Building a Garden Fence

About the Lesson

In this activity, students will use basic concepts of perimeter and area to investigate a classic problem requiring area maximization. They must decide how to build a garden fence to enclose the largest possible area. As a result, students will:

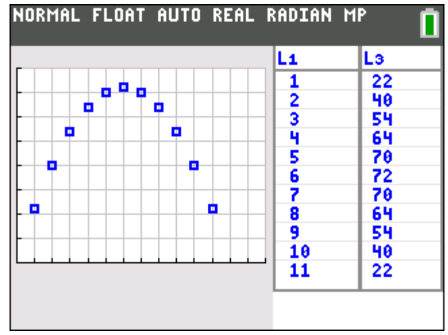
- Analyze scale drawings of a rectangular garden to compute area.
- Know and apply the formula for the area of a rectangle given specific constraints on the length and width of the figure.
- Use technology tools strategically to model a real-world problem with mathematics and maximize the area of a rectangle.

Vocabulary

- perimeter
- area

Teacher Preparation and Notes

- Some students may find it helpful to build several sample garden designs with small square tiles or with graph paper. It is important that students realize that only three sides of the rectangle require fencing.
- Students can plot the graph by hand, use technology to confirm, and analyze the data graphically and algebraically.
- To differentiate instruction, as a challenging variation on this problem, instead of using sections of fencing as 1 meter, they could be set at a length of 0.75 meters.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-84 Plus family with the latest TI-84 Plus operating system (2.55MP) featuring MathPrint ™ functionality. Slight variations to these directions given within may be required if using other calculator models.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Compatible Devices:

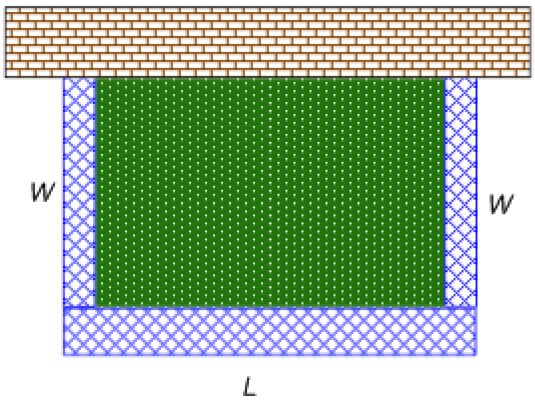
- TI-84 Plus Family
- TI-84 Plus C Silver Edition

Associated Materials:

- Building_Garden_Fence_Student.pdf
- Building_Garden_Fence_Student.doc

Tech Tip: On the TI-84 Plus C Silver Edition, turn on the GridLine by pressing [2nd] [ZOOM] to change the [FORMAT] settings. Note that the GridLine feature is unique to the TI-84 Plus C Silver Edition.

You and a friend are visiting her grandparents on their small farm. They have asked the two of you to design a small, rectangular-shaped vegetable garden along an existing wall in their backyard. They wish to surround the garden with a small fence to protect their plants from small animals.



To enclose the garden, you have 24 sections of 1-meter long rigid border fencing. In order to grow as many vegetables as possible, your task is to design the fence to enclose the maximum possible area. There are many rectangular shapes that can be formed using the 24 fencing sections and, before digging the fence posts, you should do some calculations.

1. If you were to use three sections of fencing (1-meter long each) along each of the two widths of the garden, how many sections of fencing would remain to form the length? What would be the area of this garden? Explain and show work.

Answer: Students should explain that they would have 24 – (2 × 3), or 18 sections for the length. The area would be 3 × 18 or 54 square meters.

Have students copy these values into the table, and then have them enter three more possible garden sizes into the table. Then, have students try to guess the width and length of the garden with the largest possible area. Students should compare their results with others in the class.

Possible Dimensions of Garden Fence

Width (m)	Length (m)	Area (m^2)
3	18	54
4	16	64
5	14	70
6	12	72

Sample Answer: There are actually 11 possible table entries with widths ranging from 1 m to 11 m in length. It is not important that students calculate all possible values since they will use a calculator to do this in order to determine the maximum area.

2. For this situation, if you know what the width is, how can you find the length? Write an equation that shows this relationship between length, *L*, and width, *W*.

Answer: You want to have the students determine the relationship $L = 24 - 2W$ since it will be used later in developing lists.

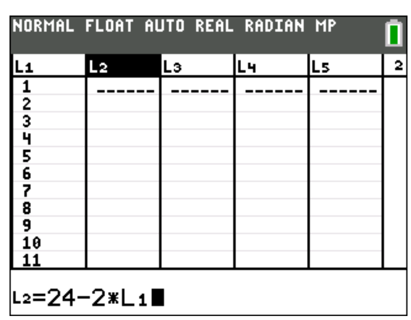
3. The smallest number of fencing pieces you can use along the garden width is one. What is the largest number of pieces that you can use along the width of the garden? Explain how you know this.

Answer: 11 pieces along each width would use 22 of the 24 pieces, leaving 2 pieces for the length. This is the maximum width, because if 12 pieces were used, there would be none left for the length.

Students will use their TI-84 to make a more complete table of possible dimensions. In L_1 they will list the possible widths, and then they will calculate the lengths in L_2 and the areas in L_3.

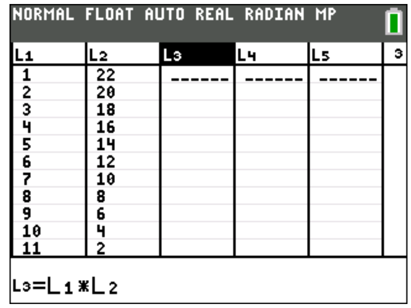
Tech Tip: Student will use the lists in the TI-84 Plus C Silver Edition to calculate all of the possibilities and explore the problem numerically and graphically. Have students press [STAT] [ENTER] to see if there are existing lists that need cleared. If so, advise students to clear them in one of three ways:

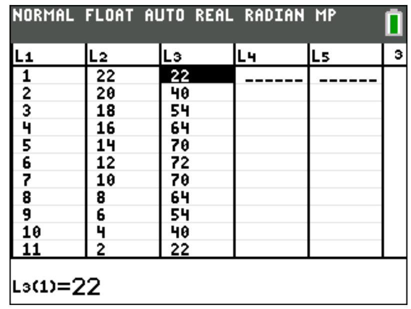
- Use ClrAllLists from the Catalog, [2nd] [0], or
- To clear the first 3 lists, press [STAT] 4:ClrList [ENTER], and then press [2nd] [L1], [2nd] [L2], [2nd] [L3] [ENTER].
- Another approach is to clear one list at a time. To do this, press [▲] to highlight the title of the list and then press [CLEAR] [ENTER].



Students will enter the values 1 through 11 in L_1. They are then instructed to press [▲] to move to the top so that L_2 is highlighted. They should enter their equation from question 2, but use L_1 instead of the variable *W*. Have students press [2nd] [1] to get [L1] for *W*, the width.

Students may discover they can use $24 - (2 \times L_1)$, or $24 - 2L_1$. The calculator will multiply when variables and coefficients are next to each other.





4. How can the values for L_3 (the areas) be determined from L_1 and L_2? Remember that L_1 stores the possible widths and L_2 stores possible lengths

Answer: $L_3 = L_1 * L_2$

5. Examine the values for area in L_3. Are the values you computed earlier contained in this list? Describe any patterns you see in the data values contained in L_3.

Answer: Students may observe that the area increases and then decreases. The rate at which it increases is not constant. It goes up by 18, then 14, then 10, 6, and 2. The decrease in area is then symmetric with the increase. If students do not make the symmetry observation by examining the list, they should when they plot the lists.

6. Examine the third list to find the dimensions of the rectangular garden area that has the largest possible area. Complete the following sentence to provide a solution to the original question:

A rectangle of a width _____meters and length _____m

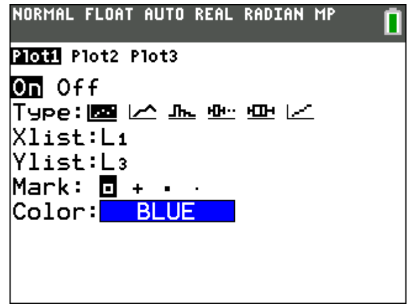
gives the largest or maximum possible garden area of _________square meters.

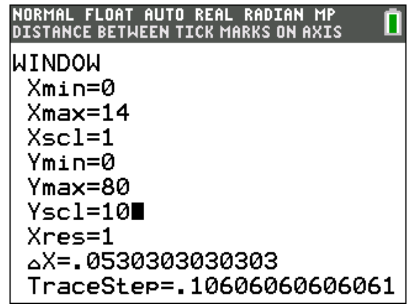
Answer: A rectangle of a width _6_meters and length _12_ m gives the largest or maximum possible garden area of _72_square meters.

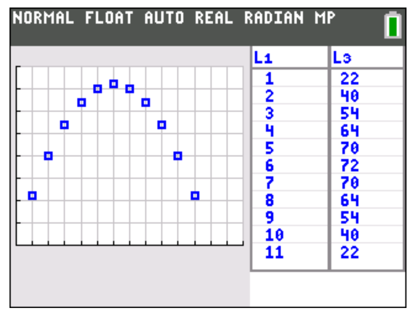
Students are then instructed to lot the area versus width on their TI-84 to help them graphically explore how the area changes as the width is increased.

Tech Tip: To plot the data in L_1 and L_3 press [2nd] [Y=] for [STAT PLOT] and turn on Plot1. Choose the Xlist to be L_1 ([2nd] [1]) and Ylist to be L_3. Press [WINDOW] and set the domain and range based on the data. Press [GRAPH] to see the plotted data.

7. When creating the scatterplot of the areas, you entered the settings shown at right for the display window. Why do you think these values were used?







Answer: Chosen minimum and maximum values enclose the range of possible *x* and *y* values and provide a frame for the graph. Scale selections are reasonable divisions for each range of data.

8. After pressing GRAPH to see the plot, press TRACE to move through the data points in the scatterplot. Which point corresponds to the maximum area? What sets it apart from the other points on the plot?

Answer: Students should note that the coordinates of the point with the largest area appear at the highest point on the scatterplot. Students should see the increase, followed by a decrease, of the areas as the width steadily increases. Algebra students may express this as a quadratic relationship between width and area based upon the relationship $A = W(24 - 2W)$.

9. How do any patterns that you observed in the lists show up in the scatterplot of the data?

Answer: Students should observe the symmetry. The power of visualizing the data graphically and numerically will help students see the maximum value of the function.

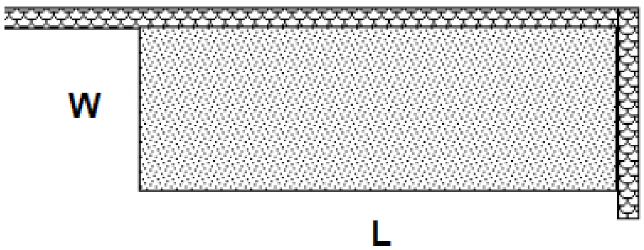
Teacher Tip: You may find it useful to have students construct a scatterplot of widths and lengths. The discussion of the needed window dimensions and the resulting linear relationship ($L = 24 - 2W$) would reinforce important concepts.

Tech Tip: Students may need to press Y= to turn on any other graphs. To display the graph and table, have students press MODE. Instead of selecting FULL, have students select the G-T or GRAPH-TABLE.

Problems for Additional Exploration

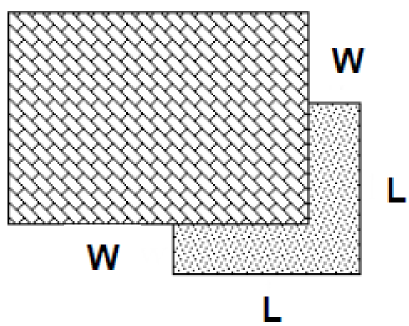
Have students use the list capabilities of their graphing calculators to investigate each of these situations. In each case, have students produce a scatterplot of the widths and areas. Students should assume that they still have the 24 sections of fencing to use in forming your border.

1. A friend suggests that you plant your grandparent's garden at a back corner of the yard so that the existing fence can border two of the four sides of your garden. What are the dimensions of the garden with the largest possible area? Is this configuration an improvement over the original plan? Explain your reasoning.



Answer: Widths may range from 1 to 23 meters with $L = 24 - W$ and $A = L \times W$. The largest possible area, 144 square meters, is formed with $W = 12$ meters and $L = 12$ meters. Since you have only two dimensions to border with the fencing, the area increases from the original plan. However, the garden is limited in that it can only be planted in corners of the yard.

2. Suppose the garden were placed at the corner of a barn so that it was positioned as shown below. What dimensions would give the largest garden area?



Answer: Widths may range from 1 to 6 meters with $L = 12 - W$ (or $L = (24 - 2W)/2$) and $A = 2LW - 2W$ (or some equivalent form). The largest possible area, 48 square meters, is formed when $W = 4$ meters and $L = 8$ meters.