Radio Station KTNS

Math Objectives

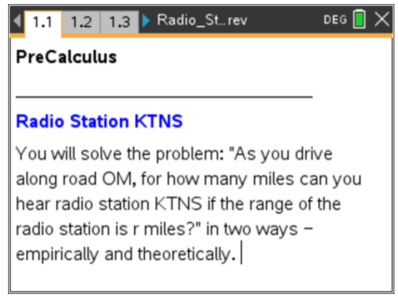
- Students will solve a problem experimentally by fitting a function to a set of data.
- Students will solve the same problem theoretically by making and verifying conjectures using algebraic and trigonometric methods.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).

Vocabulary

- Law of Sines
- Law of Cosines

About the Lesson

- This lesson involves determining the distance one can hear a radio station as a function of the range of the station.
- Note: Some portions of the activity require CAS functionality – TI-Nspire CAS Required.
- As a result, students will:
  - Solve the problem empirically by fitting a regression equation to a set of gathered data.
  - Solve the problem theoretically by finding an equation involving the Law of Cosines and the Law of Sines.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- Once a function has been graphed, the entry line can be shown by pressing ctrl G. The entry line can also be expanded or collapsed by clicking the chevron.

Lesson Files:
Student Activity
Radio_Station_KTNS_Student.pdf
Radio_Station_KTNS_Student.doc

TI-Nspire document
Radio_Station_KTNS.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Radio Station KTNS is located at point P in the figure. The range of its signal is r miles, meaning that people within r miles of P would be able to hear the station. You are driving along road OM at an angle of 30° with OP. For how many miles, d, could you hear station KTNS?

In ΔPAB, the Law of Cosines tells us that $d^2 = 2r^2 - 2r^2 \cdot \cos(\angle APB)$, so it is reasonable to assume that d^2 could be a linear function of r^2. To solve this problem, you will determine d^2 in terms of r^2 in two ways:

- Find an experimental model by gathering data and fitting an appropriate regression function to the data.
- Find a theoretical model using the Law of Sines, the Law of Cosines, and algebra.

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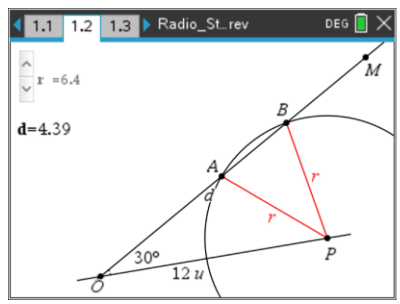

The figure is a scale drawing with 1 unit = 10 miles so that OP = 12 units or 120 miles.

1. In miles, the reasonable values of r satisfy $k < r \le 120$. What is the value of k? Why?

   <u>Answer:</u> $k = 12 \cdot \sin(30°) = 6\ miles$ since the smallest value of k occurs when r is perpendicular to OM and $d = 0$.

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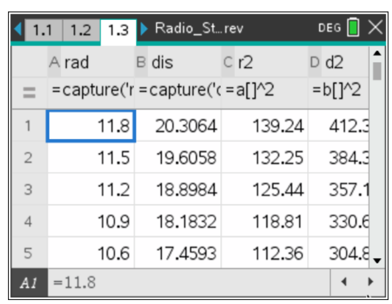
Using the slider, the following data has been gathered in the spreadsheet in the four columns: $rad(r)\quad dis(d)\quad r2 = r^2\quad d2 = d^2$



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1	11.8	20.3064	139.24	412.3
2	11.5	19.6058	132.25	384.3
3	11.2	18.8984	125.44	357.1
4	10.9	18.1832	118.81	330.6
5	10.6	17.4593	112.36	304.8

A1 =11.8

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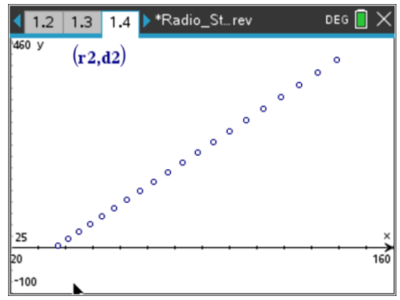
A scatterplot of the data has been drawn on this page.



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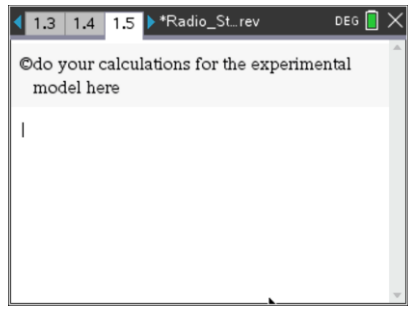

2. Fit a linear regression function to the data with $x = r2$ and $y = d2$ in units. Select **MENU > Statistics > Stat Calculations > Linear Regression (mx+b)**. with $r2$ for X List, $d2$ for Y List, and **Save RegEqn** to: $f1$.

   <u>Answer:</u> $d^2 = 4r^2 - 144.611$.

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3. Plot the regression equation on the scatterplot, and note how well it fits. Open the entry line, move back up to $f1(x)$, and press enter. According to this linear model, for how many miles, d, could you hear the station if $r = 90$ miles?
   Hint: Remember r = 9 units corresponds to r = 90 miles.

   <u>Answer:</u> $\sqrt{4 \cdot 9^2 - 144.61} \cdot 10 = 133.94$ miles.

> **Teacher Tip:** Students could use Scratchpad or the Calculator page to compute their answers for #3.

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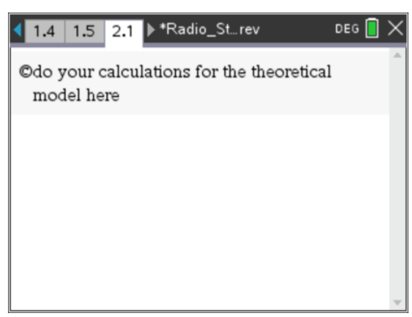
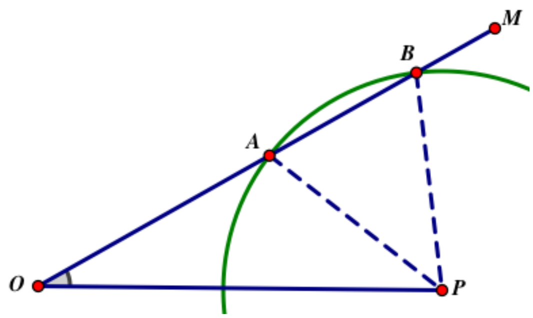
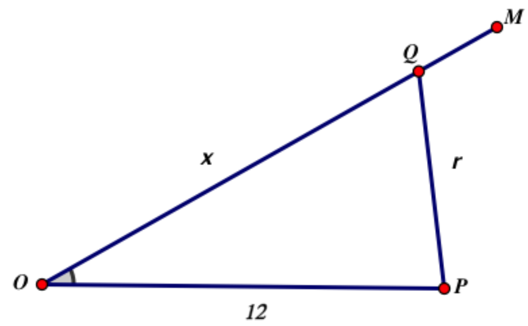


Theoretical Model

Find the theoretical function expressing d^2 in terms of r^2 by completing the argument below.





4. The figure for this problem shows an example of an ambiguous case of the Law of Sines since there are two triangles with two sides OP = 12, r, and the non-included angle of 30°. Consequently, if we apply the Law of Cosines to a triangle with sides OP = 12, r, x and angle 30°, we obtain the equation:

_________________________________ = 0.

On the scale drawing, then, the two solutions for x are OA and OB, and the distance, d, is $d = OB - OA$.

Sample Answers: By the Law of Cosines, $r^2 = x^2 + 12^2 - 2 \cdot 12 \cdot x \cdot \cos(30°)$, so that the desired equation is $x^2 - 12\sqrt{3} \cdot x + (144 - r^2) = 0$.

5. a. Find the two solutions for x of this equation.____________________________ .
Hint: You can use "solve" command. Both solutions will be functions of r^2

Sample Answers: Using paper-and-pencil or $solve(x^2 - 12\sqrt{3}x + 144 - r^2 = 0, x)$, the two solutions are $x = 6\sqrt{3} - \sqrt{r^2 - 36}$ and $x = 6\sqrt{3} + \sqrt{r^2 - 36}$.

b. Find the difference of the two solutions and express d^2 in terms of r^2 in units:

d^2 = _______________________

Answer: The difference is $d = 2\sqrt{r^2 - 36} = \sqrt{4r^2 - 144}$ so that $d^2 = 4r^2 - 144$.

6. How does your theoretical equation compare to the regression equation?

Answer: They are essentially the same with only a small difference in the constant terms.

7. According to this theoretical model, for how many miles, d, could you hear the station if

$r = 90$ miles?

Hint: Remember r = 9 units corresponds to r = 90 miles.

Answer: $\sqrt{4 \cdot 9^2 - 144.} \cdot 10 = 134.16$ miles.

8. Suppose the angle between the two roads OP and OM is changed to $\theta°$. Express d^2 in terms of r^2 and θ:

d^2 = ________________________

Answer: We want to find the square of the difference of the two solutions of $x^2 - 24x \cdot \cos\theta + (144 - r^2) = 0$. If we use 'paper-and-pencil', we will probably obtain $d^2 = 4r^2 + 576(\cos^2\theta - 1)$. Using $solve(x^2 - 24 * \cos\theta + 144 - r^2 = 0, x)$ and some rewriting yields $d^2 = 4r^2 - 576\sin^2\theta$.

Teacher Tip: Ask students why these two solutions are equivalent.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to interpret a scale drawing.
- How to fit a linear regression equation to a set of data.
- Setting up and solving an equation involving the Law of Cosines and interpreting the solutions.