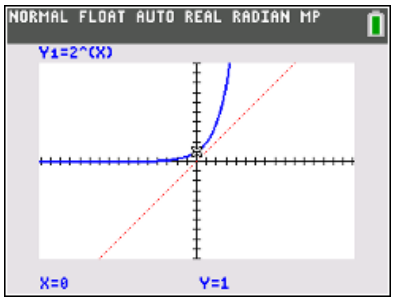
What is Log?
Student Activity

Name _____

Class _____

You may have noticed the log button on the handheld. What does *log* mean? Right above the log button is an exponential key 10^x. Why is the 10^x placed above the log button? You will investigate these questions in this activity.

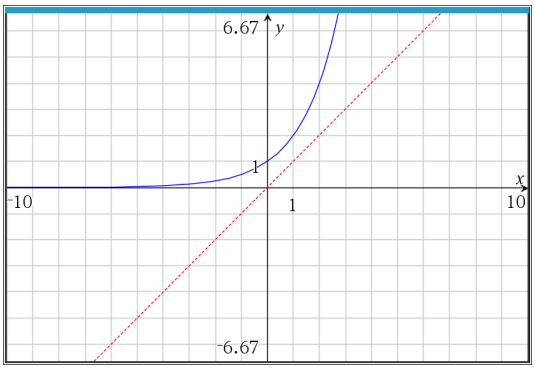


Go to the *y* = screen and follow the directions below.

1. In Y_1, graph the function $Y_1 = 2^x$.
   a. What are the domain and range of this function in Y_1?

   b. Recall that $f(x) = 2^x$ is a one-to-one function, so it has an inverse reflected over the line *y* = *x*. Graph this line into Y_2. What are the domain and range of $f^{-1}(x)$?

   c. Press **graph**, then **trace**. The coordinates you see at the bottom of the screen is a point on the function $f(x) = 2^x$. Move the cursor left and right using the arrows, on the axis below, sketch what you think the reflection over the line $y = x$ would look like. Write a corresponding equation for what you think the function is.



d. The equation $x = 2^y$ cannot be written as a function of y in terms of x without new notation. The inverse of $f(x)$ is actually $f^{-1}(x) = \log_2 x$. In general, $\log_b x = y$ is equivalent to $b^y = x$ for $x > 0$, $b > 0$ and $b \neq 1$. Why do you think x and b must be greater than 0? Why can b not be equal to 1?

e. Enter the following function into Y_3 and press graph: $Y_3 = \log_2 x$. On the graph screen, while using **trace**, use the left/right arrows to trace a function, use the up/down arrows to toggle between functions. While on the exponential function, press the number 1 then **enter**. This point has coordinates of (1, 2). The point (1, 2) on $f(x) = 2^x$ indicates that $2^1 = 2$. Move the cursor to the logarithmic function and press 2 then **enter**. This point has the coordinates $(2, 1)$. The point (2, 1) on $f^{-1}(x) = \log_2(x)$ indicates that $\log_2 2 = 1$. Use this relationship between exponential expressions and logarithmic expressions to complete the following table. (Use the trace function as necessary.)

P	***P′***	**Exponential Expression**	**Logarithmic Expression**
(1, 2)	(2, 1)	$2^1 = 2$	$\log_2 2 = 1$
(2, 4)			
	(8, 3)		
		$2^0 = 1$	
		$2^{-1} = \frac{1}{2}$	
$\left(-2, \frac{1}{4}\right)$			
			$\log_2 \frac{1}{8} = -3$

2. You have discussed the idea of reflecting the exponential function over the line $y = x$. The result of this reflection is the logarithmic function. Now we will discuss any tabular relationships that are formed between an exponential function and a logarithmic function.

Using the first and second columns from the table above, fill in the following tables.

x	$f(x) = 2^x$
-3	
-2	
-1	
0	
1	
2	
3	

x	$f^{-1}(x) = \log_2 x$
$1/8$	
$1/4$	
$1/2$	
1	
2	
4	
8	

(a) Briefly explain your process of filling in the tables on the previous page.

(b) With a classmate, discuss and describe the patterns you see in each individual column.

(c) Write down a rule for each table that you can use to classify the function as either exponential or logarithmic.

3. Solve the logarithmic equation $\log_2 32 = y$ using the patterns from questions 1 and 2. How does the exponential equation verify your result?

4. Solve the equation $\log_4 \frac{1}{256} = y$ using the patterns from questions 1 and 2. How does the exponential equation verify your result?

5. Maya solved the logarithmic equation $\log_4 16 = y$. She says the answer is 4 since $4 \times 4 = 16$. Is her answer correct? Why or why not?

6. Alex says that when solving a logarithmic equation in the form $\log_b a = y$, he can rewrite it as $b^a = y$. Is this a good strategy? Why or why not?