DO NOT OPEN the *Power.tns* file until you teacher indicates that you should do so.

Alpha determines the probability of rejecting the null hypothesis when it is true. What do we know about the probability of rejecting the null hypothesis when it is false? This lesson uses the concept of power to explore that question.

Consider the following hypotheses:
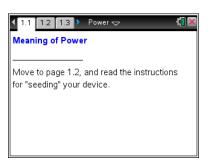
$H_0: \mu = 0$

$H_a: \mu > 0$

$\alpha = 0.1$

1. The graph to the right shows a normal curve with $\mu = 0$ and $\sigma = 1$ representing a sampling distribution of sample means taken from a population for which the null hypothesis stated above is true.

   a. What do the values of $\bar{x}$ that fall into the shaded region represent?

   b. Based on your response in part a and the graph shown, how often would you reject the null hypothesis if the null is in fact true?
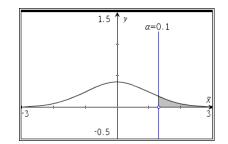
Open the TI-Nspire document *Power.tns.*
Move to page 1.2.

This activity involves generating a number of random samples from a population. In order to avoid having your results be identical to another student in the room, it is necessary to "seed" the random number generator. Read the instructions on page 1.2 for seeding your random number generator.

Move to page 2.1.

This page continues your thinking about the hypothesis test posed in Question 1. Each time you press the top (▲) arrow on page 2.1, you generate a random sample of size 10. The bottom graph shows the distribution of sample means if the null hypothesis is true. The top graph is the population corresponding to a true null hypothesis. The elements of each sample are displayed as points on the x-axis of the top graph. The vertical line represents the mean of the sample. An α of 0.1 is shaded.

2. a. What does α represent?

    b. If you drew a sample from this population, where would you expect the sample mean to fall in the bottom graph?

    c. Predict the proportion of samples that will have means that fall in the shaded region.

    d. Use the arrow (▲) on Page 2.1 to draw 25 samples of size 10. Record what you notice about the pattern of sample means in relation to the shaded region on the bottom graph.

    e. What do your observations in part d tell you about the predictions you made in parts b and c?

Move to page 3.1.

The top screen on Page 3.1 represents two populations. The dotted curve centered at 0 represents the null hypothesis. The up arrow (▲) will draw a sample of size 10 from the population you are studying. The bottom screen will accumulate the sample means from those draws. Use the top arrow to generate at least 25 different samples.

3. a. Which population seems to be the one from which you are sampling?

    b. What does your response to part a suggest about the null hypothesis?

Move to page 4.1.

If the top graph does not have a shaded region, select the top screen so a shaded region appears.

4. In the top graph, α is 0.1.

    a. What do $\bar{x}$ values in the shaded region indicate about the hypothesis test being carried out?

    b. α is the area of the "rejection region" for the null hypothesis when it is true. Predict the proportion of the sample means you think will fall in the rejection region.

    c. Draw one sample by pressing the up arrow (▲) on the top screen. Based on the displayed mean of your sample, what decision (reject or fail to reject) would you make regarding the null hypothesis?

d. Draw 25 samples, one at a time. For each sample, tally the decision you make to reject or fail to reject the null hypothesis.

e. Are your results from parts c and d consistent with your prediction in part b? Explain what might be happening.

5. The bottom curve on Page 4.1 represents the sampling distribution of sample means from a population whose mean is 1. The samples that were drawn in Question 4 actually <u>do</u> come from this population.

a. Given that fact, what decision should you make about the null hypothesis? Explain your reasoning.

b. Use the up arrow (▲) to generate 25 more samples, for a total of 50 samples. What proportion of those samples lead to a rejection of the null hypothesis?

c. The probability of rejecting the null hypothesis is known as **power**. Thus, *power* is the proportion of the set of all samples' means that fall in the rejection region. Your answer in part b approximates power. To improve that approximation for this situation, draw 50 more samples, making a total of 100. What proportion of all 100 of your sample means lead to the conclusion to reject the null hypothesis?

6. You studied type I errors earlier. A type II error is failing to reject the null hypothesis when it is false. The probability of committing a type II error is noted as beta, β.

a. What is a type I error?

b. For the 100 samples you generated in Question 5, fill out the chart.

Decision	Probability	Type of error
Reject null if it had been true		
Reject null when false		
Failing to reject null if it had been true		
Failing to reject null when false		

c. What factors do you think affect power?

Move to page 5.1.

If the top graph does not have a shaded region, select the top screen so a shaded region appears.

7. On page 5.1, α is 0.1, power is 0.830078, $n = 5$, and the true population mean is $\mu = 1.0$.

    a. Explain what each of these numbers represents.

    b. Explain the meaning of the $\bar{x}$ values in the shaded region in each of the top and bottom graphs.

    c. What do the areas in the shaded region in each of the top and bottom graphs represent and how are they connected to power?

8. The up arrow (▲) labeled **n** on this page will change the sample size.

    a. Increase the sample size, and observe the reported value for power. How does power change? Explain why this seems reasonable.

    b. Select the up arrow (▲) labeled α. How does power change? Explain why this seems reasonable.

    c. In the lower half of the screen, use the slider labeled μ_a to change the value of the true population mean. How does power change? Explain why this seems reasonable.

    d. Revisit your conjecture in Question 6 part c about the factors you think affect power. Has your answer changed? If so, how?

9. a. Describe the possible range of values for power and how you know. Explain why the size of power is important.

    b. The probability of committing a type II error (failing to reject the null hypothesis when it is false) is denoted as β. How is β related to power?

    c. How do changes in sample size, alpha, and true population mean affect β? Explain why your answers are reasonable.

10. Write an explanation of the power of a test and why power is important to a test.