Math Objectives

- Students will be able to write a power function with unit fractions as exponents using radical notion.
- Students will be able to describe the shape, end behavior, and key points for power functions with unit fractions as exponents.
- Students will be able to describe the shape, end behavior, and key points for power functions with negative integer exponents.
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

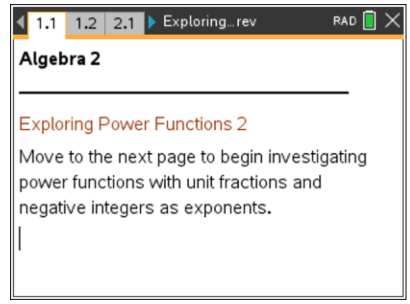
- radical functions
- rational functions
- domain
- range
- asymptotes

About the Lesson

- This lesson involves students investigating power functions by clicking on a slider.
- As a result, students will:
  - Discover the connection among power functions and both radical and rational functions.
  - Compare how different exponents impact the shape of a power function.

TI-Nspire™ Navigator™ System

- Use Screen Capture or Live Presenter to monitor student progress or have them share their investigations.
- Use Quick Poll to assess student understanding.
- Use Live Presenter to have students share their results.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the entry line by pressing ctrl G.

Lesson Materials:
Student Activity
Exploring_Power_Functions_2.pdf
Exploring_Power_Functions_2.doc
TI-Nspire document
Exploring_Power_Function_2.tns

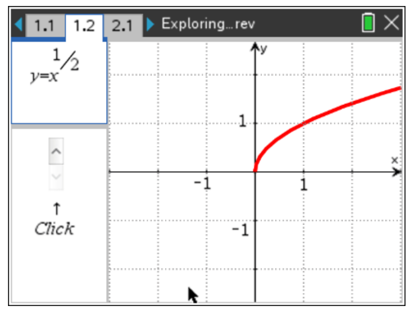
Visit www.mathnspired.com for lesson updates and tech tip videos. (optional)

Discussion Points and Possible Answers

TI-Nspire Navigator Opportunity: ***File Transfer***
See Note 1 at the end of this lesson.

Teaching Tip: Students may vary in how quickly they navigate the lesson. For students who move very quickly, you might consider asking them to investigate power functions with fractions other than unit fractions using the Scratchpad.

Move to page 1.2.



1. This page displays graphs of power functions with unit fractions as the exponent. Describe the shape of the graph of $y = x^{\frac{1}{2}}$.

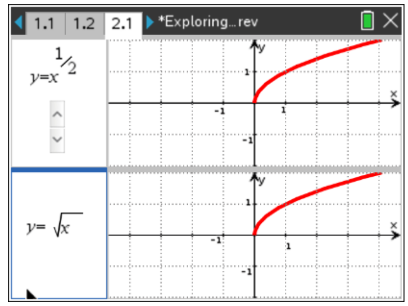
   <u>Sample answer:</u> Answers may vary, but they should relate to the graph shown on the right. Students may say that it looks like half of a sideways parabola.

2. Compare the graph of an equation where the power has an even denominator to the graph of an equation where the power has an odd denominator. Why are the graphs different?

   <u>Answer:</u> Answers should reference the fact that the graph is displayed in a different place depending on whether the power has an even or an odd denominator. The shapes are similar, but the graph appears only in the first quadrant for even powers and in the first and third quadrants for odd powers. This is because powers that are unit fractions result in radicals, and even roots of negative numbers produce complex numbers that cannot be graphed in the coordinate system. (Students may not know this information depending on the sequence of your course.)

TI-Nspire Navigator Opportunity: ***Screen Capture*** **or** ***Live Presenter***
See Note 2 at the end of this lesson.

Move to page 2.1.



3. This page compares radical equations to power functions with unit fractions as exponents. Make a conjecture about a rule that relates power functions and radical functions. Click the slider to explore.

   Answer: The denominator of the unit fraction is the order of the root. This page illustrates the exponent rule $x^{\frac{1}{n}} = \sqrt[n]{x}$. The graphs of the two equations are equivalent.

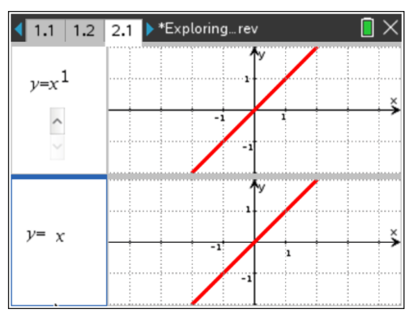
4. Why is the graph of $y = x^{\frac{1}{2}}$ displayed only in the first quadrant?

   Answer: Square roots of negative numbers produce complex numbers that cannot be displayed on the coordinate system. (Answers may vary depending on if you have addressed complex numbers yet.)

5. Compare the key information about power functions with even or odd unit fraction exponents by completing the table below:

p-value	Points on All Graphs	Function Domain	Function Range
Even	(–1, 1), (0, 0), and (1, 1)	$\{x \in \mathbf{R} \mid x \geq 0\}$	$\{y \in \mathbf{R} \mid y \geq 0\}$
Odd	(–1, –1), (0, 0), and (1, 1)	$\{x \in \mathbf{R}\}$	$\{y \in \mathbf{R}\}$

Teacher Tip: It is important to point out that $y = x$ is also a power function. Student may not think of this as being in the family of power functions.



6. Click the slider until the denominator equals 1. Why does the graph resemble a line?

   Answer: When the value of the denominator is 1, the equation has 1/1 as an exponent. This simplifies to x^1, which results in $y = x$.

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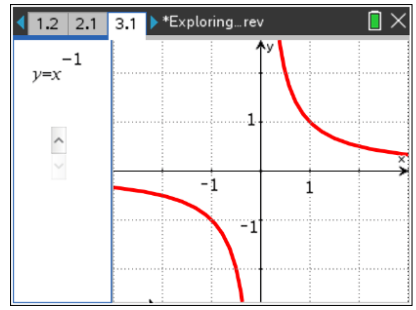
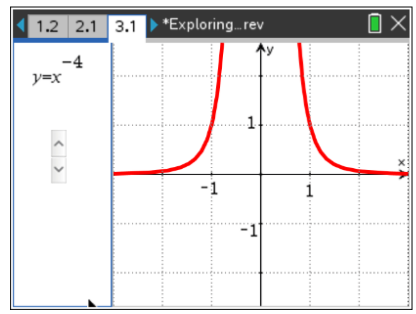


7. This page shows graphs of power functions with negative integer exponents. How is the graph of $y = x^{-1}$ different from the graphs in previous questions?

   Sample answer: Answers may vary, but should mention that the graph has "two parts," or that it has a "mirror image." Descriptions should accurately describe the graph to the right.



8. Click the slider to change the value of the exponent. The graphs never connect. At which *x*-value is there a break? Why is the break at this *x*-value?

   Sample answer: Student answers should mention the fact that the graph has "two parts" because the graph is not defined at $x = 0$, since that would result in division by zero.

9. When the value of the exponent is negative, the graph always consists of two pieces. When the exponent is even, the two branches are in the first and second quadrants. When the exponent is odd, the two branches are in the first and third quadrants. Why do the graphs behave this way?

   Sample answer: Student answers may vary but could address that even exponents and odd exponents produce different graphs because when you raise a negative *x*-value to an even integer, you get a positive result for the *y*-value, and when you raise a negative *x*-value to an odd power, you get a negative result for the *y*-value.

Move to page 4.1.

10. Compare the key information about power functions with negative integer exponents in the table below:

p-value	Points on All Graphs	Function Domain	Function Range
Evens	(1, 1) and (–1, 1)	$\{x \in \mathbf{R} \mid x \neq 0\}$	$\{y \in \mathbf{R} \mid y > 0\}$
Odds	(1, 1) and (–1, –1)	$\{x \in \mathbf{R} \mid x \neq 0\}$	$\{y \in \mathbf{R} \mid y \neq 0\}$

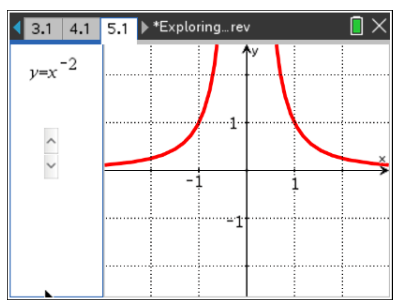
11. A rule for power functions with negative exponents is $x^{-n} = \dfrac{1}{x^n}$. How do the graphs on page 4.1 support this? Why does this rule work?

Answer: The exponent rule is supported because the graphs are equivalent on the screen. The exponent rule works because it is related to the division rule for exponents. When you have the expression $\dfrac{x^a}{x^b}$, you subtract the exponents: $\dfrac{x^a}{x^b} = x^{a-b}$. When $a = 0$, the equation becomes $\dfrac{x^0}{x^b} = x^{0-b} = x^{-b}$. Since $x^0 = 1$, you can replace the numerator with x^0 to get $\dfrac{1}{x^b} = x^{0-b} = x^{-b}$.

12. An asymptote is a line that a graph approaches but never reaches. The graphs of power functions with negative integer exponents have the *x*-axis and *y*-axis as asymptotes. Why do the graphs approach, but never reach, these lines?

Answer: The function approaches, but never touches, the *x*-axis because as *x* increases, you are dividing 1 by larger and larger numbers, which produces smaller and smaller numbers. So as the *x*-values increase, the *y*-values decrease. However, no matter how big the *x*-values get, the *y*-values can never get to zero because there will always be a 1 in the numerator. The function approaches, but never touches, the *y*-axis because you cannot put zero in for *x* in the function since that will result in division by zero, which is undefined.

Move to page 5.1.



13. This page shows power functions with integer exponents from –8 to 8. Click the slider until the exponent is 0. What is the equation when the exponent is 0, and what is the form of the graph? Why is there a hole at $x = 0$ on this graph? (Hint: Consider using the Scratchpad to do a calculation.)

Answer: The equation when the exponent is 0 is $y = x^0$. This equation is the same as $y = 1$, except *x* cannot equal zero. The graph has a hole at $x = 0$ because 0^0 is undefined as shown in the Scratchpad.

Teacher Tip: 0^0 is undefined. Students may recall that a^0 is 1 (for $a \neq 0$), so some may think 0^0 could be defined to equal 1, but 0^a is equal to 0 (for $a > 0$), so some may think 0^0 could be defined to equal 0. Since these two situations contradict each other, the quantity 0^0 is indeterminate. This is often a difficult concept for students to accept.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to compare and contrast power functions with unit fractions and negative integers as exponents.
- How to identify common points on power functions based on the exponent.
- How to identify the behavior of the graphs of a power function based on the exponent.

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Note 1

Entire Document, *File Transfer:* Use *File Transfer* to efficiently send the TI-Nspire document file to students. Using TI-Navigator will allow students to receive the file without having to leave their seats or use extra cables.

Note 2

Entire Document, *Screen Capture* or *Live Presenter:* If students experience difficulty with the operation of a file or a question, use *Screen Capture* or *Live Presenter* with TI-Navigator. You can also use this to facilitate student discussion.

Note 3

End of Lesson, *Quick Poll:* A *Quick Poll* can be given at the conclusion of the lesson. You can save the results and show a review of them at the start of the next class to discuss possible misunderstandings students may have.

The following are some sample questions you can use:

1. If (5, 0.1) is a point on a power function with an even negative integer exponent, which of the following points will also be on the graph?
   a. (–5, 0.1)
   b. (–5, –0.1)
   c. (5, –0.1)
   d. (–0.1, 5)

2. What would be the asymptote of $y = \frac{1}{x-4}$?
   a. The *y*-axis
   b. $x = -4$
   c. $x = 4$
   d. There are no asymptotes

3. Sometimes, Always, Never: The domain of a power function with a unit fraction as an exponent is all real numbers.

   Answer: Sometimes