



Problem 1 – Finding combinations

Four babies born during the same night in the same hospital were labeled with four identification bracelets. Somehow, the bracelets were mixed up, and only two are correct. How many different ways can this happen?

Create a chart with four columns labeled A, B, C, and, D. Let each letter represent a baby. Use the chart to answer the following questions.

1. How many different ways can you label 4 babies so that 2 are correct and 2 are incorrect?
2. How many different ways can you label 5 babies so that 3 are correct and 2 are incorrect? (Use a chart with 5 columns.)
3. What if there were 3 babies and 2 were labeled correctly? How many different ways can this happen?
4. Is it necessary to make a chart to answer this question? Why or why not?

Run the program **BABYLIST**. View the **BABY**, **CORR**, and **WAYS** lists generated by the program by entering these titles in new lists. The data in the lists is reproduced in the table below. Use logic and/or make charts to complete the table.

Total Babies	4	5	3	4	5	3	4	5	3	4
Babies Labeled Correctly	2	3	2	3	4	1	1	1	3	4
Different Ways	6	10								

To complete each row of a chart that lists all the different ways to label the babies, you must choose some babies to label correctly and label the rest incorrectly. The different sets that can be formed by choosing objects from a group are called **combinations**.

The problem of choosing which babies to label correctly is a combinations problem, as is any situation where we must choose members from a group and the order in which we choose does not matter.

The number of combinations of r objects, chosen from a group of n objects is written as ${}_n C_r$, and calculated it with the following formula:

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

Use the combinations formula and the **nCr** command to answer the following question.

5. A group of 4 students chooses 2 members to represent the group in a presentation. How many ways can the group choose?

Create a new list titled **COMB**. In the list **COMB**, calculate the formula **BABY nCr CORR**. Compare the **WAYS** list with the **COMB** list.

6. When (for what numbers of correctly labeled babies) can we use the combinations formula to find the number of ways?
7. When can we **not** use the combinations formula? Why?

Test your hypothesis by entering some new values in the lists **BABY** and **CORR**, then re-entering the formula at the top of the list **COMB**.

Problem 2 – Finding probabilities

You can also use combinations to find probabilities. For example, if the bracelets of 4 babies were mixed randomly, what is the probability that two will be correct and two will be incorrect?

Definition of Probability: $P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$

8. Find the number of possible outcomes using the Fundamental Counting Principle. How many different ways can 4 babies be labeled?
9. Find the number of favorable outcomes. How many different ways can 4 babies be labeled such that 2 are correctly labeled and 2 are incorrectly labeled?
10. If 4 babies are labeled at random, what is the probability that 2 are correctly labeled and 2 are incorrectly labeled?
11. Write and simplify factorial expressions to find the probability of each event listed below if 4 babies are labeled randomly. Use the **factorial** and **nCr** commands to check your answer.
 - a. $P(\text{all correct})$
 - b. $P(3 \text{ correct})$
 - c. $P(\text{exactly } 1 \text{ correct})$
 - d. $P(\text{all incorrect})$
 - e. $P(\text{at least } 1 \text{ correct})$
 - f. $P(\text{at least } 1 \text{ incorrect})$

Extension – Writing a piecewise function

Using factorial notation, write and simplify a piecewise function $f(x)$ that gives the number of ways to label 4 babies so that x are correctly labeled. Then write a function $g(x)$ that gives the number of ways to label 5 babies so that x are correctly labeled.