What's the Difference?

ID: 12556

Time Required
15–20 minutes

Activity Overview

Students explore the angle difference formula for cosine. Students will apply the formula and compare their results to interactive unit circle diagrams that assist the student in visualizing the problems involved. The derivations of the angle difference and sum formulas for cosine are optional extensions included with this activity.

Topic: Cosine Difference Identity

- *Angle Sum and Difference Identity Derivation (optional extensions)*
- *Unit Circle*
- *Sine and Cosine values*
- *Verification of Equivalence by Graphing*

Teacher Preparation and Notes

- *If the extensions are used during class, the activity will take approximately 30–45 minutes to complete.*
- *It will be necessary to load the UNITCIRC Cabri jr. files to the graphing calculators before beginning this activity.*
- *The first and second problems engage students in an exploration of the difference formula for cosine. Problem 1 is devoted to unit circle review and developing an understanding of the angle difference diagram included in the activity.*
- *Problem 2 engages students in the application of the angle difference formula for cosine. Students find the cosine for angles such as 15° from well-known angles on the unit circle, such as 45° and 60°.*
- *The extensions of this activity have students derive the angle sum and difference formulas for cosine.*
- ***To download the Cabri Jr. Files and student worksheet, go to education.ti.com/exchange and enter "12556" in the quick search box.***

Associated Materials

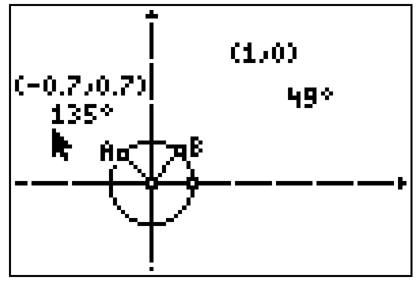
- *PrecalcWeek29_CosDiff_Worksheet_TI84.doc*
- *UNITCIRC.8xv*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Proof of Identity (TI-Nspire technology) — 9847*
- *Round and Round She Goes…(TI-Nspire technology) — 12386*

Problem 1 – Exploring the Angle Difference Formula for Cosine



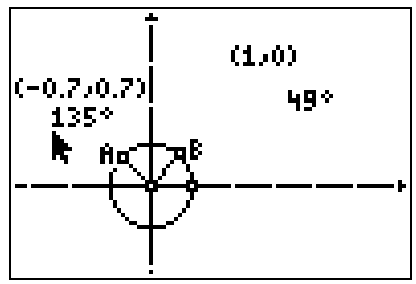
One of the great things about using the unit circle is that the *y*-coordinate is always the sine of the angle and the *x*-coordinate is always the cosine of the angle.

The *Cabri Jr.* file titled **UNITCIRC** is useful in exploring this topic and then for exploring the angle difference formula for cosine.

Students will press ALPHA to grab points *A* and *B* and move them around the circle. Remind them that their results are limited to the resolution of the sketch. In other words, just because the *x*- and *y*-coordinates are only displayed to the tenth, they really go on for a long while.

In this part of the activity, the students answer a variety of questions related to the angle difference diagram.

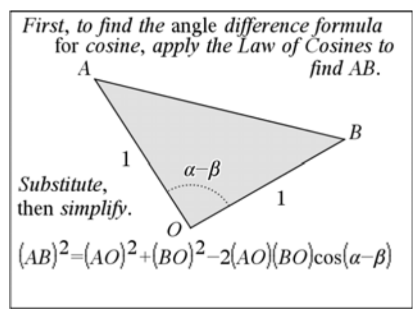
Problem 2 – Applying the Angle Difference Formula



Students find cosine values for angle measures such as 15° and 105°, which take advantage of angles with well known values (for many students) of sine and cosine.

Again, remind the students about the *Cabri Jr.* application only measuring the nearest tenth to account for any discrepancies between their calculated results and their graphical results. Also, be sure to have the students set their graphing calculators to **Degree** mode.

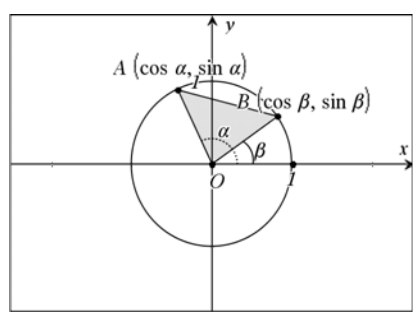
Extension – Deriving the Angle Difference Formula for Cosine



Students use the Law of Cosines to derive the angle difference formula for cosine. A unit circle representation is provided to help students visualize the problem and to provide the necessary background to set up the derivation.

Guidance regarding how to begin the derivation will be helpful to students. Show students how to set up their work for the first derivation and students should be able to follow that example for the remaining derivations in this activity.

Extension – Angle Sum Formula for Cosine



It is important to take some time here to discuss with students why $\cos(-y) = \cos(y)$ and $\sin(-y) = -\sin(y)$ and explain these two situations involved in this formula derivation.

Student Solutions

1. cosine

2. sine

3. 0.98

4. –0.17

5. 0.34

6. 0.94

7. 0.98

8. 0.17

9. answers may vary—relationship is not easy to quickly obtain from the interactive graph page

10. 0.97

11. 0.26

12. –0.26

13. $(AB)^2 = AO^2 + BO^2 - 2 \cdot AO \cdot BO \cdot \cos(AOB)$
$= 1 + 1 - 2\cos(\alpha - \beta)$
$= 2 - 2\cos(\alpha - \beta)$

14. $(AB)^2 = (\cos(\alpha) - \cos(\beta))^2 + (\sin(\alpha) - \sin(\beta))^2$
$= \cos^2(\alpha) - 2\cos(\alpha)\cos(\beta) + \cos^2(\beta) + \sin^2(\alpha) - 2\sin(\alpha)\sin(\beta) + \sin^2(\beta)$
$= 1 - 2\cos(\alpha)\cos(\beta) + 1 - 2\sin(\alpha)\sin(\beta)$
$= 2 - 2\cos(\alpha)\cos(\beta) - 2\sin(\alpha)\sin(\beta)$

15. $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

16. $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$