Advanced Algebra Nomograph

ID: 8720

Time required
30 minutes

Activity Overview

*This activity is similar to the idea of a function machine. There are two levels of the manipulative (called a **nomograph**). The first is comprised of two vertical number lines, input on the left and output on the right. The second has three number lines to accommodate displaying the composition of two functions. At the first level, students try to find the rule of a hidden function by entering domain values and observing how they are transformed to new (range) values. The transformation is illustrated dynamically by an arrow that connects a domain entry to its range value. At the second level, students investigate composite functions. Inverse functions are treated as special cases of composition.*

Topic: Sequences, Series, & Functions

- *Calculate the value of a function f(x) defined by an algebraic expression at any real value of x.*

Teacher Preparation and Notes

- *This activity is appropriate for students in Algebra 2 or Precalculus.*
- *Prerequisites are: an introduction to functions (including the terms domain and range), function notation ("y=" and "f(x)="), and experience graphing linear functions using slope and y-intercept. It is important that the model be demonstrated to students prior to them exploring the program on their own. (Perhaps work through Problem 1 as a class.)*
- *This activity is designed to have students explore **individually and in pairs**. However, an alternate approach would be to use the activity in a whole-class format. By using a ViewScreen™ panel and the questions found on the student worksheet, you can lead an interactive class discussion on functions and their inverses.*
- ***To download the NOMOGRPH program file and student worksheet, go to education.ti.com/exchange and enter "8720" in the keyword search box.***

Associated Materials

- *AdvAlgebraNomograph_Student.doc*
- *NOMOGRPH.8xp (program)*

Suggested Related Activities

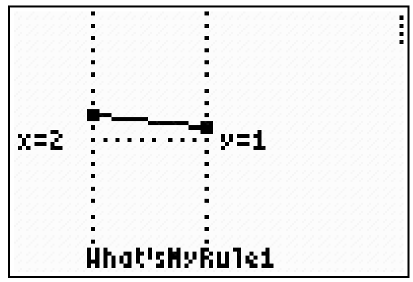
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Radical Functions (TI-84 Plus family) — 8977*
- *What is the Inverse of a Function (TI-84 Plus family) — 8211*

A ***nomograph*** is similar to a function machine in that it relates a number from one set (the domain) to a number in a second set (the range). Each set of numbers is represented in a pair of vertical number lines; the domain is on the left, and the range is on the right. According to the function rule, an element of the domain is mapped to its corresponding range element, and this mapping is depicted by an arrow.

Prior to beginning Problem 1, review domain and range, and ensure that students understand how to use the model.

Problem 1 – "What's my Rule?"



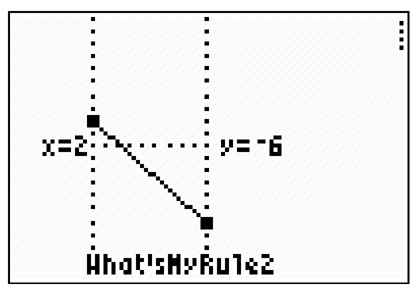
The first several problems are "What's my Rule?" activities. Input values are entered, one at a time, when prompted. The nomograph displays the input and its corresponding output. By repeatedly entering different inputs, the student should be able to discover the function's rule.

For example, if domain values 1, 2, 5, and 7 and their respective range values 3, 5, 11, and 15 are observed, the rule $f(x) = 2x + 1$ should be identified. When students have conjectured a rule, they should record it on their worksheets and check it. The rule is checked by selecting an input number, applying the rule, and predicting the output number.

Solution:

- $\boldsymbol{f(x) = 3x - 5}$

Problem 2 – A more difficult "What's my Rule?"



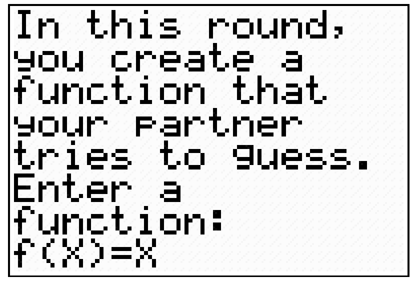
This nomograph (**What'sMyRule2**) follows a quadratic rule. Students are guided through the same steps to determine the rule. Encourage students to record on their worksheets several of the ordered pairs they observed. This will help them in determining the function's rule.

Solution:

- $\boldsymbol{f(x) = x^2 - 10}$

Problem 3 – The "What's my Rule?" Challenge

Instruct students to create their own functions of the form $f(x) = ax + b$ or $f(x) = ax^2 + b$ (where a and b are integers). Each student should enter the **RuleChallenge** section of the program define f to their own function when prompted. Students should then exchange handhelds with a partner. It is the partner's task to use the nomograph to identify the mystery function. Encourage students to repeat this activity several times.

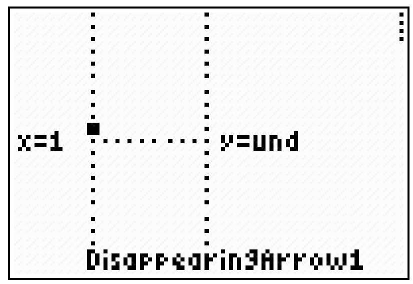


Solutions:

- Functions will vary.

Problem 4 – The case of the disappearing arrow

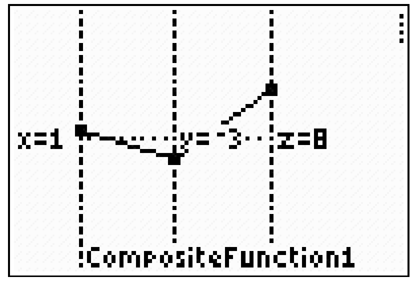
The nomograph in **DisappearArrow > Disappear1** displays a function with restricted domain: $f(x) = \sqrt{x^2 - 4}$. If the x-value entered is outside the domain, the function arrow disappears. Students are asked to explain when and why this happens for this specific function. To avoid confusion, have students begin with a value of x such that $|x| > 2$ to make sure the arrow is visible.



Solutions:

- **$-2 < x < 2$**
- Possible answer: **The square root of a negative number is undefined.**

Problem 5 – Composite functions: "wired in series"



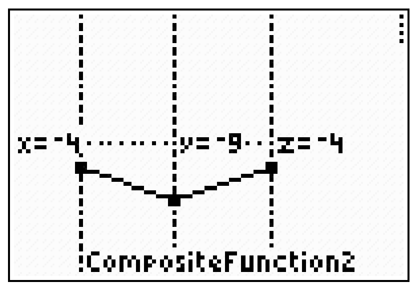
The nomographs in **CompositeFunc > CompositeFunc1** and **MakeYourOwn** enable students to explore the meaning of the composition of functions.

Be sure students are familiar with both notations for composite functions: $f_2 \circ f_1$ and $f_2(f_1(x))$. The input for the first function is given by the student, and an arrow connects x to its output, y. The point y is used as input for a second function, and connected by a second arrow to its corresponding output, z.

Solutions:

- $\boldsymbol{f_3(x) = -6x + 14}$
- $\boldsymbol{f_2(f_1(x)) = 2(x - 1)^2 + 3}$; $\boldsymbol{f_1(f_2(x)) = (2x + 2)^2}$

Problem 6 – A well-behaved composite function

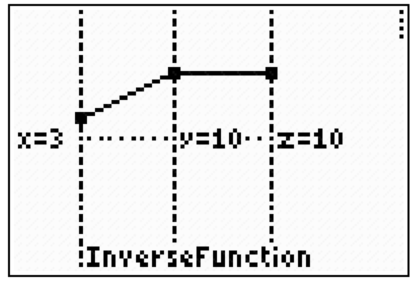


The concept of an inverse function is introduced. The nomograph (**CompositeFunc2**) shows the function $f_1(x) = 3x + 3$ and its mystery inverse f_2, left for the student to determine. Here, they should find that $f_1 \circ f_2$ gives the same value as $f_2 \circ f_1$. However, as they saw in Problem 5, this is not always the case. Have them consider an additional pair of functions $f_1(x) = x + 2$ and $f_2(x) = x^2$ with **MakeYourOwn** to see that the order of composition does matter. (The order does not matter *if and only if* f_1 and f_2 are inverses.)

Solutions:

- Possible answer: **The final output *z* is equal to the initial input value *x*.**
- $f_2(x) = \frac{1}{3}(x - 3)$
- Possible answer: **For each x, $f_1(f_2(x)) = f_2(f_1(x)) = x$.**

Problem 7 – Inverse functions

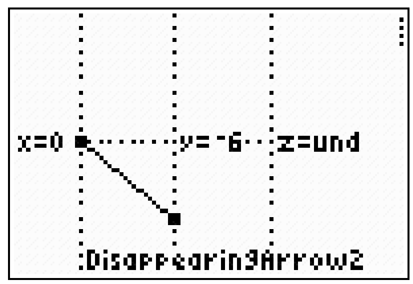


The formal definition of inverse functions is given here (**InverseFunc > InverseFunc**). You may wish to provide students with several (linear) functions and have them identify each function's inverse. Encourage them to identify the inverse of an equation by swapping *x* and *y* in the equation and then solving for *y*. Also, you should reinforce that they need to find both $f(g(x))$ **AND** $g(f(x))$ to determine if two functions *f* and *g* are inverses. In Problem 9, students will explore some of these subtleties.

Solution:

- $f_2(x) = \frac{1}{2}(x - 4)$

Problem 8 – Disappearing arrows in a composition function



Domain restrictions on composite functions are examined in **DisappearArrow > Disappear2**. Arrows disappear when $f_1(x) = 2x - 6$ fails to be in the domain of $f_2(x) = \sqrt{x}$.

Solutions:

- **second arrow (for f_2)**
- Possible answer: **It disappears when $2x - 6 < 0$ or $x < 3$ because the square root of a negative number is undefined.**

Problem 9 – "Almost" inverses and more missing arrows

In **InverseFunc > AlmostInverse1** and **AlmostInverse2**, the composition of functions $f(x) = \sqrt{x}$ and $g(x) = x^2$ are compared. For $x \geq 0$, *g* appears to be the inverse of *f*, because $g(f(x)) = f((g(x)) = x$. But for $x < 0$, $g(f(x))$ is undefined because $f(x)$ is undefined. Both arrows disappear and thus *f* and *g* are not inverses.