Solving Systems Using Row Operations 2

Math Objectives

- Students will find and interpret the reduced row-echelon answer matrix for the solution to the system of three equations with three unknowns.
- Students will interpret a consistent and independent system of three equations with three unknowns resulting in an intersection point.
- Students will interpret a consistent and dependent system of three equations with three unknowns resulting in an intersection line.
- Students will interpret an inconsistent system of three equations with three unknowns resulting in no common intersections.

Vocabulary

- augmented matrix
- reduced row-echelon form
- main diagonal
- consistent system of equations
- inconsistent system of equations
- dependent system of equations
- independent system of equations

About the Lesson

- This lesson involves using matrices as a tool to solve a system of three equations with three unknowns
- As a result, students will:
  - Enter the coefficients of a system into an augmented matrix.
  - Find the reduced row-echelon form of the matrix using the **rref( )** command on the TI-Nspire.
  - Translate the answer matrix into a solution of the system, both algebraically and geometrically.

TI-Nspire™ Navigator™ System

- Use Screen Capture/Live Presenter to demonstrate the process of entering a system into matrix form and using the matrix operation to obtain the reduced row-echelon form. You may also display and discuss the solutions to various systems.
- Use Quick Poll to assess students' understanding of the solution matrix to solve a system of equations.

TI-Nspire™ Technology Skills:

- Create a document
- Insert new page
- Enter an augmented matrix
- Use the menu to find the reduced row-echelon solution matrix

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Materials:
Student Activity
Solving_Systems_Using_Row_Operations_2_Student.doc
Solving_Systems_Using_Row_Operations_2_Student.pdf

Visit www.mathnspired.com for lesson updates.

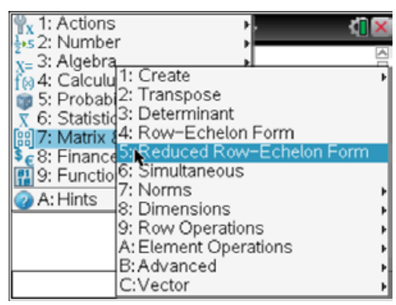
This activity uses the reduced row-echelon form of a matrix to solve systems of three equations with three unknowns. To follow a step by step process for row reduction, see the activity: *Solving Systems Using Row Operations 1.*

TI-Nspire Navigator Opportunity: ***Screen Capture/Live Presenter***
See Note 1 at the end of this lesson.

Teacher Tip: Before you begin this lesson, review the three types of solutions to systems with two equations and two unknowns. How do you know there is one solution, no solution, or a dependent solution? After solving systems of equations algebraically and interpreting the results, you may use this lesson to look at a number of systems of three equations with three unknowns. You may want to do the first example as a class, using the elimination method or begin with the activity *Solving Systems Using Row Operations 1*, showing the reduced row-echelon procedure. Students should already be familiar with the vocabulary, the types of solutions possible, and the interpretation of the results. Students should have experience with a system of three equations with three unknowns.

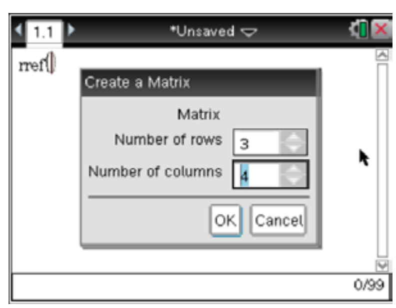
1. Open a new document and choose a Calculator page by pressing on **> New Document > Add Calculator.**

2. Press **Menu > Matrix & Vector > Reduced Row-Echelon Form.**



3. Press the key and choose the 3 X 3 matrix template.

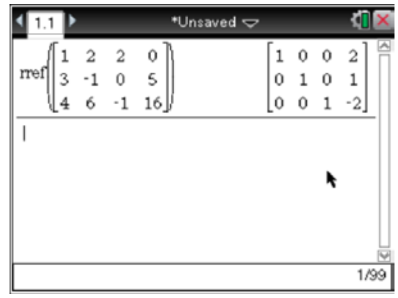
4. Make sure the number of rows is 3. Tab to the number of columns and change it to 4, tab to OK, and press enter.



5. Enter the coefficients into the matrix, using the tab key to move from element to element. The matrix shown is for the system of equations:

$$\begin{aligned} x \quad +2y \quad +2z &= 0 \\ 3x \quad -y \quad\quad\quad &= 5 \\ 4x \quad +6y \quad -z &= 16 \end{aligned}$$

Press enter.



6. The resulting solution matrix will be displayed.

7. To enter another system of equations you may repeat the above Steps 2 through 5 or press ▲ twice to highlight the command. Press enter. Go to each element and press del to delete the previous value and enter the new value. Use the tab key to move from element to element. When completed press enter.

Discussion Points and Possible Answers

Tech Tip: The calculator is being used to quickly and correctly solve a system of three equations and three unknowns.

8. Identify the system as consistent (dependent or independent) or as inconsistent. Give the geometric interpretation of the solution as a point, line or having no common intersections.

Given System	Reduced Row-Echelon Form	Type of System	Solution	Geometric Interpretation
$\begin{aligned} x \quad +2y \quad +2z &= 0 \\ 3x \quad -y \quad\quad\quad &= 5 \\ 4x \quad +6y \quad -z &= 16 \end{aligned}$	$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$	Consistent and independent	(2, 1, –2)	point

Teacher Tip: You may want to have students rewrite the reduced row-echelon solution matrix as a system with coefficients of 1 and zero. This will allow students to make the connection between the solution matrix and the original system. For example, the first row-eschelon form would be written as:

$1x + 0y + 0z = 2$

$0x + 1y + 0z = 1$ Therefore, the solution is $\{x, y, z\} = (2, 1, -2)$.

$0x + 0y + 1z = -2$

9. How do you know when a matrix is in reduced row-echelon form?

   <u>Answer:</u> A matrix is in reduced row-echelon form when the main diagonal consists of 1s and they are the only non-zero numbers in each of those columns.

10. What does the last column of the reduced row-echelon matrix represent?

    <u>Answer:</u> The last column of the reduced row-echelon matrix represents the right sides of the equations represented by the rows. In a consistent and independent system, the last column is the solution to the system.

11. Solve the following systems. Identify the system as consistent (dependent or independent) or as inconsistent. Give the geometric interpretation of the solution as a point, line or having no common intersections.

Given System	Reduced Row-Echelon Form	Type of System	Solution	Geometric Interpretation
a. $\begin{array}{llll} x & +y & +z & =6 \\ -3x & +2y & +z & =4 \\ x & -3y & +2z & =1 \end{array}$	$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$	Consistent and independent	(1, 2, 3)	point
b. $\begin{array}{llll} 2x & -y & +z & =1 \\ x & +2y & -z & =3 \\ x & +7y & -4z & =8 \end{array}$	$\begin{bmatrix} 1 & 0 & \frac{1}{5} & 1 \\ 0 & 1 & \frac{-3}{5} & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	Consistent and dependent	$\left(\frac{5-t}{5}, \frac{5+3t}{5}, t\right)$	line
c. $\begin{array}{llll} x & +2y & +3z & =4 \\ 2x & -3y & +z & =5 \\ 3x & -y & +4z & =9 \end{array}$	$\begin{bmatrix} 1 & 0 & \frac{11}{7} & \frac{22}{7} \\ 0 & 1 & \frac{5}{7} & \frac{3}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$	Consistent and dependent	$\left(\frac{22-11t}{7}, \frac{3-5t}{7}, t\right)$	line
d. $\begin{array}{llll} 2x & +y & -z & =3 \\ -3x & +2y & +z & =4 \\ 4x & +2y & -2z & =8 \end{array}$	$\begin{bmatrix} 1 & 0 & \frac{-3}{7} & 0 \\ 0 & 1 & \frac{-1}{7} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	Inconsistent	No solution	No common intersections

12. In a reduced row-echelon solution matrix:
   a. Why does a last row of 0 0 0 1 indicate no solution?

   Answer: A last row of 0 0 0 1 is interpreted as $0x + 0y + 0z = 1$, which is clearly impossible. Therefore, there is no solution to the system.

   b. Why does a last row of 0 0 0 0 indicate a dependent system?

   Answer: A last row of 0 0 0 0 is interpreted as $0x + 0y + 0z = 0$, which will be true for any values of the variables. One variable stays as an unknown, traditionally the variable *t*, and the other variables will be solved in terms of this one.

   c. Explain what types of systems result when the main diagonal does not consist entirely of 1's.
   Answer: If the system is consistent but dependent, the last row will be all zeros. This puts a zero in the main diagonal. The other rows may have non-zeros to the right of the main diagonal, not considering the last column. If the system is inconsistent, one row will have all zeros except for a 1 in the last column. This also puts a zero in the main diagonal. The main diagonal is all 1's only for a consistent, independent system.

13. For each of the following types of systems, fill in the blank with the appropriate geometric interpretation. When solving a system of three equations with three unknowns,
   a. a consistent and independent system results in a/an **intersection point.**

   b. a consistent and dependent system results in a/an **intersection line (or plane).**

   c. an inconsistent system results in **no common intersections.**

> **Teacher Tip:** Physical models of three planes would help students visualize what is happening and the different possibilities. Consistent systems are three planes intersecting in a point (independent), or in a line or a plane (dependent). Inconsistent systems are three planes with no common intersection. They all may be distinct and parallel to each other, two may be distinct and parallel, or the planes intersect in pairs but the three intersection lines are parallel to each other.

TI-Nspire Navigator Opportunity: ***Quick Poll***
See Note 2 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to interpret the reduced row-echelon matrix form of a solution to a system of three equations with three unknowns.
- A consistent and independent system of three equations with three unknowns results in an intersection point.
- A consistent and dependent system of three equations with three unknowns results in an intersection line.
- An inconsistent system of three equations with three unknowns results in no common intersections.

Assessment

Sample Questions:

1. The solution matrix to a system of equations is $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}$. This system is:

a) consistent and independent with the solution the point $(5,0,3)$.

b) consistent and dependent with the solution the line $(5t,t,3t)$.

c) inconsistent with no common intersection points.

2. The solution matrix to a system of equations is $\begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. This system is:

a) consistent and independent with the solution the point $(-2,1,0)$.

b) consistent and dependent with the solution the line $(-2-t,1-t,t)$.

c) inconsistent with no common intersection points.

3. The solution matrix to a system of equations is $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. This system is:

a) consistent and independent with the solution the point $(3,-2,1)$.

b) consistent and dependent with the solution the line $(3t,-2t,t)$.

c) inconsistent with no common intersection points.

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Note 1
Question 1, *Screen Capture/Live Presenter*: You may use the TI-Nspire Navigator to demonstrate the process to start a new document. Students will need to translate a system into a matrix, find the reduced row-echelon form of the matrix, and interpret the solution matrix to find the solution to the system of equations. *Live Presenter* could help students follow the steps.

Note 2
Question 13, *Quick Poll*: You may use the TI-Nspire Navigator to assess students' understanding of the solution matrix of a system of equations both algebraically and geometrically. Sample questions are included above.