Properties of Parabolas

ID: 8854

Time required
60 minutes

Activity Overview

This investigation offers an approach to show students the basic definition of a parabola as the locus of all points equidistant from a fixed point (focus) and a fixed line (directrix). Students will also interpret the equation for a parabola in vertex form and gain a visual understanding of a parabola's focal width.

Topic: Quadratic Functions and Equations

- *Observe the changes in the equation of a quadratic function under a translation and/or a stretch.*
- *Approximate the real zeros, vertex and extrema of a quadratic function graphically.*

Teacher Preparation and Notes

- *This activity is designed for use in an Algebra 2 classroom as an introduction to the parabola as a conic section. It may also be used in Precalculus as a review of properties of parabolas.*
- *Students should be familiar with quadratics as transformations and should have some experience using tools on their calculators. Encourage students to ask for help if they are unsure how to complete a certain task.*
- *This activity is intended to be mainly **teacher-led**, with breaks for students to work in pairs. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their graphing calculators.*
- *To exit the **PROPPARA** program, instruct students to press the ON button, then select **1:Quit**.*
- ***To download the student worksheet and calculator program, go to education.ti.com/exchange and enter "8854" in the keyword search box.***

Associated Materials

- *PropertiesOfParabolas_Student.doc*
- *PROPPARA.8xp (program)*
- *PARA.8xv (Cabri Jr file)*

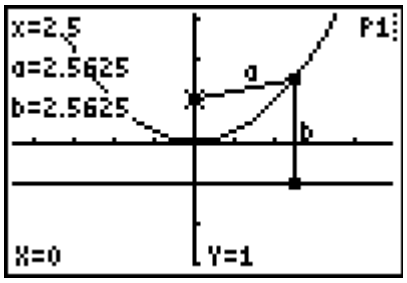
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Graphing Quadratic Functions (TI-84 Plus family) — 9406*
- *Motorcycle Jump (TI-84 Plus family) — 8893*
- *Quadratic Regression with Transformation Graphing (TI-84 Plus family) — 8206*

Problem 1 – Focus and directrix

In the first problem, students will use a graph to better understand the definitions of focus and directrix.

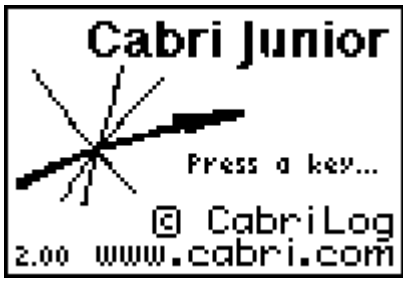


When students select a point on the parabola, a line segment, *a*, is drawn from this point to the point on the *y*-axis, and a second line segment, *b*, from the point to the horizontal line. The lengths of *a* and *b* are also displayed in the top left corner of the screen.

Students should make the conjecture that *a* is equal to *b* all along the parabola. They should also give a definition of focus as the fixed point and the directrix as the line. The distance from the vertex to the focus is one unit.

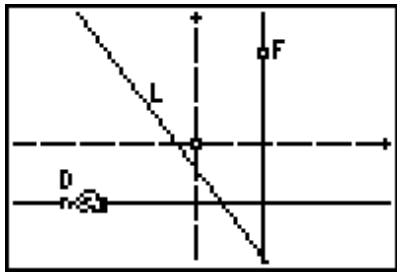
Problem 2 – Creating a parabola

In this problem, students will use a given focus and directrix to create a parabola with the *Cabri Jr.* app.

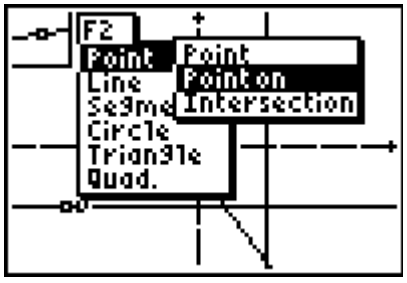


The following steps provide extra instruction not included on the worksheet.

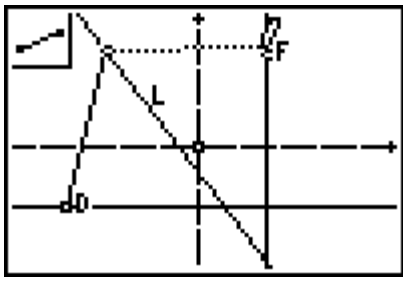
Step 1: Move your cursor to point *D*. You will know that you have reached it when the cursor turns white. Press [ALPHA] to "grab" point *D*. Use the left and right arrow keys to drag it along the horizontal line.



Step 2: Place a point on line *L*. Press [CLEAR] to let go of point *D*. Then press [WINDOW] to open the **F2:Creation** menu, and select **Point > Point On**. Move your cursor to line *L*, and press [ENTER] to place a point.

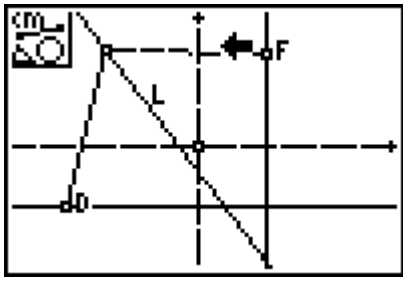


Step 3: Draw a line segment from *F* to your point and then again from your point to *D*. To draw a line segment, press WINDOW to open the **F2:Creation** menu and choose **Segment**. Move your cursor to the first endpoint and press ENTER. Then move your cursor to the second endpoint and press ENTER again.



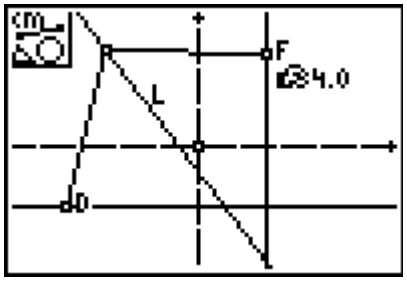
Step 4: Next, you will measure each segment. Press GRAPH to open the **F5:Appearance** menu and choose **Measure > Distance & Length**.

Move the cursor to one of the line segments. You will know that you have reached it when the cursor turns into a thick black arrow.



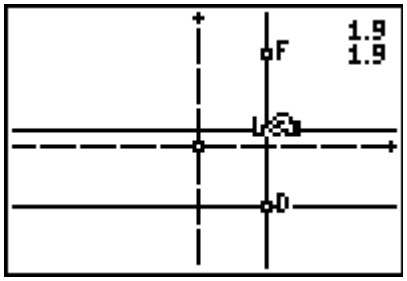
Press ENTER to choose the segment. Use the arrow keys to adjust the position of the measurement, and then press ENTER again to place it. Repeat with the other line segment.

Grab and drag point *D* again.



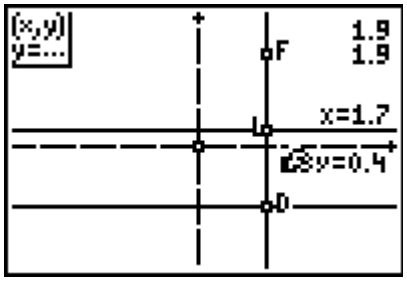
Step 5: Next, find the distance from the vertex to the focus of the parabola you are creating. Grab point *D*, and move it so that it is on the line that goes through point *F*.

Then grab the point that you drew on line *L* and move it so that it is also on the line that goes through point *F*.



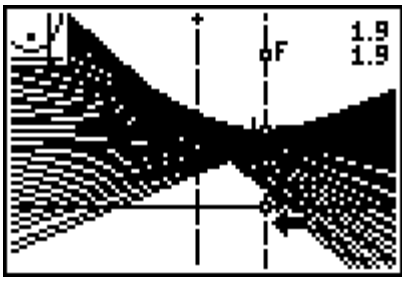
To get the *x*-value of the vertex, use the coordinates and equations tool (**F5:Appearance > Coord. & Eq.**) to display the equation of the vertical line that goes through *F*.

To get the *y*-value, use the coordinates and equations tool to display the equation of the horizontal line *L*.

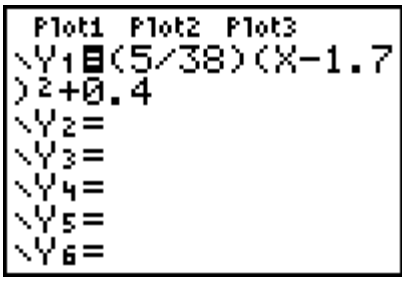


Then find the coordinates of point *F*.

Step 6: Use the **Locus** tool to create a parabola. Go to **F3:Construction > Locus**. Click on the line *L*. Move the cursor to the point *D*. The shape of the cursor turns into a double arrow. Press [ENTER].



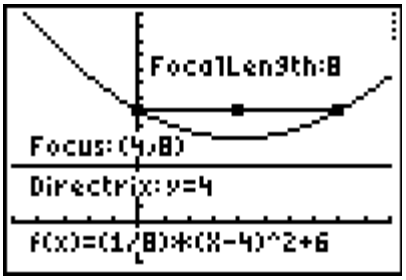
Step 7: To check your equation, press [2nd] [QUIT] to exit the **CabriJr** app. Then press [Y=]. Clear out any equations and turn off any stat plots. Enter in **Y1** the equation you found and graph it.



Students should see that line *L* follows the shape of a parabola, and that the distances from the point they create to the focus and the directrix are equal, even when they move point *D* along the directrix. The vertex of the parabola is (1.7, 0.4). The coordinates of the focus are (1.7, 2.3). Students should see the shape of a parabola on their handheld screen. They will find that the further away the focus is from the vertex, the "wider" the parabola.

Extension: Focal Width

In the extension, students will choose the second option **FocalWidth** of the program **PROPPARA**. They will see a parabola graphed with an *a*-value of $\frac{1}{8}$, along with its focus and directrix. There is also a segment through the focus parallel to the *x*-axis. The length of this segment is given.



When students press [ENTER], they will be asked to enter a value for *a*. After investigating the graph for different values of a, students should realize that the length of the focal width is 8 and the *a*-value is $\frac{1}{8}$; these values are reciprocals. The distance from the vertex to the focus is 2. The focal width is 4 times the distance from the vertex to the focus. This is why you multiply by 4 and take the reciprocal to determine the *a*-value.

Student worksheet – possible responses

1. the lengths of *a* and *b* are equal
2. the fixed point
3. the line
4. 1 unit
5. a parabola
6. the measurements are equal
7. 1.9
8. (1.7, 0.4)
9. (1.7, 2.3)
10. many lines that make up a parabola
11. the parabola gets narrower
12. the parabola gets wider
13. $y = \frac{5}{38}(x - 1.7)^2 + 0.4$