203.57935…
 Xscl=1
 Ymin=⁻26.67441…
 Ymax=216.27051…
 Yscl=1
 Xres=1
```

Figure 25

22. To find a more useful viewing window, consider the relationship between the scale on the **X**- and **Y**- axes. It will also be helpful if you can guarantee reasonably “friendly” numbers (numbers without a lot of decimal places) when you trace along the lines. The **X**-values displayed as a graph is traced are determined by the window settings. Friendly **X**-values can be found by making the range on your window 94 or some number times 94. The **Y**-values are calculated from the equations. If you want the graph drawn to scale, then make the spread of **Y**-values equal 62 when the spread on the **X**-values is 94. To make sure all the data points are in the window, the spread cannot be reduced, only expanded.

23. The current window was set up by the calculator to assure all points would be displayed. Since the **Ys** have the biggest values, start with them and then adjust the **Xs**. The current spread on the **Ys** is approximately 242. It ranges from -26 to +216. The closest multiple of 62 that would include all the data points would be 4 * 62 = 248. As a result, you would therefore need 4 * 94 = 376 for the **Xs**. **See Figure 26.**

```
4*62
                 248
4*94
                 376
```

Figure 26

24. It will be helpful to have some negative numbers in the picture to display the axes so that when you are tracing the lines, the display of the coordinates of the points along the bottom of the screen will not get in the way of viewing the points. Set your window to match the screenshot provided.
See Figure 27.

```
WINDOW
 Xmin=⁻20
 Xmax=356
 Xscl=10
 Ymin=⁻20
 Ymax=248
 Yscl=10
 Xres=1
```

Figure 27

25. Press GRAPH. This graph demonstrates symmetry much better than when the calculator chose the ranges for the window. Suggest that students work in pairs or groups so they can allow one calculator to trace **Y1** and use another calculator to trace **Y2**. **See Figure 28.**

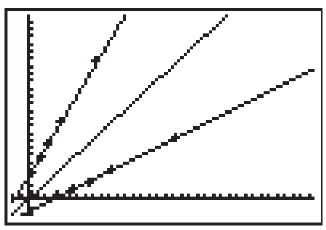


Figure 28

26. Press TRACE and press the down arrow until **Y1** is displayed in the upper left corner of the screen. Scroll to a point with friendly coordinates. In this example, the point is (60, 140). **See Figure 29.**

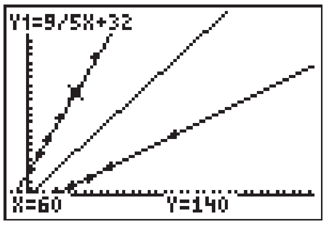


Figure 29

27. Press the down arrow key on a second calculator to move to **Y2**. At the bottom of the screen you will see that the **X**-value is 60. It is the same value as in the previous screen. Scroll to the right until you get to a point whose **X**-coordinate is the same as (or as close as possible to) the **Y**-coordinate from the previous screen. In this example, the value is 140. Examine the **Y**-value. Is it the same value as the **X**-value on the previous screen? **See Figure 30.**

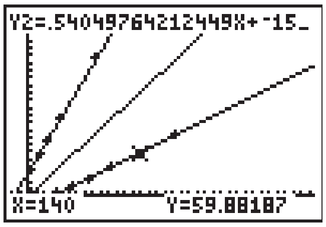


Figure 30

28. Recall that one of the regression equations was the conversion equation found in a book and the other one was specifically fit to your data. The amount of error should be small enough to accept as human error. **See Figure 31.**

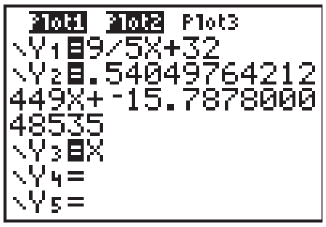


Figure 31

29. Solve the conversion equation in **Y1** algebraically for **X**.
Given **Y1 = 9/5X+32**, rewrite this as F = 9/5 C + 32. Then,
F – 32 = 9/5 C and C = 5/9 (F – 32). Position the cursor to highlight the equal sign beside **Y2** and then press ENTER. **See Figure 32**. This will turn off that equation so you won't lose the information, but it will not be displayed in the graph.

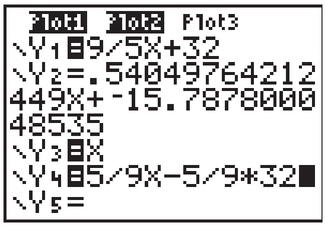


Figure 32

30. Enter **5/9X–5/9*32** into **Y4. See Figure 32**. Press GRAPH and examine how closely **Y4** matches the data from **Plot2**. **See Figure 33.**

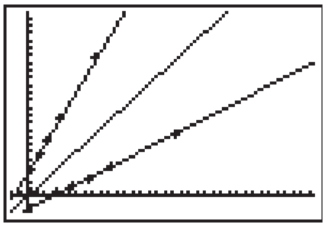


Figure 33

31. Repeat the tracing procedure in steps 26 and 27 above. Working in pairs, have students scroll to see if they can find points whose coordinates are whole numbers. Have them scroll to confirm that the point where these two coordinates are reversed, sits on the other line. Stress that each point on the first line has a corresponding point on the second line whose coordinates are the reverse of the first point. **See Figure 34.**

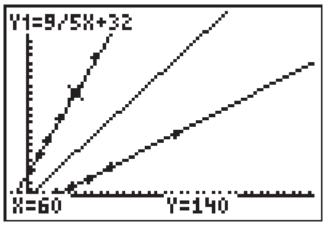


Figure 34

32. Encourage your students to visually confirm that if they folded the graph along the line Y = X, the two lines would lie on top of each other. **See Figure 35.**

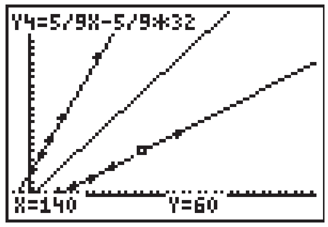


Figure 35

EXPLORATION

Consider stepping through this procedure with your students using the standard conversion equations and then have students repeat the procedure with their own conversion equations. Your decision to include this Exploration should depend on your students' background in symmetry. If they have already taken geometry, this will be a good review. This section also uses the built-in Equation Solver.

1. To prove symmetry, show that the distance from a point to the line of symmetry is the same as the distance from its corresponding point to the line of symmetry. The distance from a point to a line must be measured along a perpendicular line. The next procedure will be to find the perpendicular line and use it to find the distance from two corresponding points to that line. The plan is to prove that **HI = IJ. See Figure 36.**

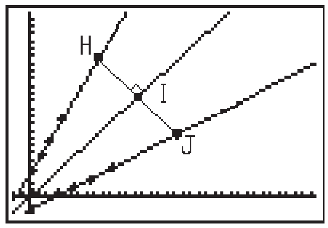


Figure 36

2. The first task will be to identify the line that contains **H** and is perpendicular to the line Y = X. The slope of Y = X is one, so the slope of the line perpendicular to Y = X is –1. Scroll along the **Plot1** to the last point. **See Figure 37**. The line must go through this point and have a slope of –1. Remind the students that if the slope-intercept form of a line is used, then the line would be y = –1x + b. The coordinates of the point you are on will lie on that line. This means that the formula becomes 185.449 = –1 * 84.700 + b for the example we are using. Use the Solve feature of the calculator to find the value of b.

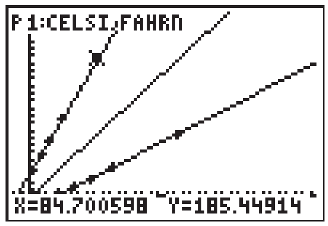


Figure 37

NOTE To use the solver, you must have a zero on one side of your equation. Transform Y = MX+B to 0 = Y–MX–B.

3. Press the MATH key and scroll down until **0:Solver** is highlighted and press ENTER. **See Figure 38.**

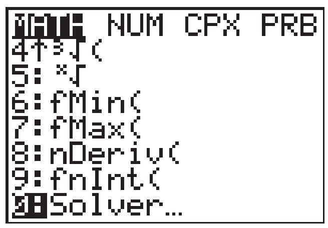


Figure 38

4. You will be taken to a screen with an equation at the top. Press ENTER then clear away any equation there and type in Y–MX–B. The **X**- and **Y**-values from the point your cursor had traced are filled in. The **M**-value comes from the last time you used **M** and most likely has no relevance for this problem. **See Figure 39.**

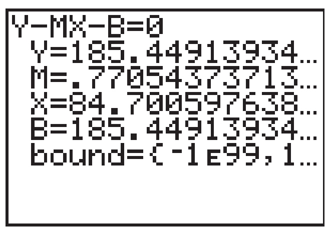


Figure 39

5. Position the cursor beside **M** and enter –1. **See Figure 40.**

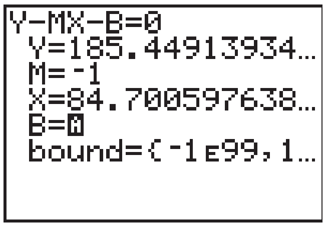


Figure 40

6. Position the cursor beside **B** and press [ALPHA] [ENTER] to access [SOLVE]. You will be given the **B** value that makes this equation true. Explain that it is the **Y**-intercept for the line perpendicular to Y = X. **See Figure 41.**

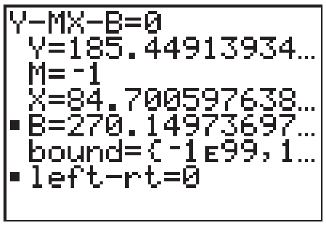


Figure 41

7. Go to the [Y=] window. Beside **Y5**, enter -1X+ the **B**-value rounded to the nearest hundredth. The equation in this example is **-1X+270.15**. **See Figure 42.**

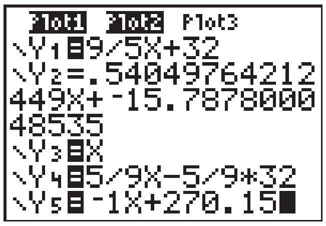


Figure 42

8. Press [GRAPH] to see the perpendicular line drawn. **See Figure 43.**

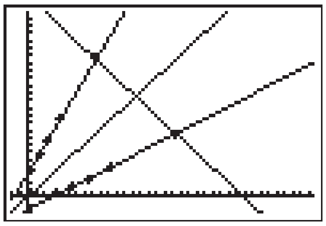


Figure 43

9. Remember that the plan is to prove that **HI = JI**. Find the distance from **H** to **I** and the distance from **I** to **J** to make sure that the distances are equal. Also, **J** will be the point whose coordinates are the reverse of the coordinates for **H. See Figure 44.**

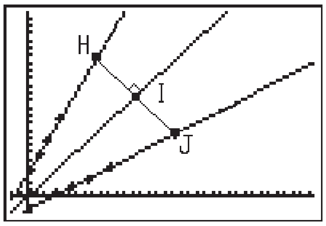


Figure 44

10. First, identify the points **H**, **I**, and **J**. Use a built-in feature of the calculator to accomplish this. Press [2nd] [TRACE] to access the [CALC] menu; arrow down to highlight **5:intersect** and press [ENTER]. **See Figure 45.**

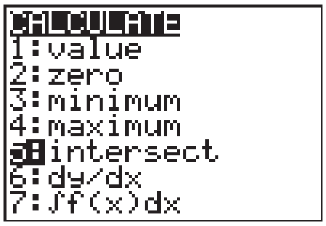


Figure 45

11. You will be taken to the graph screen and asked to identify the first equation. Point **H** is the intersection of **Y1** and **Y5**. **Y1** is already shown in the upper left corner of the screen. Press [ENTER] to select it. **See Figure 46.**

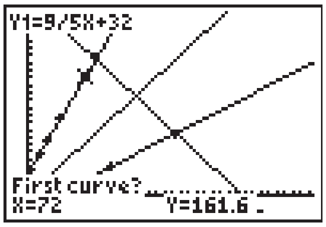


Figure 46

12. The cursor will jump to the second line that is turned on in the [Y=] window. Press the down arrow key until you see **Y5** in the upper left corner. The cursor is positioned on **Y5** and the question changes to ask for the second curve. Press [ENTER] to select it. **See Figure 47.**

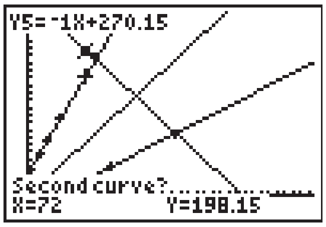


Figure 47

13. The question on the screen asks for a guess. Scroll close to the point of intersection and press [ENTER]. **See Figure 48.**

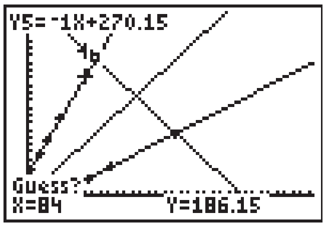


Figure 48

14. The point of intersection will be displayed at the bottom of the screen. This is point **H** in the sketch from the previous screens. **See Figure 49.**

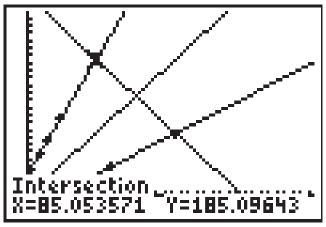


Figure 49

15. Press [2nd] [MODE] to [QUIT] and return to the home screen and to store the **X**- and **Y**-coordinates of the point of intersection into the variables **A** and **B**. To store the **X**-coordinate in **A**, press [X,T,Θ,*n*] [STO▸] [ALPHA] **A** [ENTER]. For **Y**, press [ALPHA] **Y** [STO▸] [ALPHA] **B** [ENTER]. **See Figure 50.**

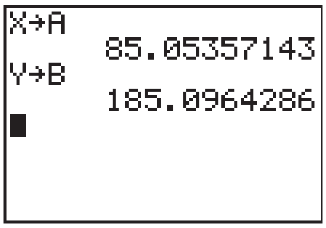


Figure 50

16. Repeat this procedure to find the other points of intersection. **I** is the intersection of **Y3** and **Y5**. Store these values in **C** and **D**. **See Figures 51–52.**

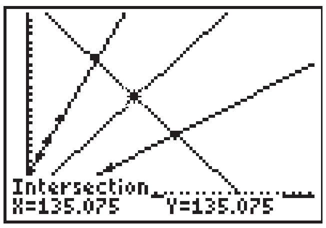


Figure 51

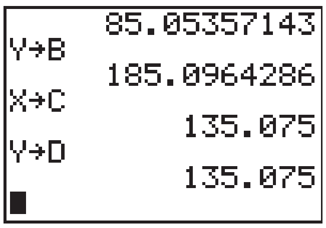


Figure 52

17. Point **J** is the intersection of **Y4** and **Y5**. Store those values in **E** and **F**. Compare **E** and **F** to **A** and **B**. The coordinates of **H** and **J** are the reverse of each other. **See Figures 53a–b.**

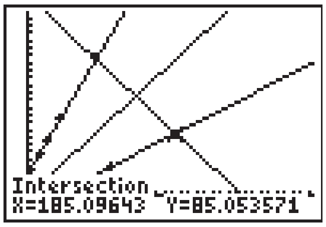


Figure 53a

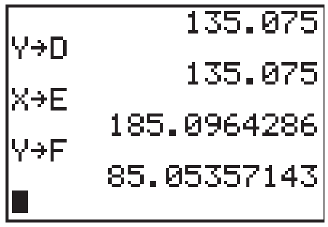


Figure 53b

18. Next, use those stored values to find the distance from **H** to **I**. Write the distance formula as shown in the screenshot on the right. Be careful to put in the extra set of parenthesis surrounding the expression under the radical. **See Figure 54.** After finding **HI** press [2nd] [ENTER] to re-write the equation used.

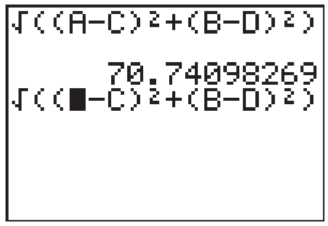


Figure 54

19. Position the cursor on the **A (See Figure 54)** and replace it with **E**. Repeat to replace **B** with **F**. Press [ENTER] to find **IJ**. These equivalent values prove that **HI = IJ. See Figure 55.**

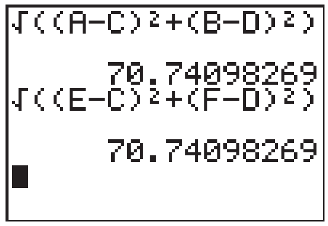


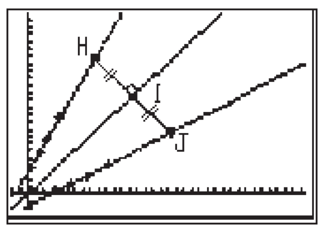
Figure 55

WORKSHEET ANSWERS

1. Y = 9/5 X + 32
2. Something close to 5/9 X – 5/9 * 32 = .5555 X – 17.7777.
3. Answers will vary depending on the regression equation used. Answers should be close to these.

ºF	ºC
-10	-23.33
0	-17.78
32	0
65	18.333
110	43.333
212	100

4. Look for understanding of the concept that any ordered pair (X, Y) on the original equation will have a correspnding (Y, X) on the inverse.
5. Y = X
6. Negative reciprocals of each other. (Their product equals negative one.)
7. The procedure described should include the drawing of the line perpendicular to the line Y = X and intersecting the regression equations. The student should identify the points of intersection and find the distances from both regression equations to the line of symmetry.
8. Answers will vary, but should be equal to each other.
9. Answers will vary. Human error, calibration error. . . . and/or rounding error.
10. See graph on right.



ACTIVITY 11

Name: ____________________

Inverse of Two Temps

Math Objectives:

- Graph scatter plots
- Analyze and graph linear equations
- Compute and model slope
- Derive and apply a conversion equation
- Analyze inverse relations

Materials:

- TI-83/TI-84 Plus Family
- Data from Activity 10

OVERVIEW

In Activity 10, you found a conversion equation that calculated the Fahrenheit temperature for any Celsius temperature. In this activity, you will find a conversion equation that will calculate the corresponding Celsius temperature for any given Fahrenheit temperature. You will then explore the relationship between these two equations, both numerically and graphically. Your teacher will outline the procedure for you.

1. From Activity 10, recreate the scatter plot with Celsius in the **Xlist** and Fahrenheit in the **Ylist**. Recall the conversion equation and make sure it is in **Y1**. Write it here. ____________________

2. Create a second scatter plot with Fahrenheit in the **Xlist** and Celsius in the **Ylist**. Find the regression equation for that data and put it in **Y2**. Write it here. ____________________

3. Use your new conversion equation and the **Table** feature of the calculator to fill in the table for the various temperatures.

4. Use your own words to explain how to tell if two equations are inverses of each other. ____________________

____

____

ºF	ºC
-10	
0	
32	
65	
110	
212	

5. If you graph a function and its inverse on the same coordinate axes, they will have symmetry with respect to what line? ____________________

6. What is the relationship between the slopes of two lines that are perpendicular to each other? ____________________

____

7. To tell if the two lines in this exercise have symmetry with respect to a line, you have to show the distances from the corresponding points to the line of symmetry are equal to each other. Describe the procedure for this. ____________________

____

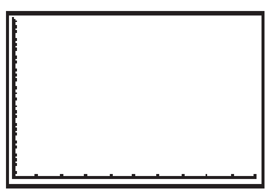
____

8. What were these distances in your activity? ________ and ________

9. Were these distances the same for your regression equations? ________ If not, what would be a reasonable explanation why they are not the same? ____________________

____

10. Draw a sketch of the two scatter plots. Draw their regression equations and the line of symmetry. Mark the two distances that have to be equal in order to prove symmetry.



NOTES