Introduction to Absolute Value

About the Lesson

In this activity, students will numerically explore absolute values using the calculator, plot points to graph $y = |x|$, and use the Transform application to perform transformations with absolute value functions. Students will explore:

- Vertical shifts performed upon the graph of $y = |x|$
- Horizontal shifts performed upon the graph of $y = |x|$
- Stretching/shrinking performed upon the graph of $y = |x|$
- Reflections about the x-axis performed upon the graph of $y = |x|$

Finally, the general absolute value equation, $y = a|bx + d| + c$, will be introduced and connections will be made to the various transformation based upon the values of a, b, c, and d.

Vocabulary

- absolute value
- piecewise function
- transformation of function

Teacher Preparation and Notes

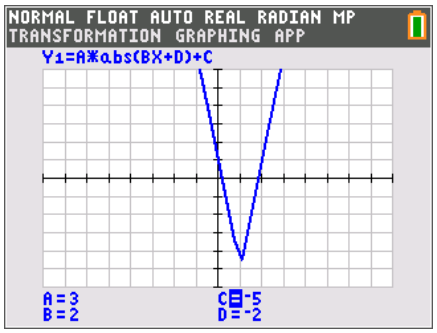
- The student worksheet that accompanies this activity provides instructions and questions to guide the inquiry and focus the observations. This works well as a student-led activity.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Introduction_to_Absolute_Value_Student.pdf
- Introduction_to_Absolute_Value_Student.doc

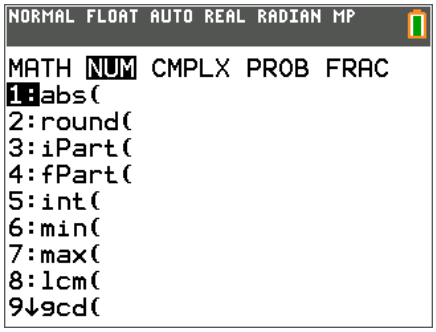
In this activity, students will explore the properties of the absolute value function. In the first problem, students will analyze an absolute value data set. In the second problem, they will analyze the absolute value function and apply what they have learned.

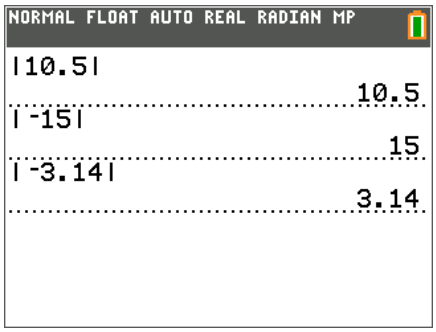
Problem 1 – Definition and Plotting Absolute Value

Students explore what the absolute value function does in relation to the piecewise definition and answer some general and specific questions.

Explain to students that the absolute value function always results in a nonnegative number, even if the inputs are negative. They should understand that $y = |x|$ means that if x is greater than zero, then $y = x$; if x is less than zero, then $y = -x$; if x is equal to zero, then $y = 0$.

Using the piecewise notation, absolute value can be defined as $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$.





1. Use the **abs** command on the Home screen to complete the following.

a. |10.5| = _____ **b.** |–15| = _____ **c.** |–3.14| = _____

d. |–12| = _____ **e.** |8| = _____ **f.** |2.71| = _____

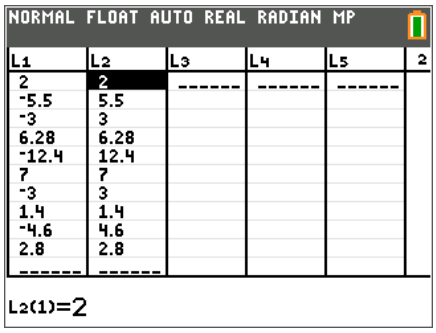
Answers: 10.5; 15; 3.14; 12; 8; 2.71

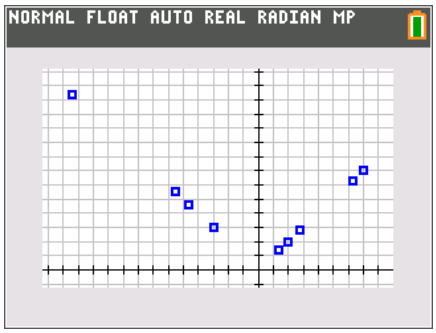
2. What happens to the absolute value of numbers that are negative (to the left of zero on a number line)? How does this relate to the definition stated above?

Answer: Even when the numbers are negative, the absolute value stays positive. This is because when $x < 0$, $-x > 0$.

Students are asked to generate ten random numbers (including negatives and decimals), input them in **L1**, and apply the **abs()** function to each value and store it in **L2**.

Students produce the scatter plot of the data and observe its shape. They can explore it further by entering more data points to test their conjecture regarding its shape.





Note: Depending upon the choices for the values of **L1,** the window size of the resulting stat plot may vary. After pressing zoom and select **ZoomStat**, students may manually edit their graphing window by pressing window and choosing appropriate values for Xmin, Xmax, Xscl, YMin, YMax and Yscl.

Tech Tip: If your students are using the TI-84 Plus CE have them turn on the GridLine by pressing 2nd zoom [format] to change the graph settings. If your students are using TI-84 Plus, they could use GridDot.

3. **L1** represents the *x*-values and **L2** represents the *y*-values. What trend do you notice about the *y*-values for the negative *x*-values? Describe the shape of the graph to the left of $x = 0$.

Answer: The shape is a line with a negative slope. The *y*-values are positive for every negative *x*-value.

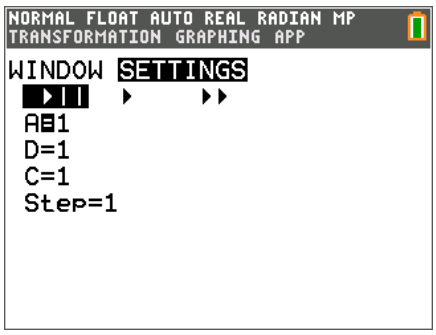
4. Describe the shape of the graph of $y = |x|$. Enter new numbers into the list, press zoom, and select **ZoomStat** to check the shape you described.

Answer: The shape of the graph is a V.

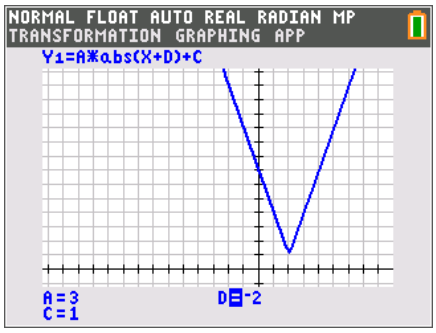
Problem 2 – Exploring Functions With Absolute Value

Have students turn off **Plot1** before beginning. To change the window setting shown at the right, press window ▲ (up arrow).

In the problem, the constants that define the general form of the absolute value function are explored to see how they affect the shape of the graph. The slope and the vertex are especially considered.



To change the values of the constants, the students will use the ▲ and ▼ arrow keys to select the desired variable and then use the ◀ and ▶ arrow keys to change the value.



5. What happens to the graph when *a* is negative? When *a* is positive?

Answer: When *a* is negative, the V-shaped graph opens down. When *a* is positive, the V-shaped graph opens up.

6. In general, what effect does *a* have on the graph?

Answer: In general, *a* affects the slope of the graph and whether the V-shape of the graph is pointing up or down.

7. **a.** What happens to the graph when *d* is positive? When *d* is negative?

Answer: When *d* is positive, the graph moves left. When *d* is negative, the graph moves right.

b. What happens to the graph when *c* is positive? When *c* is negative?

Answer: When *c* is positive, the graph moves up. When *c* is negative, the graph moves down.

c. For this general function $y = |x + d| + c$, what are the coordinates of the vertex?

Answer: $(-d, c)$

Problem 3 – Matching Equations to Graphs

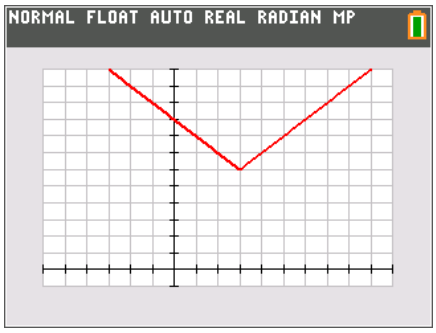
Students are to use what they have learned in the previous problems to determine the equation of the graph shown. If students are having trouble, tell them to identify the vertex first.

Choose the correct equation from the options below for each graph shown.

a. $y = |x - 6| + 3$ **b.** $y = -|x + 3| - 6$

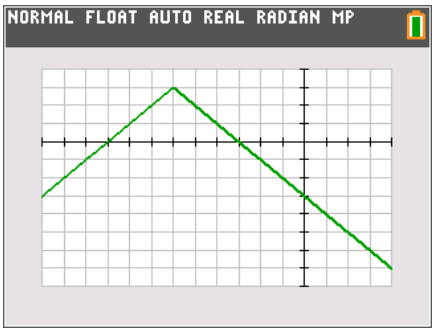
c. $y = |x - 3| + 6$ **d.** $y = -|x + 6| + 3$

8.



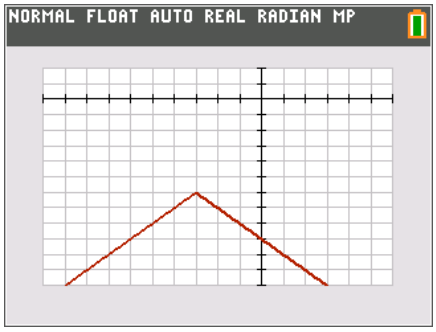
Answer: c

9.



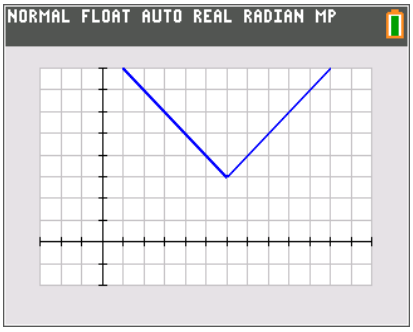
Answer: d

10.



Answer: b

11.

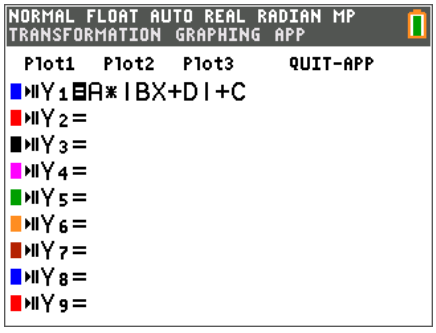


Answer: a

Extension – General Absolute Value Function

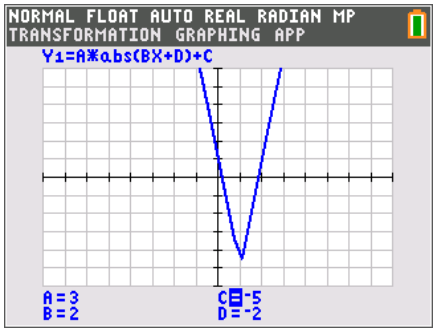
Students will explore all four variables that make up the general absolute value function. Note that *b* is included here, but was not explored in the previous problems.

12. Using the Transformation Graphing App, explore the graph of **Y1=A∗abs(BX+D)+C**. What does the graph look like when *a* is zero? What about when *b* is zero? Explain why.



Answer: When either *a* or *b* is zero, the graph is the horizontal line $y = c$. This happens because the *x* is eliminated and all that is left is a constant.

13. List any other observations. For example, how is the slope related to *a* and *b*? Is the vertex always (–*d*, *c*)?



Sample Answers: The slope is the product of *a* and *b*. The vertex is $\left(-\frac{d}{b}, c\right)$.

Point out some things to look at. For example, ask students to increase *a* and decrease *b* and observe the effect. Consider that the minimum or maximum point will occur when the expression inside the absolute value is zero.