

# Volume of a Watermelon

## Teacher Notes

7 8 9 10 11 12



TI-84PlusCE™



Activity



Student



50 min

### Teacher Notes:

#### Equipment Requirements:

- Watermelon – Try to purchase a melon that is approximately elliptical, some watermelons are more elliptical than others.
- Camera, computer and USB cable – Take photographs of the watermelon along both the major and minor axis. A sample image is included with these activity documents, however, in order to complete the task with students, a fresh watermelon is required. The image needs to be transferred to student calculators, this can be done by the teacher or students.
- Large bucket, water and measuring jug – To measure the volume of the watermelon, students place the watermelon into a bucket full of water, remove the watermelon and refill the bucket using the measuring jug in order to ascertain the volume (litres) of water displaced.  
**Note:** Watermelons tend to float, this can be part of the challenge! Alternatively, students can place a line around the centre of the melon and immerse only half of the melon. This can help reduce the size of the bucket required and the impact of trying to immerse and retrieve the watermelon from the bucket.
- Knife and chopping board – The watermelon will be cut into slices for students to use. It is difficult to keep the slices uniform; however, this can form part of the discussion with students.
- Ruler – The major and minor axis of the watermelon needs to be measured for scaling in the image. Students also need to measure the dimensions of their slice of watermelon to help form a volume estimate.
- Plates, serviettes & hand sanitiser – This lesson is going to be messy!



## Introduction

This activity looks at three different ways to measure the volume of a watermelon.

- Archimedes' principle (measurement by displacement of water)
- Approximation – Cutting the watermelon into slices that can be approximated by cylinders
- Calculus – Determining an equation to model the shape of the watermelon and rotating it around an axis.

### Part 1 - Archimedes

For this measurement you will need:

- Watermelon
- Bucket – big enough to submerge your watermelon\*.
- Measuring jug
- Water

\*If you don't have a bucket big enough, try one that will hold half your watermelon.

Archimedes' principle states:

A body at rest in a fluid is acted upon by a force pushing upward called the buoyant force, which is equal to the weight of the fluid that the body displaces. **If the body is completely submerged, the volume of fluid displaced is equal to the volume of the body.**

Completely fill the bucket with water. Place the watermelon in the bucket, you will most likely need to nudge the watermelon to ensure it is completely submerged, avoid displacing too much water with your hand. Once the water has settled, remove the watermelon from the bucket, again, try and avoid displacing any additional water. Once the watermelon is removed from the bucket, use your measuring jug to refill the bucket and therefore determine the volume of water that was displaced by the watermelon.

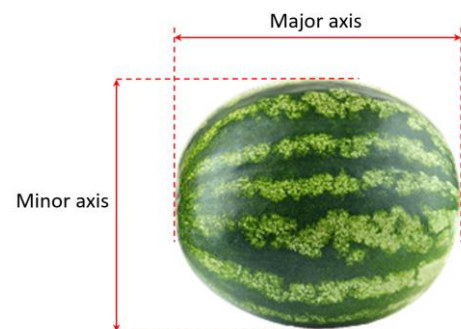
### Question: 1.

According to your measurements and Archimedes' principle, what is the volume of your watermelon.

Note: 1 litre = 1000cm<sup>3</sup>

**Answer:** Answers will vary.

**Teacher Notes:** This is messy but lots of fun. This component of the activity is best done as a whole class. Forks pushed into the watermelon make it easier to push into the water and retrieve without significant impact to the volume measured.



## Part 2 – Preparing your watermelon

For this section you will need:

- Watermelon (same)
- Camera / phone
- Computer + TI-Connect™ CE software + USB cable
- Ruler / Tape measure

It is important that the image be an accurate scaled version of the actual watermelon. Measure and record the major and minor axis lengths carefully.

### Step 1:

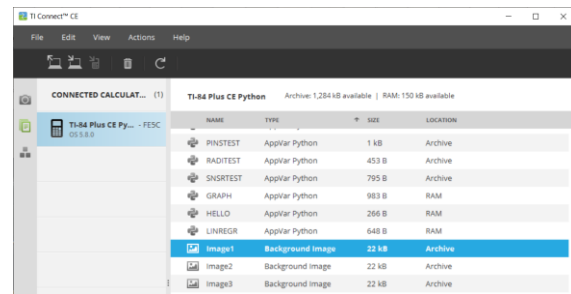
Photograph your watermelon. Make sure the image is 'square' to the melon. Try and keep the background as plain as possible to make it easier to see the outline once the image has been sent to your calculator.

### Step 2:

The calculator screen has a 4:3 aspect ratio, the background image however is mapped to 83:133. To ensure the watermelon doesn't get distorted, use imaging software to crop your watermelon image to the 83:133 ratio. Try and crop as close to the edge (major axis) of your watermelon image. The actual watermelon's dimensions are unlikely to be in the same ratio as your calculator screen, this will most likely leave space at the top and bottom of the image. Don't worry too much about the resolution of your image, the TI software will reduce the image resolution to suit the calculator.

**Step 3:**

Launch the TI-ConnectCE software and connect your calculator to the computer. Select your handheld in the software and the contents icon immediately below the camera icon. A complete list of the content on your calculator is displayed, including any images currently stored on your calculator.



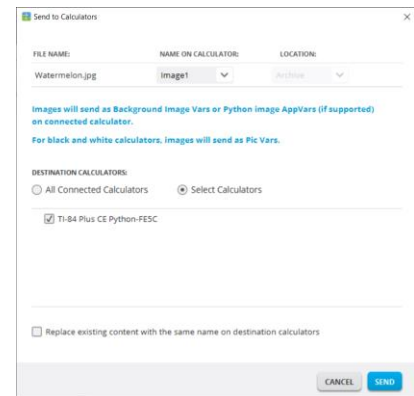
Locate the picture on your computer and drag it into the contents list. You will be prompted to select the image number, then select SEND.

**Step 4:**

The watermelon image can now be set up as a background for the graph screen. On the calculator press:  $\boxed{2\text{nd}}\boxed{\text{format}}$ . Navigate down to **Background** and select your image.

Once your image has been selected, press the  $\boxed{\text{graph}}$  key to the axis and image.

**Note:** The axis scale **must** be set up to accurately reflect the physical dimensions of the watermelon. The window settings must also remain 'square', that is to say the scale in both the x and y direction must be the same.

**Step 5:**

Measure and record the length (major axis) and height (minor axis) of your watermelon.

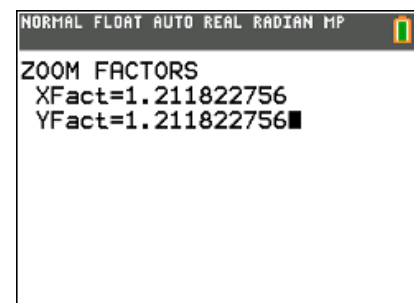
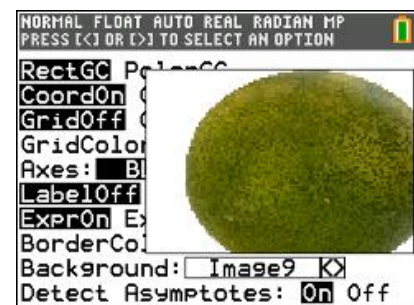
On the calculator, press:  $\boxed{\text{zoom}}\boxed{6}$  (standard) then  $\boxed{\text{zoom}}\boxed{5}$ , this will ensure the cartesian plane has the same scale in the x and y direction and that the origin is in the centre of the screen.

Whilst on the graph screen, press the  $\boxed{\text{trace}}$  key and navigate to the left most point of the watermelon image, record the x co-ordinate. In the sample shown here the x co-ordinate is: -12.195

Now navigate to the right most point of the watermelon image, once again record the x co-ordinate.

Divide the measurement for the major axis (cm) by the difference in the two x co-ordinates, the result is the scale factor for your window settings.

The window settings can be changed directly, alternatively the zoom factors can be set to make the adjustments easier. Press  $\boxed{\text{zoom}}$  and navigate across and select: **memory**, select option  $\boxed{4}$  Set Factors. Make sure the XFact and YFact are the same, this ensures zooming in or out will remain 'square'. When the zoom-in or zoom-out options are used the calculator will automatically adjust the x and y scales accordingly.



Return to the graph screen, place the cursor at the origin, then press: **zoom** [3] zoom-out and press **enter**.

To confirm that the scale has been set correctly, use the Trace option again to locate the x coordinates of the extremities of the watermelon. The difference between these extreme values should be equal to the length of the major axis. Do the same for the extremities of the y coordinates for the watermelon, the difference between these values should be equal to the measurement for the minor axis of the watermelon.



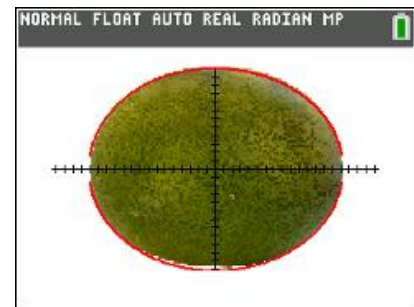
**Question: 2.**

The general equation for an ellipse is given by:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Transpose the equation to make Y the subject. Determine an equation to model the edge of the watermelon storing the corresponding values in 'a' and 'b'.

**Note:** As the equation is a relation, use  $Y_1$  to model the top of the watermelon and  $Y_2 = -Y_1$  to model the bottom.



You may also need to translate your function, alternatively, move the origin to the 'centre' of the watermelon by adding/subtracting values to the Xmin, Xmax, Ymin, Ymax values.

**Answer:** If the entire class is using the same watermelon all students should arrive at the same equation. Storing the values in memory locations A and B make the equation more compact and will help with calculations later in the activity.

**Question: 3.**

Your teacher will cut the watermelon into slices such that each slice resembles 'short' cylinder, a cross-section of the watermelon. Use a ruler to measure the dimensions of your cylinder (watermelon slice) and determine the corresponding volume.



**Answer:** Answers will vary.

**Teacher Note:** Students may ask which 'diameter' to measure, particularly on slices that are close to the end of the watermelon. This is all part of the learning, helping students realise that 'thinner' slices would improve the 'estimate'. It is not important to get all the slices the same thickness, it is more important to try can keep the thickness of each uniform, which is difficult in itself! Again, this is a practical measurement, if each slice varies in thickness, students can 'average' the thickness of their slice.

**Question: 4.**

Collect the volume of watermelon each slice from each of your classmates and hence determine an approximate volume for the original watermelon.

**Answer:** All students should obtain the same answer if only one watermelon is being used.

**Teacher Note:** At this point it should also be easy to see if a student has calculated the volume incorrectly.

**Question: 5.**

Determine the location of **your** slice of watermelon on the graph. Explain how the radius of your slice of watermelon relates to the function.

**Answer:** Students solve  $Y_1 = R$  (radius measurement). The purpose is for students to identify that Y represents the radius, therefore the expression  $\pi y^2 \Delta x$  will generate the volume of each slice.

**Question: 6.**

Explain what this expression is doing:

$$\pi \sum_{-a}^a y^2,$$

where 'a' represents the nearest integer value for the x-axis intercepts of the function modelling the watermelon.

Try this on your calculator (see opposite)

	NORMAL	FLOAT	AUTO	REAL	RADIAN	MP
	00/27/2007/001					
	1.211822756					
Y→B	12.11822756					
X→A	15.22167608					
	$\pi \sum_{x=\text{int}(A)}^{\text{int}(B)} (Y_1^2)$					
	9363.727144					

**Answer:** The formula is calculating the volume of the watermelon by adding up the volume of each slice (cylinder), assuming that each slice is 1 unit in thickness.

**Teacher Notes:** It is important that students include reference to the fact that it is 'assumed' that each slice has a thickness of 1 unit. The summation command on the calculator increments in 1 unit amounts!

**Question: 7.**

Explain how this expression (below) relates to the expression in the previous question.

$$\pi \int_{-a}^a y^2 dx$$

**Answer:** The summation assumes a thickness of 1 unit (cylinder height) and provides an approximation for the volume. Integral calculus uses cylinder heights that are 'infinitely' small, providing a much better *approximation* for the actual volume. It is still an approximation as the function itself is modelling the shape of the watermelon.

	NORMAL	FLOAT	AUTO	REAL	RADIAN	MP
	00/27/2007/001					
	12.11822756					
Y→B	15.22167608					
X→A	$\pi \sum_{x=\text{int}(A)}^{\text{int}(B)} (Y_1^2)$					
	9363.727144					
	$\pi \int_{-A}^A (Y_1^2) dx$					

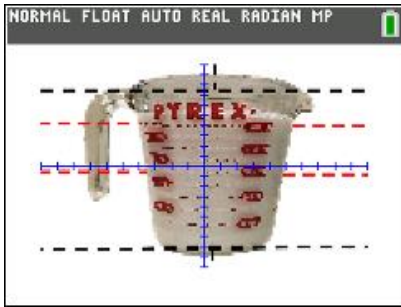
**Application**

A measuring jug (shown) contains measurements in 100ml increments, up to 500ml.

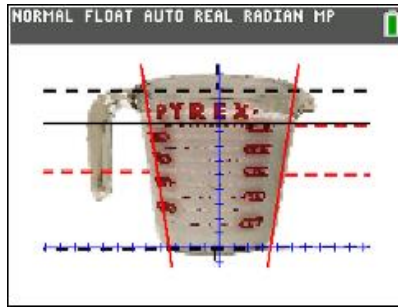
- Insert a copy of the image into your calculator.
- Scale your calculator window settings accordingly.
- Model the side(s) of the measuring jug using a linear function.
- Rotate the function around the y – axis to form a solid of revolution, and check the approximated volume for the 500ml marking.
- Determine the distance between the 100ml – 200ml – 300ml – 400ml markings.



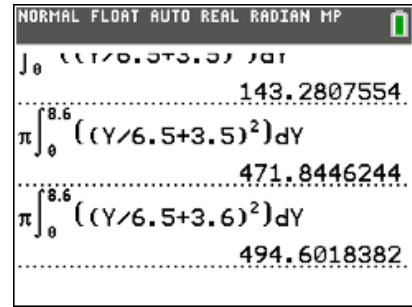
Answers:



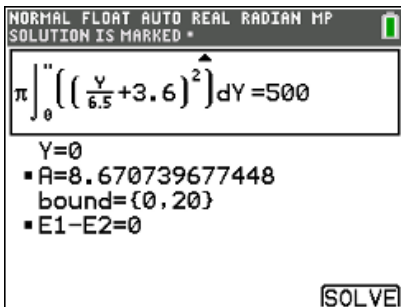
Jug images inserted with 'standard' window settings.



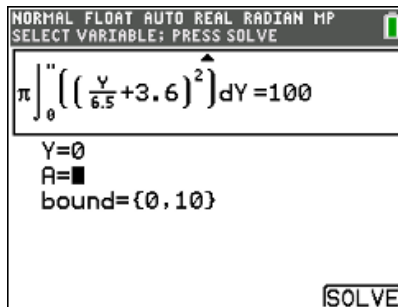
Linear functions modeling the centred and scaled jug image.



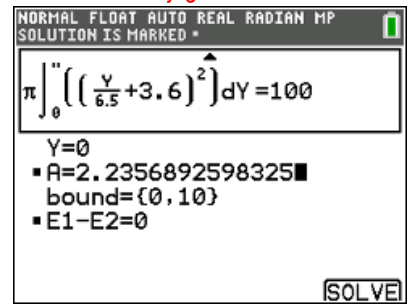
Volume calculation for 500ml. Notice the sensitivity with regards to a small change in the function(s) modelling the sides of the jug.



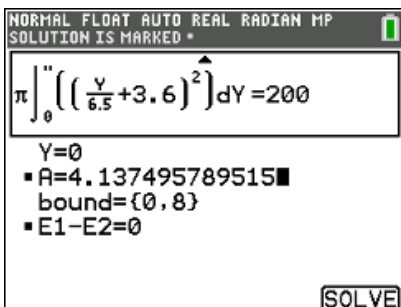
Solve can be used to check the 500ml calculation.



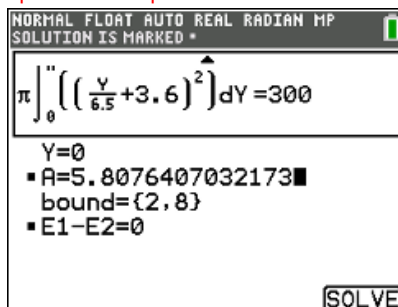
Change the boundary condition over which the integral is computed to speed up the answer process.



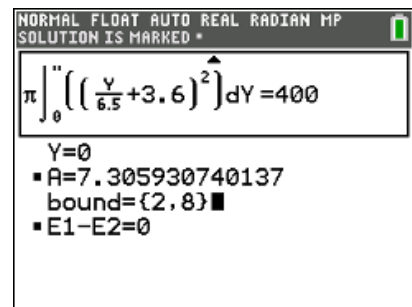
The 100ml marking should be approximately 2.236cms, based on the integration technique.



The volume has changed but the height does not, because as the measuring jug gets wider towards the top.



The first 100ml went up by approximately 2.3cm, the second 100ml went up by 1.90cm and the third by 1.50cm.



Based on these measurements and changes, the next increment should be approximately 1.1, resulting in a height of 8.5, close to the computed value of 8.6.